ACCELERATING WAVEGUIDES FOR HIGH-ENERGY ACCELERATORS

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Summary

Accelerating waveguides are used for both high-energy and low-energy electron accelerators and can also be applied to synchrotrons. High-energy synchrotrons have a very narrow frequency band(due to the high energy of injection), use a high frequency supply and require a significant energy gain per turn and therefore non-tuned accelerating waveguides can be used. This application was proposed at CERN and then was extended to high intensity accelerated beams. For the last case, heavy beam loading, bunched in an equilibrium phase, must be taken into account. Uniform waveguides are considered and some recommendations from the energetical point of view are discussed. Some general remarks are made concerning non-uniform waveguides.

Introduction

In high-energy synchrotrons the frequency of the accelerating voltage is increased to assure resonant acceleration for an equilibrium particle. Owing to the dispersion of non-tuned waveguides, a shift of the equilibrium particle relative to the travelling wave created by a high-frequency generator exists inside the waveguide. Such process practically takes place during the whole time of acceleration, excepting only one specific moment when the shift is equal to zero.

When the intensity of the accelerated beam is as high as it is for modern synchrotrons, it is quite necessary to take into account the action of the field having the same pattern as the field of generator, but irradiated by bunches. This field irradiated by each separate bunch is stationary with respect to the bunch and has a different main frequency in comparison with the generator frequency. Both frequencies only coincide at the very same moment mentioned above, when the bunch velocity and the phase velocity of the travelling wave become equal.

The frequency of the irradiated field after a build-up time becomes equal to the generator frequency. Therefore the bunches accelerated inside the non-tuned waveguide are shifting relative to their own irradiated field. Finally, there is the essential difference between the waveguides implemented for linear electron accelerators: the bunches are located at the crest of the accelerating wave, while in synchrotron waveguides the bunches are in a certain equilibrium phase.

The application of non-tuned waveguides to synchrotrons with variable frequency was first suggested by Schnell at CERN[1]. In this article the phase shift of bunches with respect to the travelling wave was also considered. But at the same time, attenuation of the electron-magnetic field inside the waveguide and beam loading were neglected. On the other hand, Lichtenberg[2] took into account the field irradiated by the bunches located in the certain equilibrium phase. But in the last article, since it was concerned with linear electron accelerators, the author neglected the shifting of bunches with respect to the field created by the high-frequency generator as the field irradiated.

In previous investigations fulfilled by the author of this report in co-operation with his collaborators, both the shifting of bunches relative to the generator field and the irradiated one and the location of the bunches in the equilibrium phase were considered. To be convinced in the correctness of the main preconditions, some experiments were performed at the linear accelerator for 360 MeV of the Kharkov Physical-Technical Institute of the Ukrainian Academy of Sciences. There, special conditions were artificially created for shifting bunches relative to the irradiated field. Close accordance was shown between theory and experiment for both frequencies and amplitudes of the irradiated fields. Results of these investigations were published as monograph[3].

Using the model presentation of physical processes inside the waveguides, one can solve problems concerning energetical efficiency, find out conditions of optimal efficiency, and choose accelerating structure for a certain synchrotron. The very same calculations were used for one alternative version of accelerated systems, for the "cybernetic accelerator", with output energy up to 1000 BeV.

Field superposition

As it is known[2] for an achievement of the right presentation for energetic field balance of generator and irradiated fields, the vector superposition is used. For accelerating waveguides of synchrotrons it is more convenient to employ the standard presentations and replace the waveguide for the equivalent accelerating gap with a monoharmonic voltage U.

For uniform waveguides such equivalent voltage can be easily deduced by integrating the generator electric field E_g and induced field E_i:

$$E_g = E_{go} \exp(-\alpha Z) \left[\cos(\varphi_o + K_s Z) + i \sin(\varphi_o + K_s Z)\right] = \mathrm{Re}\, E_g + \mathrm{Im}\, Eg \qquad (1)$$

$$E_i = \left\{ I R_s Z \Big/ \left[(\alpha Z)^2 + (K_s Z)^2 \right] \right\} \Big[(\alpha Z - \alpha Z \exp(-\alpha Z) \cdot \cos K_s Z + K_s Z \exp(-\alpha Z) \sin K_s Z) + i\,(K_s Z - K_s Z \cdot \exp(-\alpha Z) . \cos K_s Z - \alpha Z \exp(-\alpha Z) \sin K_s Z) \Big] = R_e E_i + \mathrm{Im}\, E_i \qquad (2)$$

The amplitude of the generator equivalent voltage can be determined by:

$$U_{og} = \sqrt{(\mathrm{Re} U_g)^2 + (\mathrm{Im} U_g)^2} \qquad (3)$$

where $\mathrm{Re}U_g = \int_0^{\ell} \mathrm{Re}\ Eg\ dz$; $\mathrm{Im}U_g = \int_0^{\ell} \mathrm{Im}\ Eg\ dz$

The same can be done for the induced field. Here the index "o" is used to indicate the amplitude of fields and voltages.

