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A new neutrino mass sum-rule from inverse seesaw

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Abstract. We propose a new model based on the S_4 flavor symmetry that leads to a new neutrino mass sum-rule and discuss how to generate a nonzero value for the reactor angle θ_{13} indicated by recent experiments, and the resulting correlation with the solar angle θ_{12}. The model implies a lower bound on the effective neutrinoless double beta mass parameter, even for normal hierarchy neutrinos.

1. Introduction

Although nonvanishing neutrino masses have been confirmed by the discovery of neutrino oscillations, their nature, if they are Dirac or Majorana particles, is still an unanswered question. While there is no way to probe the Dirac nature of neutrinos, a confirmation of the Majorana nature would be the observation of neutrinoless double beta decay ($0\nu2\beta$) [1]. The mass matrix characterizing Majorana neutrinos is a symmetric mass matrix whose parameters are restricted by the experimental data: the neutrino oscillation parameters as well as the limits on the $0\nu2\beta$ effective mass parameter [2, 3].

The general neutrino mixing matrix can be parametrized in different equivalent ways [4, 5, 6]. The Tribimaximal Mixing Matrix (TBM) [7] is a particular *ansatz* of the mixing matrix in which the θ_{13} angle has a zero value. However, the recent experiments [8, 9, 10, 11] indicated a non-zero value of the θ_{13} angle, but taking into account that the θ_{13} value can receive corrections from charged lepton diagonalization and/or from renormalization effects, the TBM can still be used as a good first aproximation.

In Ref. [12] is pointed out that a two-parameter neutrino mass matrix implying a particular mixing matrix form can be obtained from several flavor models based in non-Abelian discrete symmetries. There it was noted that in these models only the following mass relations can be obtained,

$$\chi m_2^\nu + \xi m_3^\nu = m_1^\nu, \tag{1}$$

$$\frac{\chi}{m_2^\nu} + \frac{\xi}{m_3^\nu} = \frac{1}{m_1^\nu}, \tag{2}$$

$$\chi\sqrt{m_2^\nu} + \xi\sqrt{m_3^\nu} = \sqrt{m_1^\nu}, \tag{3}$$

$$\frac{\chi}{\sqrt{m_2^\nu}} + \frac{\xi}{\sqrt{m_3^\nu}} = \frac{1}{\sqrt{m_1^\nu}}, \tag{4}$$

where χ and ξ are free parameters that characterize each specific model. Also, a classification of all models predicting TBM mixing which generate mass relations similar to the first three is presented there, but the last case correspond to a completely new case. In this work we present a model implementing the inverse seesaw mechanism [13, 14] as well as a non-Abelian flavor symmetry [15], along the lines of Ref. [16], but adopting S_4, instead of A_4.

The characteristic which distinguish the inverse seesaw scheme from other schemes is that it allows a low-scale seesaw scheme [17] with naturally light neutrinos. The particle content of the Standard Model (SM) is extended by adding a pair of two component gauge singlet leptons, ν_i^c and S_i, with i running over the three generations. The fermion singlets S_i have opposite lepton number with respect to that of the three singlets ν_i^c associated to the "right-handed" neutrinos.

In the ν, ν^c, S basis the 9×9 neutral lepton mass matrix M_ν has the form:

$$M_\nu = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M^T \\ 0 & M & \mu \end{pmatrix}, \tag{5}$$

where m_D and M are arbitrary 3×3 complex matrices, while μ is symmetric due to the Pauli principle.

Following the seesaw diagonalization method in [18] one obtains the effective light neutrino mass matrix $m_\nu \sim m_D^T M^{T-1} \mu M^{-1} m_D$, with the entry μ being very small.

If m_D and μ are both proportional to the identity, and

$$M \sim M_{TBM} = \begin{pmatrix} x & y & y \\ y & x+z & y-z \\ y & y-z & x+z \end{pmatrix}, \tag{6}$$

in the basis where the charged lepton mass matrix is diagonal, then there is a specific (complex) relation among the parameters x, y and z [19], leaving only two free complex parameters, and we obtain the mass sum-rule in Eq. (4).

Our model is presented in the next section, and the predictions regarding the lower bound on the $0\nu2\beta$ amplitude are presented in section III, where we also discuss possible departures from tribimaximality, including a finite θ_{13} value.

