
Article

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Abstract: In a paper published in 1939, Albert Einstein argued that Black Holes (BHs) did not exist “in the real world”. However, recent astronomical observations indicate otherwise. Does this mean that we should also expect White Holes (WHs) to exist in the real world? In classical General Relativity (GR), a WH refers to the time reversed version of a collapsing BH solution that allows the crossing of the BH event horizon inside out. Such solution has been disputed as not possible because escaping an event horizon violates causality. Despite such objections, the Big Bang model is often understood as a WH (the reverse of a BH collapse). Does this mean that the Big Bang breaks causality? Recent measurements of cosmic acceleration indicate that our Big Bang solution is not really a WH, but a BH. Events decelerate when the expansion accelerates and this prevents the crossing of the event horizon from inside out. We present a general explanation of why this happens; the explanation resolves the above causality puzzle and indicates that such apparent WH solutions have a regular Schwarzschild BH exterior.

Keywords: cosmology; dark energy; general relativity; black holes

1. Introduction

Decades after Einstein [1] concluded that BHs did not exist, observations have shown that they are real astronomical objects [2–4]. A Schwarzschild Black Hole (BH) solution:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -[1 - r_S/r] dt^2 + \frac{dr^2}{[1 - r_S/r]} + r^2 d\Omega^2, \quad (1)$$

represents a singular point source of mass M . The gravitational radius $r_S \equiv 2GM$ corresponds to an event horizon and prevents us from seeing inside r_S . The Schwarzschild solution applies to the exterior of any BH, no matter what the interior solution is, as long as we can approximate the outer region as empty space. The Hawking–Penrose’s theorems [5,6] tell us that nothing can come out of r_S . This has created the BH information lost paradox [7,8]. One possible way around this is to introduce the concept of maximally extended Schwarzschild solution using the Kruskal–Szekeres coordinates $T = T(t, r)$ and $X = X(t, r)$ (see Figure 1), where the future BH event horizon becomes the past White Hole (WH) horizon. Information can escape r_S in a WH. There are two disconnected exterior spaces which could be connected inside with an Einstein–Rosen bridge or Schwarzschild wormhole [9].

If we throw a particle into a BH, the WH solution corresponds to the traveling of that particle back in time to us (from our past), before the particle was sent. Such trajectory might be formally possible (because there is no arrow of time at the fundamental level), but it violates causality, so it makes no physical sense as a classical solution (quantum mechanics effects might provide some way around this [10]). This is related to the example of retarded and advanced potentials in classical electrodynamics: both are mathematical solutions of the wave equations, but only one of them connects cause and effect. The mirror image of the top quadrant in Figure 1 has the arrow pointing downward and not



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upward. This shows that the time reversed solution (the mirror image) is still a BH (where the particle falls into the gravitational radius r_S) and not a WH (as indicated in the figure).