Below one can find the list of terms:

- Z – co-ordinate along the waveguides axes
- φ_o – particle phase in respect to the travelling wave crest for $Z = o$;
- α – attenuation coefficient ;
- K_s – shifting coefficient ;

$$K_s = \frac{2\pi}{\lambda}\left(\frac{1}{\beta_w} - \frac{1}{\beta_p}\right) ; \qquad (4)$$

- λ – free-space wavelength ;
- β_w – relative wave velocity (in units of the light velocity – c) ;
- β_p – relative particle velocity ;
- I – accelerated current ;
- R_s – waveguide series resistance ;

$$R_s = \frac{E_o^2}{2P} ; \qquad (5)$$

- P – high-frequency power.

The voltage acting to any particle of the bunch is given by:

$$U = U_{og}\cos(\varphi_o + \gamma) - U_{oi}\cos(\varphi_o - \varphi_{os} + \delta) =$$

$$= U_o \cos(\varphi_o + \theta) \qquad (6)$$

The corresponding vector diagramme is shown in Fig.1. Values of U_{og} and U_{oi} obtained by integrating can be found from the next equations:

$$U_{og} = (2R_sP)^{\frac{1}{2}}\ell\ A(X,Y) ; \qquad (7)$$

$$U_{oi} = IR_s\ell^2\ G(X,Y) ; \qquad (8)$$

$$A(X,Y) = \left\{\left[1 - 2\exp(-X)\cos Y + \exp(-2X)\right]/(X^2 + Y^2)\right\}^{\frac{1}{2}}$$

$$G(X,Y) = \left[2/(X^2+Y^2)\right]\cdot\left\{Y^2 - (1-X)^2 - 2\exp(-X)\ x\right.$$

$$\left. x\ \left[(1-X)\cos Y + Y\sin Y\right] - \exp(-2X)\right\}^{\frac{1}{2}}$$

Here $X = \alpha\ell$ and $Y = K_s\ell$ – the total attenuation and total phase shift correspondently. The phase angles can be determined with the help of the next expressions:

$$\gamma = \tan^{-1}\frac{Y - \exp(-X)(X\sin Y + Y\cos Y)}{X - \exp(-X)(Y\sin Y - X\cos Y)} \qquad (9)$$

$$\delta = \tan^{-1}\frac{Y(X^2+Y^2) - 2XY - \exp(-X)\cdot}{X(X^2+Y^2) + Y^2 - X^2 - \exp(-X)}\ \frac{\cdot\left[(Y^2-X^2)\sin Y - 2XY\cos Y\right]}{\cdot\left[(Y^2-X^2)\cos Y + 2XY\sin Y\right]} \qquad (10)$$

$$\theta = \tan^{-1}\frac{U_{og}\sin\gamma - U_{oi}\sin\delta}{U_{og}\cos\gamma - U_{oi}\cos\delta} \qquad (11)$$

The voltage applied to the equilibrium particle in accordance with Fig.1, is given by:

$$U_s = U_{og}\cos(\varphi_{os} + \gamma) - U_{oi}\cos\delta = U_o\cos\varphi_s \qquad (12)$$

The analogical relations can be obtained for non-uniform waveguides if the well-known supposition about linear dropping of the group velocity along the waveguide is applied to have the constant electric field acting to the bunch in any point of the waveguide. This can be done either for the zero accelerated current or for a certain nominal value. For both cases shifting of the bunch with respect to the accelerating wave becomes non-linear and voltages are described by much more complex formulae.

If for linear accelerator waveguides one considers that increasing of the waveguide length is useless beyond the point where the acting electric field is equal to zero, for synchrotron waveguides this condition can be written in the form $-\cos\varphi_s < 1$. This requirement is determined by the whole accelerator and the equilibrium phase can be found out by

$$\varphi_s = \tan^{-1}\left\{\frac{1}{U_s}\left[\pm\sqrt{U_{og}^2 - (U_s + U_{oi}\cos\delta)^2} - U_{oi}\sin\delta\right]\right\} \qquad (13)$$

Significantly, the last expression includes δ, U_{oi}, U_{og} which are dependent on the total phase shift Y and, consequently, the equilibrium phase will be varied if the high-frequency power is constant during the accelerating cycle. The equations considered allow these variations to be investigated.