2. The model

We follow Table I given in Ref. [16], where some possible schemes realizing the TBM pattern are summarized for the inverse seesaw case. From these, adopting the S_4 flavor symmetry instead of A_4, we will implement case 2)

$$M_D \propto \mathcal{I} \qquad \mu \propto \mathcal{I}, \qquad M \propto \begin{pmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & C & B \end{pmatrix}. \tag{7}$$

In order to obtain the S_4-based inverse seesaw model we assign the charge matter fields as in Table 1 (see [20] for S_4 multiplication rules). To generate the desired mass matrix structures five flavon fields, ϕ_ν, ϕ'_ν, ϕ_l, ϕ'_l, ϕ''_l are introduced, and the extra symmetries Z_3 and Z_2. The introduction of a scalar field σ is required to break lepton number through the coupling with S_iS_j terms, and when this field acquire its vacuum expectation value (VEV) it will be responsible of the mass terms. In order to keep the renormalizability of the Lagrangian we add Frogatt-Nielsen fermion, singlet under the weak $SU(2)$ gauge group, χ and its conjugate χ^c [21, 22, 23, 24]. The quantum numbers under these extra symmetries are shown in Table 2.

The renormalizable Lagrangian relevant for neutrinos is

$$\mathcal{L}_\nu = Y_{D_{ij}} \overline{L}_i \nu_{R_j} h + Y^k_{\nu ij} \nu_{R_i} S_j \phi_{\nu_k} + Y'_{\nu ij} \nu_{R_i} S_j \phi'_\nu + \mu_{ij} S_i S_j \sigma, \tag{8}$$

Table 1. Fields and transformation properties under $SU(2)$, the S_4 flavor symmetry, and global lepton number $U_l(1)$

	$\overline{L}$	ν_R	l_R	h	S	ϕ_ν	ϕ'_ν	ϕ_l	ϕ'_l	ϕ''_l	σ	χ	χ^c
$SU(2)$	2	1	1	2	1	1	1	1	1	1	1	1	1
S_4	3_1	3_1	3_1	1_1	3_1	3_1	1_1	3_1	3_2	1_1	1_1	3_1	3_1
$U_l(1)$	-1	1	1	0	-1	0	0	0	0	0	2	1	-1

Table 2. Fields and their transformation properties under the Z_3, and Z_2 flavor symmetries

	$\overline{L}$	ν_R	l_R	h	S	ϕ_ν	ϕ'_ν	ϕ_l	ϕ'_l	ϕ''_l	σ	χ	χ^c
Z_3	ω^2	ω	1	1	1	ω^2	ω^2	ω	ω	ω	1	ω	ω^2
Z_2	+	+	+	+	−	−	−	+	+	+	+	+	+

while the renormalizable Yukawa terms involving the messenger fields are

$$\mathcal{L}_\chi = M_\chi \chi\chi^c + \bar{L}h\chi + \chi^c l_R \phi_l + \chi^c l_R \phi'_l + \chi^c l_R \phi''_l. \tag{9}$$

The effective Lagrangian for charged leptons, after integrating out the messenger fields χ, takes the form

$$\mathcal{L}_l = \frac{y_l}{\Lambda}(\bar{L}l_R)h\phi_l + \frac{y'_l}{\Lambda}(\bar{L}l_R)h\phi'_l + \frac{y''_l}{\Lambda}(\bar{L}l_R)h\phi''_l, \tag{10}$$

being Λ the effective scale. This effective Lagrangian is responsible for charged lepton mass generation.

To obtain the desired neutrino mixing matrix the flavon fields must have the following alignments:

$$\langle\phi_\nu\rangle = v_\nu(1,0,0), \quad \langle\phi_l\rangle = v_l(1,1,1), \quad \langle\phi'_l\rangle = v'_l(1,1,1), \tag{11}$$

where we define $\langle\phi'_\nu\rangle = v'_\nu$, $\langle\phi''_l\rangle = v''_l$, $\langle\sigma\rangle = v_\sigma$ and $\langle h\rangle = v$. We do not write here explicitly the potential (see [25] for more details) but we have verified that it is possible to find parameters for which these alignments provide a minimum of the potential. With the previous alignments the mass matrices take the form

$$(\mu) = \begin{pmatrix} \mu v_\sigma & 0 & 0 \\ 0 & \mu v_\sigma & 0 \\ 0 & 0 & \mu v_\sigma \end{pmatrix}, \quad M_D = \begin{pmatrix} Y_D v & 0 & 0 \\ 0 & Y_D v & 0 \\ 0 & 0 & Y_D v \end{pmatrix}, \quad M = \begin{pmatrix} Y'_\nu v'_\nu & 0 & 0 \\ 0 & Y'_\nu v'_\nu & Y_\nu v_\nu \\ 0 & Y_\nu v_\nu & Y'_\nu v'_\nu \end{pmatrix}, \tag{12}$$

and

$$M_l = \begin{pmatrix} y''_l v''_l & y_l v_l - y'_l v'_l & y_l v_l + y'_l v'_l \\ y_l v_l + y'_l v'_l & y''_l v''_l & y_l v_l - y'_l v'_l \\ y_l v_l - y'_l v'_l & y_l v_l + y'_l v'_l & y''_l v''_l \end{pmatrix} \frac{v}{\Lambda}. \tag{13}$$