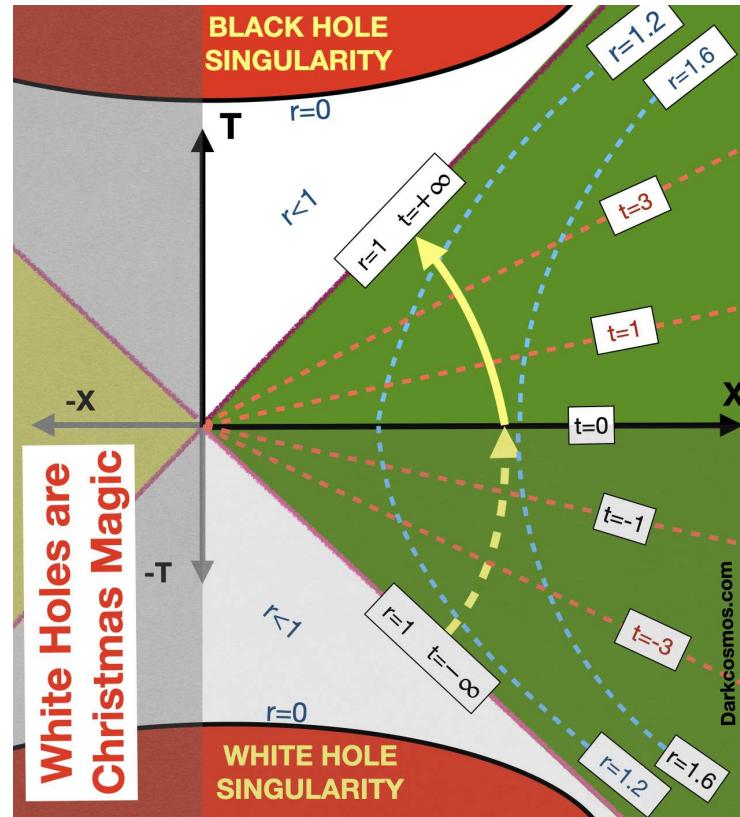


Figure 1. Relation between the Schwarzschild coordinates (t, r) (in units of $r_S = 1$) and the Kruskal-Szekeres coordinates (T, X) . The top right quadrant ($T > 0, X > 0$) is the regular BH solution (the green region is external to r_S). Lines of constant r are the cyan hyperbolic dashed lines. Lines of constant time t are orange dashed straight lines ($T = 0$ is also $t = 0$). If we radially throw a test particle at $t = 0$, it will follow the continuous yellow arrow. This solution can be formally extended into the bottom right quadrant, which corresponds to a WH, and is the time reversed (horizontal flip) of the BH solution. A particle can escape the event horizon of the WH (dashed yellow arrow), but only before it is thrown! This violates causality and is therefore not a physical solution. With a vertical flip, the solution could be maximally extended to a negative radius $X < 0$. But this generates a disconnected external space (yellow region), which is also not part of the original solution.

Here we will study the more realistic case of classical Lemaitre-Tolman-Bondi (LTB) solutions, which include the FLRW metric, the Oppenheimer-Snyder BH [11] and the thin shell BH [12] as particular cases. For some reason, i.e., the difficulty of a black-to-white hole bounce (see [13,14]), these solutions are usually investigated only as BH collapsing solutions. As we will show, the same solutions also exist, in principle, as WH solutions. The most famous of this is the expanding Big Bang model originally proposed by Friedman in 1922 [15] and Lemaitre in 1927 [16]. However, at closer inspection, these solutions need to be modified to include a surface term. After that correction, we show that the WH correspond, in fact, to BH expanding solutions. Our argument is supported by considering surface terms in the Einstein-Hilbert action of classical GR and also by the recent observation that our cosmic expansion is accelerating.

2. LTB Solutions

The most general metric with spherical symmetry in spherical coordinates $dx^\mu = (dt, dr, d\theta, d\phi)$ can be written as [17]:

$$ds^2 = -A(t, r) dt^2 + B(t, r) dr^2 + R^2(t, r) d\Omega^2. \quad (2)$$

Other common notation is: $A \equiv e^\nu$, $B \equiv e^\lambda$ and $R^2 \equiv e^u$ [11,18,19]. An alternative to this uses proper time $dx^\mu = (d\tau, d\chi, d\theta, d\phi)$:

$$ds^2 = -d\tau^2 + e^{\lambda(\tau, \chi)} d\chi^2 + r^2(\tau, \chi) d\Omega^2, \quad (3)$$

where the radial coordinate χ can be comoving or not (because its evolution can be encoded in the λ and r functions). This last metric is sometimes called the *Lemaitre–Tolman metric* [16,18] or the *Lemaitre–Tolman–Bondi* or *LTB metric*. A metric such as this one, expressed with $g_{00} = 1$ and $g_{0\mu} = 0$ is called *synchronous* (or in a *synchronous frame*) because time lines are geodesics. Either way, it is possible to express the spherical symmetric metric with two functions. The best form in each case depends on the energy content and the observer's frame. In all cases, this is a *local metric* around a reference central point in space which we have set to be the origin ($\vec{r} = 0$).

The advantage of using the proper time and an observer moving with a perfect fluid is that the stress tensor becomes diagonal: $T_\mu^\nu = \text{diag}[-\rho, p, p, p]$, where $\rho = \rho(\tau, \chi)$ is the energy density and $p = p(\tau, \chi)$ is the pressure. We will focus here in the matter-dominated case $p = 0$ for simplicity, but we expect similar results to apply to more general situations (see [20]). The solution to the field equation $8\pi GT_0^1 = G_0^1 = 0$ is $\dot{\lambda}r' = 2\dot{r}'$, where dots and primes correspond to time τ and radial χ partial derivatives. This equation can be solved as $e^\lambda = Cr^2$, where $C = C(\chi)$ is an arbitrary function of χ . The choice $C = 1$ corresponds to the particular flat geometry case:

$$ds^2 = -d\tau^2 + [\partial_\chi r]^2 d\chi^2 + r(\tau, \chi)^2 d\Omega^2, \quad (4)$$