Optimization and choice of accelerating structure

Using the relations obtained, the optimal efficiency of the accelerating waveguides to achieve the minimum of high-frequency power supply can be found. As a result, we determine the condition which corresponds to maximum transformation of high-frequency power into beam power. For uniform waveguides, for instance, we can find the requirements on the accelerating structure. For the highest efficiency the accelerating structure would have the series resistance:

$$(R_s)_{opt} = \frac{2U_o}{I\ell^2 G(X,Y)} \qquad (14)$$

which gives the efficiency:

$$\eta_{max} = \frac{2 A^2(X,Y) \cos \varphi_s}{G(X,Y) \left[1 + \cos(\varphi_s - \delta) \right]} \qquad (15)$$

A very helpful simplification can be obtained supposing that X = 0 and Y = 0. In this case $\delta = 0$ and $A(X,Y) = G(X,Y) = 1$. For non-uniform waveguides, the corresponding expressions have rather complicated forms[3] and therefore are omitted here.

Parameters contained in the equation written above are not mutual independent. For example, the phase shift γ depends not only on the waveguide length but on the group velocity (β_g). If the value of β_w and the mode of oscillation are fixed, it appears that for any high-frequency structure the dependence R_s (β_g) can be plotted.

On the other hand, if the values of the parameters for the accelerating waveguide are fixed (length ℓ, X, φ_s, U_s etc) the equation (14) can be plotted in the same form. An intersection of two curves, one of which belongs to the accelerating system of the synchrotron and the other to the structure, presents the optimal waveguide.

For example, for one of the variants of the 1000 BeV "cybernetic accelerators", optimal systems are: "clover-leaf" and "corrugated" waveguide with additional radial slits. The cylindrical corrugated waveguide employed for linear electron accelerators is not a high-frequency structure providing optimal conditions (14).

At the same time the possibility to achieve optimal conditions by this waveguide is not excluded if other parameters of the accelerating system are suitably chosen.

It is necessary to notice that there is not an essential benefit in efficiency for accelerating waveguides without feedback if the attenuation tends to zero. For instance, the waveguide efficiencies for X = 0 in comparison with X = 0,25 have the benefit not more than 10% and this circumstance does not allow the conclusion that the application of superconducting waveguides is advantageous.

Conclusion

The above analysis shows that in principle accelerating, non-tuned waveguides can be applied with high efficiency for synchrotrons with variable frequency. Nevertheless, some phenomena which have not been considered here must be taken into account. The first of these is a variation of the high-frequency voltage, connected with changes of shifts for particles relative to the accelerating wave. The second major problem is connected with achieving the proper performance at the transition energy, because of the long build-up time for the waveguides. Special attention must be paid to the possibility of generating asymmetrical waves, which can destroy the accelerated beam. In linear electron accelerators it is known to produce a shortening of the beam pulse, (the "beam blow-up effect"). It cannot be eliminated by focusing magnets, because of the high-frequency modulated structure of the beam. This effect must be further studied, especially if the structure used is asymmetric and excites the asymmetric type of electromagnetic wave. Implementation of non-uniform waveguides decreases the last effect.

References

1. Schnell, CERN Report AR/Int. SG/65-16, 1965.
2. Lichtenberg, RSI, Vol. 33, p.395, 1962.
3. O.A. Valdner, A.V. Shalnov, A.N. Didenko "Accelerating Waveguides", Moscow, Atomizdat, 1973.

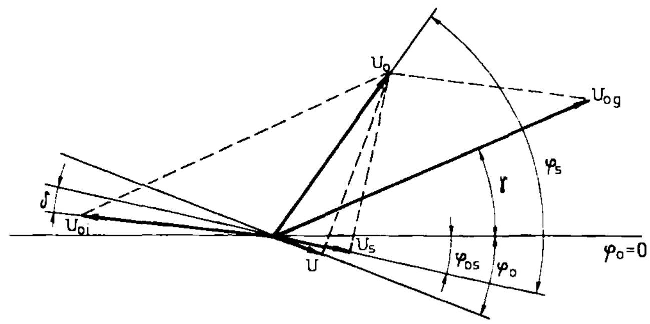

Figure 1.