This matrix M_l is diagonalized by the "magic" matrix" U_ω[16, 25]. On the other hand, after diagonalization, the light neutrino mass matrix takes the form

$$M_\nu = \begin{pmatrix} \frac{1}{a^2} & 0 & 0 \\ 0 & \frac{a^2+b^2}{(b^2-a^2)^2} & -\frac{2ab}{(b^2-a^2)^2} \\ 0 & -\frac{2ab}{(b^2-a^2)^2} & \frac{a^2+b^2}{(b^2-a^2)^2} \end{pmatrix} \tag{14}$$

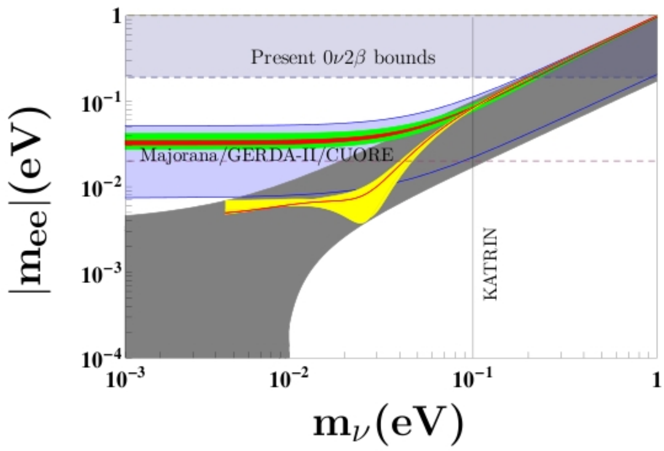

Figure 1. $|m_{ee}|$ as a function of the lightest neutrino mass corresponding to the mass sum-rule in Eq. (4). The bands in gray and blue correspond to generic normal and inverse hierarchy regions, while the yellow and green bands correspond to our flavor prediction varying the values of oscillation parameters in their 3σ C.L. range. The thin red bands correspond to the TBM limit. The upper band in lavender corresponds to the present bounds on $0\nu 2\beta$.

where $a = Y'_\nu v'_\nu/(\sqrt{\mu v_\sigma} Y_D v)$ and $b = Y_\nu v_\nu/(\sqrt{\mu v_\sigma} Y_D v)$. In the basis where charged lepton mass matrix is diagonal, the light neutrino mass matrix is diagonalized by the TBM form, and the corresponding eigenvalues are given by $m_1 = 1/(a+b)^2$, $m_2 = 1/(a-b)^2$ and $m_3 = 1/a^2$, and with these eigenvalues we obtain the desired neutrino mass sum-rule $\frac{1}{\sqrt{m_1}} = \frac{2}{\sqrt{m_3}} - \frac{1}{\sqrt{m_2}}$.

3. Phenomenology

3.1. Neutrinoless double beta decay

We can write the general expression of the mass parameter $|m_{ee}|$ which determines the $0\nu 2\beta$ decay amplitude as

$$|m_{ee}| = \left|\sum_j U_{ej}^2\, m_j\right| = \begin{cases} \left|c_{12}^2 c_{13}^2\, m_1 + s_{12}^2 c_{13}^2\, m_2\, \mathrm{e}^{\frac{1}{2}i\alpha_{21}} + s_{13}^2\, m_3\, \mathrm{e}^{\frac{1}{2}i(\alpha_{31}-2\delta)}\right| & \text{(PDG [4])}, \\ \left|c_{12}^2 c_{13}^2\, m_1 + s_{12}^2 c_{13}^2\, m_2\, \mathrm{e}^{2i\phi_{12}} + s_{13}^2\, m_3\, \mathrm{e}^{2i\phi_{13}}\right| & \text{(symmetrical[5, 6])}. \end{cases} \tag{15}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, m_i, $i = 1, 2, 3$, are the neutrino masses, and ϕ_{12} and ϕ_{13} are the two Majorana phases.