The solution for r in this case is easily found:

$$H^2 \equiv r_H^{-2} \equiv \left(\frac{\dot{r}}{r}\right)^2 = \frac{2GM}{r^3} \quad (5)$$

$$M \equiv 4\pi \int_0^\chi \rho(\tau, \chi) r' r^2 d\chi = M(\chi) \quad (6)$$

The above expression reproduces the Newtonian energy conservation in free fall: $\frac{1}{2}\dot{r}^2 = GM/r$ [21] and corresponds to an expanding or collapsing relativistic spherical ball. When $\rho = \rho(\tau)$ is uniform, we find $r = a(\tau)\chi$ so that Equation (4) reproduces the flat FLRW metric:

$$ds^2 = -d\tau^2 + a^2(\tau) [d\chi^2 + \chi^2 d\Omega^2], \quad (7)$$

and Equation (5) reproduces the corresponding solution $3H^2 = 8\pi G\rho$. The next simplest solution to Equation (5) is that of the FLRW uniform cloud with a fixed total mass M_T :

$$M_T \equiv \int_0^\infty \rho 4\pi r^2 (\partial_\chi r) d\chi \quad (8)$$

The solution is $r = a\chi$ as in the standard FLRW metric but with a boundary at $R(\tau) \equiv a(\tau)\chi_*$ above which ($\chi > \chi_*$) we have empty space: $\rho = 0$.

This is a consequence of *Birkhoff's theorem* [22] (or *Gauss' law* in non relativistic mechanics), since a sphere cut out of an infinite uniform distribution conserves the same spherical symmetry and the solutions are independent of what is outside. If the outer region is empty space, we just recover the static Schwarzschild solution outside and the FLRW metric inside. Thus, the FLRW metric is both a solution to a global homogeneous

(i.e., $M_T = \infty$) uniform background and also to the inside of a local (finite M_T) uniform sphere centered around one particular point. The local solution is called the FLRW cloud (FLRW*) [20]. As we will show next, the LTB solution could in principle be viewed as a BH or a WH, depending on whether the FLRW metric is expanding or collapsing.

A timelike radial geodesic ($d\chi = 0$) in the FLRW cloud has a mass-energy M inside χ which is independent of τ . Such fixed comoving coordinate $\chi = \chi_*$ corresponds to a system with a fixed mass M_T inside (see also [23]), which expands or collapses following the Hubble–Lemaître law of Equation (5).

From Equation (5), we have $H = H_S(a/a_S)^{-3/2} = \pm 1/\tau$, where H_S is just the value at some arbitrary time ($a = a_S$), when R intersects r_S , so that $r_S = a_S\chi_* = 1/H_S = \pm 2GM_T$. This solution is time reversible and the evolution can cross r_S in both directions. This is a well known solution which includes the Oppenheimer–Snyder BH collapse [11] and the thin shell BH [12]. However, note that when $R < r_S$, we have $R > r_H$ (or $\dot{R} > 1$), which creates a region between $R > r > r_H$ which is acausal during expansion (this is the well known horizon problem in the standard Big Bang cosmology). We can also reproduce the same LTB (or FLRW*) solution using junction conditions to verify that the exterior of r_S is indeed a classical (Schwarzschild) BH despite the looks of Equation (4). The original derivation [20] is reproduced here in Appendix A (with some typos corrected) for reference.

To show that this solution actually crosses the gravitational radius r_S , we can estimate the event horizon (EH), R_{EH} , of the FLRW* metric. This is the maximum distance that a photon emitted at time τ can travel following an outgoing radial null geodesic [24]:

$$R_{EH} = a(\tau) \int_{\tau}^{\infty} \frac{d\tau}{a(\tau)} = a \int_a^{\infty} \frac{da}{H(a)a^2} \quad (9)$$

For $H \sim a^{-3/2}$, we have $R_{EH} \sim a^{3/2}$, which grows unbounded with a and therefore crosses r_S , as shown by the dashed red line in Figure 2.