The parameter $|m_{ee}|$ can be plotted in terms of the lightest neutrino mass, varying the neutrino oscillation parameters in their allowed range, and depending on which is the lightest neutrino one can have normal or inverse hierarchy, having a lower bound in the latter case. However, as was noted in Ref. [12], the neutrino mass sum-rule can be interpreted geometrically as a triangle in the complex plane, giving its area a measure of the Majorana CP violation, see Ref. [12] for details.

In the present scheme, as a result, there is a lower bound on $|m_{ee}|$ even in the case of normal hierarchy, and since the allowed ranges for normal and inverse hierarchy are much more constrained than in the generic case, it becomes possible to distinguish the neutrino mass hierarchy even for lighter neutrinos.

3.2. Quark sector

For the quark sector, we will only mention here that its possible to fit quark masses and mixing assigning charges as in Table 3 and adding flavons $\phi_{D,S}$ in doublet and singlet representations of the S_4. As in the charged lepton sector the dimension five operators can be given in terms of renormalizable interaction by introducing suitable messenger fields, and the alignment required for the VEV of ϕ_D is $\langle\phi_D\rangle \sim (-\sqrt{3}, 1)$ (see [25] for further details).

Table 3. Quark sector and their transformation properties under the Z_3, and Z_2 flavor symmetries

	$\overline{Q}_D$	$\overline{Q}_S$	u_{R_D}	u_{R_S}	d_{R_D}	d_{R_S}	ϕ_D	ϕ_S
$SU(2)$	2	2	1	1	1	1	1	1
S_4	2	1_1	2	1_1	2	1_1	2	1_1
Z_3	ω	ω	ω^2	ω^2	ω	ω	ω	ω
Z_2	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$-$

3.3. Finite θ_{13} value

Although the model leads to the TBM pattern we can obtain corrections from the charged lepton sector by coupling an extra S_4-doublet flavon, inducing in this way a nonzero values of θ_{13} as recently suggested by experiments [8, 9, 10, 11].

For instance, consider a flavon scalar doublet under S_4, $\phi \sim \mathbf{2}$, transforming as $(\omega, +)$ under $Z_3 \times Z_2$. In the Lagrangian we must then include the term $(\bar{L} l_R) h \phi$, which is a dimension five operator that can be obtained from a renormalizable Lagrangian by means of the messenger fields χ, χ^c.

Assuming that ϕ acquires VEV $\langle \phi \rangle = (u_1, u_2)$, a natural vacuum alignment, consistent with the previous alignments, is $u_1 = -\sqrt{3} u_2$. Then, one finds that the contribution from this term to the charged lepton mass matrix is

$$\delta M_l = \begin{pmatrix} -\sqrt{\frac{2}{3}} v u_2 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} v u_1 + \sqrt{\frac{1}{6}} v u_2 & 0 \\ 0 & 0 & -\sqrt{\frac{1}{2}} v u_1 + \sqrt{\frac{1}{6}} v u_2 \end{pmatrix}, \tag{16}$$

which modifies the diagonal entries in the charged lepton mass matrix, M_l, so that the total $M_l + \delta M_l$ is no longer diagonalized by U_ω. In this way one can induce a potentially large value for θ_{13} and also potential departures of the solar and atmospheric angles from their TBM values. Besides, one finds relations among these three neutrino mixing angles. The most interesting of these is the correlation involving the solar and reactor angles, as illustrated in Fig. 2. The

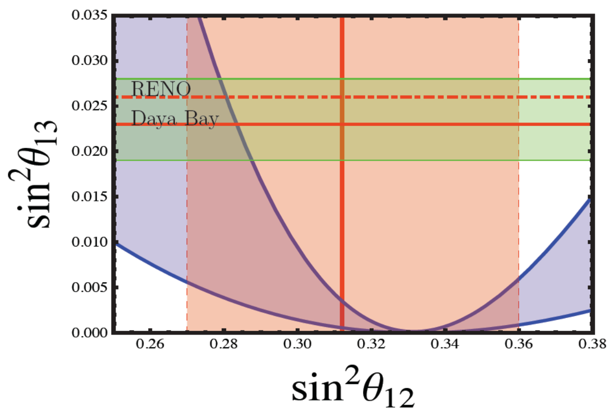

Figure 2. Correlations between reactor and solar neutrino mixing angles. See text for explanations.

horizontal green band represents the 2σ Daya Bay measurement [8], and its central value is indicated by the horizontal red line, while the horizontal dot-dashed line indicates the central value of the recent RENO measurement [11]. On the other hand, the vertical band, delimited by dotted lines, corresponds to the 2σ region for $\sin^2\theta_{13}$ found in the global analysis in Ref. [26], and the vertical line corresponds to the central value. The region in lavender shows the correlation between the reactor and solar angles. We observe that the deviation of θ_{13} from zero can be substantial provided the departure of θ_{12} from its TBM value is also large. Moreover, the model is consistent with the measurements of the two recent reactor experiments, only if the solar angle lies substantially below the TBM prediction (at 2σ).