The case $H_S < 0$ corresponds to a collapsing solution, and therefore, a BH. This collapsing solution is protected by the Equivalence principle, as a free fall test particle at $r > R$ is equivalent to a particle moving in empty space and can therefore cross r_S . The case $H_S > 0$ is expanding and is what we have labeled as a WH solution. It just corresponds to a fluid expanding inside r_S . However, what is strange about this solution is that information can actually escape from the interior to the exterior of r_S , which is contrary to all that we have learned about BHs and causality. How is that possible?

The standard objection to this paradox is that this expanding configuration can never be achieved. This is reflected in the fact that $R > r_H$ is not causally connected to its past (the so called horizon problem), which is a similar objection to the one for WH interpretation of the Schwarzschild solution, as discussed in the introduction. Note that this expanding solution corresponds to the matter-dominated Big Bang solution, which is very close to current observations¹. This is why it is often said that the Big Bang is a WH².

Here, we argue that this expanding WH solution is not correct. This is not because it cannot be achieved (as illustrated by the existence of our own observed universe). But because the gravitational radius r_S should be interpreted as a boundary that separates the interior from the exterior manifold. This is strictly the case if the exterior is empty space (as we have assumed here)³. Such boundary requires that we change the GR field equations. Appendix B reproduces the original calculation in [20,25] that shows that the Gibbons–Hawking–York (GHY) boundary in the action corresponds to an effective Λ term: $\Lambda = 3/r_S^2$. We will show next how this boundary term transforms the WH solution into a BH solution.

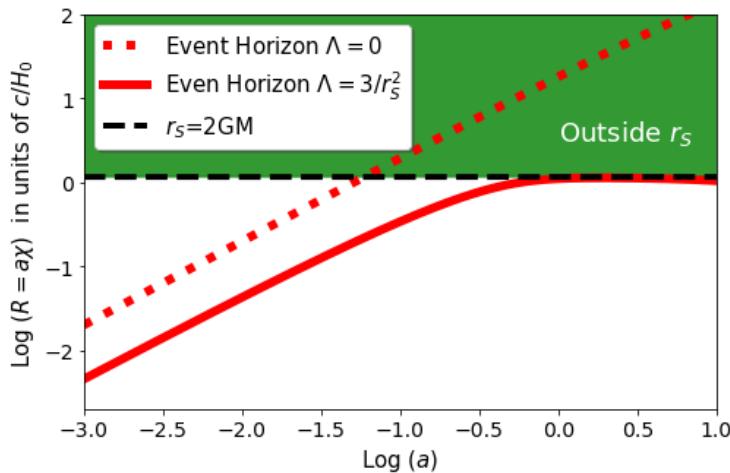


Figure 2. Event horizon in Equation (9) as a function of cosmic time (given by the scale factor a) for a matter-dominated FLRW metric ($\Omega_m = 1, \Omega_\Lambda = 0$, dashed red line) and for one which also has a $\Lambda = 3/r_S^2$ term ($\Omega_m = 0.25, \Omega_\Lambda = 0.75$, continuous red line).

How a WH Turns into a BH

Let us next consider how the derivation presented in Appendix A, which assumed $\Lambda = 0$, changes when including an effective Λ term: $\Lambda = 3/r_S^2$ inside r_S (as suggested by the GHY boundary argument given above). Such Λ term does not change the form of the FLRW metric itself, but (as it is well known) it changes the field equations and therefore the solution to expansion rate $3H^2 = 8\pi G\rho + \Lambda$. But the Λ term does change the form of the Schwarzschild solution and metric inside. The solution now is the deSitter–Schwarzschild metric: $F = 1 - r_S/R - R^2/r_S^2$. Thus, to find the new junction, we just need to replace F in the definition of β in Equation (A4). The new second junction condition then becomes:

$$R = \left[\frac{r_H^2 r_S^3}{r_S^2 - r_H^2} \right]^{1/3} \quad \text{or} \quad H^2 = \frac{r_S}{R^3} + \frac{1}{r_S^2} \quad (10)$$

which is exactly the new Hubble law with $\Lambda = 3/r_S^2$ and a constant mass $M = 4/3\pi\rho R^3$ in Equation (6). This shows that the LTB (or FLRW*) expanding metric is also a solution to the new field equations with the r_S boundary. However, this solution is no longer a WH, but has become a BH. We can check this by estimating the new EH in Equation (9), now including the effective Λ term in H . The new estimation for R_{EH} is displayed as a red continuous line in Figure 2. As can be seen, the EH is trapped inside r_S , which indicates that no information can escape. The WH solution has now turned into a BH.

3. Conclusions

We have shown that classical WH solutions in GR can be turned into an expanding BH solutions once we account for the fact that the gravitational radius r_S corresponds to a boundary condition in the action of GR.

The matter-dominated case study here is a very good approximation for our universe, because in the later stages of its evolution, it is totally dominated by matter and the effective $\Lambda = 3/r_S^2$. This could also be in general a good approximation for stellar or supermassive BHs with uniform density and pressure because as $a \rightarrow \infty$ inside, matter and Λ always dominate. The characteristic gravitational time is quite short:

$$\tau \sim GM \simeq 1.1 \times 10^{-13} \frac{M}{M_\odot} \text{yr}, \quad (11)$$

so, even for a super massive BH ($M \sim 10^9 M_{\odot}$), time is measured in seconds or hours. In astronomical time-scales, the evolution is quickly dominated by the effective $\Lambda = 1/r_S^2$ term inside. This, by the way, explains the coincidence problem in our Universe [26].

If we think of experimental cosmology before the year 2003 (i.e., ignore cosmic acceleration for a minute), the LTB expanding WH solution in Equation (5) (with $H > 0$) agrees very well with all the observations at that time, which favored a matter-dominated universe (the so called EdS universe with $\Omega_m = 1$). This is why some people still say that the Big Bang is a WH. However, today, we know that the universe has an effective Λ term, and this indicates that we are inside a BH [20,27]. Here, we interpret the observed Λ to be an effective term that corresponds to the gravitational radius $r_S = \sqrt{3/\Lambda} = 2GM_T$ of our local universe. Such BH Universe (BHU) could be within a larger background that may or may not be totally empty. In the later case, r_S will increase because of accretion from the outside. This case needs to be studied in more detail, but it could result in an effective Λ term that slowly decreases with time.

In terms of the proper coordinate radius r in Equation (5), the universe seems to enter a phase of cosmic acceleration because of the effective $\Lambda = 3/r_S^2$ term. However, this description is coordinate (or gauge) dependent. In terms of the more physical R_{EH} radius in Equation (9), the effect of r_S (or Λ) is, in fact, to decelerate events and bring cosmic expansion to a halt or frozen state.

This is illustrated in Figure 2. The case with a dashed line ($\Lambda = 0$) represents the always accelerating solution ($\ddot{R}_{EH} > 0$), whereas the case of the continuous line (with $\Lambda \neq 0$) becomes a decelerating solution that asymptotically stops ($\ddot{R}_{EH} < 0$) at $R_{EH} = r_S$. Thus, events in the universe decelerate (and not accelerate) because of Λ . It is therefore more appropriate to say that our physical universe is decelerating and it is described by an expanding metric inside a BH and not by a WH solution. This same conclusion also applies to the most general BH (or time reverse WH) solutions described by the spherically symmetric metric of Equation (4) with finite mass in Equation (6): the solutions are always trapped BH and not WH solutions.

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Appendix A. Timelike Junction

To match the FLRW metric in Equation (7) to the Schwarzschild metric in Equation (1), we chose a timelike 3D surface Σ that is fixed in comoving coordinates at $\chi = \chi_*$. Thus, Σ corresponds to a spherical shell with radius $R = a\chi_*$ that follows a radial geodesic trajectory. The interior of such shell corresponds to a FLRW cloud of fixed mass M_T that is expanding or contracting. We use 3D indexes (α, β) to label the matching shell. We can then take the 3D subset of FLRW coordinates $dy^\alpha = (d\tau, d\delta, d\theta)$ as coordinates in the shell, so that the induced metric, $h_{\alpha\beta}^-$, is:

$$ds_{\Sigma^-}^2 = h_{\alpha\beta}^- dy^\alpha dy^\beta = -d\tau^2 + a^2(\tau) \chi_*^2 d\Omega^2 \quad (\text{A1})$$