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References

[1] Schechter J and Valle J Wen F 1982 *Phys. Rev.* D **25** 2951; Duerr M, Lindner M and Merle A 2011 On the Quantitative Impact of the Schechter-Valle Theorem *J. High Energy Phys.* JHEP 1106(2011)091 (*Preprint* hep-ph/1105.0901)
[2] Rodejohann W 2011 *Int.J.Mod.Phys.* E **20** 1833 (*Preprint* hep-ph/1106.1334)
[3] Barabash A 75 years of double beta decay: yesterday, today and tomorrow (*Preprint* nucl-ex/1101.4502v1)
[4] Nakamura K *et al* 2010 *Journal of Physics G: Nuclear and Particle Physics* **37** 075021
[5] Schechter J and Valle J. W. F. 1980 *Phys. Rev.* D **22**, 2227
[6] Rodejohann W and Valle J 2011 *Phys.Rev.* D **84** 073011 (*Preprint* hep-ph/1108.3484)
[7] Harrison P F, Perkins D H and Scott D H 2002 *Phys. Lett.* B **530** 167
[8] An F P *et al* (DAYA-BAY Collaboration) 2012 *Phys. Rev. Lett.* 108 171803 (*Preprint* hep-ex/1203.1669)
[9] Abe K *et al* (T2K Collaboration) 2011 *Phys. Rev. Lett.* **107** 041801
[10] Abe Y *et al* (DOUBLE-CHOOZ Collaboration) 2012 *Phys. Rev. Lett.* 108 131801 (*Preprint* hep-ph/1112.6353)
[11] Ahn J K *et al* (RENO Collaboration) *Phys. Rev. Lett. 108, 131801* (*Preprint* hep-ex/1204.0626)
[12] Dorame L, Meloni D, Morisi S, Peinado E and Valle J W F 2012 *Nucl. Phys.* B **861** 259 (*Preprint* hep-ph/1111.5614)
[13] Mohapatra R N and Valle J W F 1986 *Phys. Rev.* D **34** 1642
[14] Gonzalez-Garcia M C and Valle J W F 1989 *Phys. Lett.* B **216** 360
[15] Hirsch M *et al* (*Preprint* hep-ph/1201.5525)
Ishimori H *et al.* 2010 *Prog. Theor. Phys. Suppl.* **183** 1 (*Preprint* hep-ph/1003.3552)
[16] Hirsch M, Morisi S and Valle J W F 2009 *Phys. Lett.* B **679** 454 (*Preprint* hep-ph/0905.3056)
[17] Valle J W F 2006 *J. Phys.*: Conf. Series **53** 473-505 (*Preprint* hep-ph/0608101)
[18] Schechter J and Valle J W F 1982 *Phys. Rev.* D **25** 774
[19] Ma E 2006 *Phys. Rev.* D **73** 057304
[20] Hagedorn C, Lindner M and Mohapatra R 2006 S4 Flavor Symmetry and Fermion Masses: Towards a Grand Unified theory of Flavor *J. High Energy Phys.* JHEP0606(2006)042 (*Preprint* hep-ph/0602244)
[21] Froggatt C D and Nielsen H B 1979 *Nucl. Phys.* B **147** 277
[22] Barbieri R, Hall L J, Raby S and Romanino A 1997 *Nucl.Phys.* B **493** 3 (*Preprint* hep-ph/9610449)
[23] Varzielas I de M and Ross G G 2006 *Nucl.Phys.* B **733** 31 (*Preprint* hep-ph/0507176)
[24] King S F and Malinsky M 2006 "Towards a Complete Theory of Fermion Masses and Mixings with SO(3) Family Symmetry and 5d SO(10) Unification" *J. High Energy Phys.* JHEP0611(2006)071
[25] Dorame L, Morisi S, Peinado E, Valle J W F and Rojas A D 2012 *Phys. Rev.* D **86** 056001 (*Preprint* hep-ph/1203.0155)
[26] Schwetz T, Tortola M and Valle J W F 2011 *New J. Phys.* **13** 063004; Schwetz T, Tortola M and Valle J W F 2011 *New J. Phys.* **13** 109401