Since we have spherical symmetry, we take the solid angle $d\Omega$ and the angular coordinates to be the same in the FLRW and the Schwarzschild metrics. Thus, the only remaining variable is the FLRW comoving time τ . For the outside Schwarzschild frame with coordinates $dx^\mu = (dt, dr, d\delta, d\theta)$ in Equation (1), the same junction Σ^+ is described by some unknown functions $r = R(\tau)$ and $t = T(\tau)$, where t and r are the time and radial coordinates in the physical Schwarzschild frame of Equation (1). We then have:

$$dr = \dot{R}d\tau ; \quad dt = \dot{T}d\tau, \quad (\text{A2})$$

where the dot here refers to τ time derivatives. The induced metric h^+ estimated from the outside BH Schwarzschild metric (in Equation (1)) becomes:

$$\begin{aligned} ds_{\Sigma^+}^2 &= h_{\alpha\beta}^+ dy^\alpha dy^\beta = -F dt^2 + \frac{dr^2}{F} + r^2 d\Omega^2 \\ &= -(F\dot{T}^2 - \dot{R}^2/F) d\tau^2 + R^2 d\Omega^2 \end{aligned} \quad (\text{A3})$$

where we have used $F \equiv 1 - r_S/R$ to simplify the following equations. Comparing Equation (A1) with Equation (A3), we can see that the matching condition $h^- = h^+$ results in

$$R(\tau) = a(\tau)\chi_* ; \quad F\dot{T} = \sqrt{\dot{R}^2 + F} \equiv \beta, \quad (\text{A4})$$

so that given $a(\tau)$ and χ_* in the FLRW metric, we can simply find both $R(\tau)$ and $\beta(\tau)$ from the above equations. The second matching condition is that the derivative of the two metrics must also be continuous at Σ . This requires that the extrinsic curvature K^\pm normal to Σ is the same from each side of the hypersurface (Σ^\pm) as

$$K_{\alpha\beta} = -[\partial_a n_b - n_c \Gamma_{ab}^c] e_\alpha^a e_\beta^b \quad (\text{A5})$$

where n_a is the 4D vector normal to Σ and $e_\alpha^a = \partial x^\alpha / \partial y^\alpha$. Thus, $u^a = e_\tau^a = (1, 0, 0, 0)$ is the outward 4D velocity and $n^- = (0, a, 0, 0)$ is the normal to Σ^- on the inside. On the outside, $u^a = (\dot{T}, \dot{R}, 0, 0)$ and $n^+ = (-\dot{R}, \dot{T}, 0, 0)$. We thus have $n_a u^a = 0$ and $n_a n^a = +1$, as expected for a timelike surface, from both from the inside n^- and from the outside n^+ . The extrinsic curvature, K^- , estimated for the inside FLRW metric is then:

$$\begin{aligned} K_{\tau\tau}^- &= -(\partial_\tau n_\tau^- - a\Gamma_{\tau\tau}^\chi) e_\tau^\tau e_\tau^\tau = 0 \\ K_{\theta\theta}^- &= a\Gamma_{\theta\theta}^\chi e_\theta^\theta e_\theta^\theta = -a\chi_* = R \end{aligned} \quad (\text{A6})$$

where we have used the Christoffel symbols for the FLRW:

$$\begin{aligned} \Gamma_{\tau\tau}^\tau &= \Gamma_{\tau\chi}^\tau = \Gamma_{\chi\tau}^\tau = \Gamma_{\chi\chi}^\tau = 0 ; \quad \Gamma_{\theta\theta}^\tau = -a^2 \chi_*^2 H \\ \Gamma_{\tau\chi}^\chi &= \Gamma_{\chi\chi}^\tau a^{-2} = -H ; \quad \Gamma_{\theta\theta}^\chi = \chi_* \end{aligned} \quad (\text{A7})$$

The extrinsic curvature K^+ estimated for the outside Schwarzschild metric:

$$\begin{aligned} K_{\tau\tau}^+ &= \ddot{R}\dot{T} - \dot{R}\ddot{T} + \frac{\dot{T}r_S}{2R^2F}(\dot{T}^2F^2 - 3\dot{R}^2) = \frac{\dot{\beta}}{\dot{R}} \\ K_{\theta\theta}^+ &= \dot{T}\Gamma_{\theta\theta}^r = \dot{T}FR = \beta R \end{aligned} \quad (\text{A8})$$

where we have used the definition of β in Equation (A4) and the Christoffel symbols for the Schwarzschild metric:

$$\begin{aligned} \Gamma_{tt}^t &= \Gamma_{tr}^r = \Gamma_{\theta\theta}^t = 0 ; \quad \Gamma_{\theta\theta}^r = FR ; \\ \Gamma_{tr}^t &= -\Gamma_{rr}^r = \Gamma_{tt}^r F^{-2} = \frac{r_S}{2FR} \end{aligned} \quad (\text{A9})$$

Note how $K_{\delta\delta} = \sin^2\theta K_{\theta\theta}$, so that $K_{\delta\delta}^- = K_{\delta\delta}^+$ follows from $K_{\theta\theta}^- = K_{\theta\theta}^+$. Comparing Equation (A6) with Equation (A8), the second matching condition: $K_{\alpha\beta}^- = K_{\alpha\beta}^+$ requires $\beta = 1$, which using Equation (A4), results in

$$R = [r_H^2 r_S]^{1/3} \quad (\text{A10})$$

This just reproduces the LTB (or FLRW*) in Equation (5) with $M = M_T$ inside R in Equation (6). The time equation is $\dot{T} = (1 - R^2 H^2)^{-1}$.

Appendix B. The GHY Boundary Term

The Einstein–Hilbert action is given by [17,28–30]:

$$S = S_4 \equiv \int_{V_4} dV_4 \left[\frac{R - 2\Lambda}{16\pi G} + \mathcal{L} \right], \quad (\text{A11})$$

where V_4 is the volume of the 4D spacetime manifold, $dV_4 = \sqrt{-g}d^4x$ is the invariant volume element, $R = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar and \mathcal{L} is the energy-matter Lagrangian. Einstein's field equations can be obtained for the metric field $g_{\mu\nu}$ by requiring S to be stationary ($\delta S = 0$) under arbitrary variations of the metric $\delta g^{\mu\nu}$. The solution is well known [17,30,31]:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \equiv -\frac{16\pi G}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}},$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$. This solution requires that boundary terms vanish (e.g., see [17,19,32]). Otherwise, we need to add a Gibbons–Hawking–York (GHY) boundary term [33–35] to the action $S = S_4 + S_{GHY}$, where:

$$S_{GHY} = \frac{1}{8\pi G} \oint_{\partial V_4} d^3y \sqrt{-h} K. \quad (\text{A12})$$

where h is the induced metric and K is the trace of the extrinsic curvature at the boundary ∂V_4 . Since the expansion inside an isolated BH is bounded by the event horizon $r < r_S$, we need to add this GHY boundary term S_{GHY} to the action. The integral is over the induced metric at ∂V_4 , i.e., Equation (A1) with $\partial V_4 = \Sigma^-$ at $R = r_S$:

$$ds_{\partial V_4}^2 = h_{\alpha\beta} dy^\alpha dy^\beta = -d\tau^2 + r_S^2 d\Omega^2 \quad (\text{A13})$$

We use Equation (A6) to estimate K :

$$K = K_\alpha^\alpha = \frac{K_{\theta\theta}}{R^2} + \frac{K_{\delta\delta}}{R^2 \sin^2 \theta} = -\frac{2}{R} = -\frac{2}{r_S}. \quad (\text{A14})$$

We can now estimate S_{GHY} :

$$S_{GHY} = \frac{1}{8\pi G} \int d\tau 4\pi r_S^2 K = -\frac{r_S}{G} \tau \quad (\text{A15})$$

The contribution of Λ to the action in Equation (A11) is

$$S_\Lambda = -\frac{\Lambda}{8\pi G} V_4 = -\frac{r_S^3 \Lambda}{3G} \tau \quad (\text{A16})$$

where we estimated the 4D volume V_4 as that bounded by ∂V_4 inside $r < r_S$: $V_4 = 2V_3\tau$. The factor of two here accounts for the fact that $V_3 = 4\pi r_S^3/3$ can be covered twice (both during collapse and during expansion).

We can then see that we need $\Lambda = 3r_S^{-2}$, or equivalently, $r_\Lambda = r_S$, to cancel the boundary term. In other words: a Λ term in the field equations can be induced by the evolution inside a BH event horizon.

Notes

- 1 This is the case only if we ignore cosmic acceleration or if we consider an observer in a galaxy far away (say at $z = 2$), when matter domination was an excellent approximation.
- 2 Note that both a WH and a BH require a finite total mass. If M_T is infinitely large, then $r_S = \infty$ and there is no WH or BH. This in fact the standard Big Bang assumption. But this assumption is impossible to implement, even with Inflation: using local laws of gravity we have to create a uniform space of infinite extend within a finite amount of time.

3 Even if the exterior is not totally empty and there is some small accretion from the outside, the value of r_S will slowly increase as the BH mass increases. But the r_S boundary still needs to be taken into account to evaluate the action inside.

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