EWSB FROM A BULK HIGGS

M. QUIRÓS
Institut de Fisica d'Altes Energies, Universidad Autonoma de Barcelona
08193 Bellaterra, Barcelona, Spain
and
Institucio Catalana de Recerca i Estudis Avançats (ICREA), Catalonia, Spain

We propose a model with a warped extra dimension where the brane distance is stabilized by a Golberger-Wise mechanism with an exponential potential. The brane distance depends exponentially on the fixed values of the scalar field at the branes and it is thus naturally stabilized. The potential generates a naked curvature singularity outside the brane interval. The vicinity of the singularity can lead to a reduction of the number of degrees of freedom at the infrared brane in the holographic theory which can in turn reduce the contribution of KK modes to the precision electroweak observables. We have quantified this phenomenon and shown it leads to lower bounds on the KK masses as low as $\mathcal{O}(1)$ TeV in the presence of a bulk custodial symmetry.

1 Introduction

Warped extra dimensions are useful to solve long-standing problems: hierarchy, flavor,... In particular for a five-dimensional (5D) model the metric does not factorizes but it can be written in proper and conformally flat coordinates as

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 = e^{-2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) \tag{1}$$

and $A(y) = ky$ is the AdS solution [1], where k is the AdS curvature, for which the model is conformal invariant. Furthermore the AdS/CFT correspondence [2] can deal with non-perturbative theories, as technicolor, QCD,... On top of the ultra-violet (UV) brane at $y = 0$ which provides the UV cutoff of the theory, conformal invariance is normally broken by an infra-red (IR) brane at $y = y_1$ as in the model of Ref. [1] (hereafter referred to as RS1). The brane distance can be stabilized by the Goldberger-Wise (GW) mechanism [3] which requires the introduction of a scalar propagating in the bulk with a *quadratic potential* in the bulk and backreacting on the gravitational metric but without generating any singularity.

Another possibility constitute the so-called soft-wall models. The scalar in the bulk with an *exponential potential* does generate a singularity at a finite distance and the extra dimension is non-compact but of finite length:

$$\int e^{-A(z)}dz \equiv y_s = \int_0^{y_s} dy < \infty \tag{2}$$

while the metric is AdS near the UV brane. This implies that there is a naked curvature singularity at $y = y_s$ where $A(y_s) \to \infty$. Soft-walls have been proposed [4]: for AdS/QCD models, to describe unparticles as fields propagating in the bulk and as alternatives to RS1 for solving the EW hierarchy.

2 The model

We will use the superpotential method[5] to solve the gravitational equations of motion (EOM). It it based upon introducing a superpotential $W(\phi)$ such that solving the second order gravitational EOM is equivalent to solving the set of first order equations

$$\begin{aligned} A'(y) &= W(\phi), \quad \phi'(y) = \partial W/\partial\phi \\ V(\phi) &= 3(\partial W/\partial\phi)^2 - 12W^2 \end{aligned} \tag{3}$$

where $V(\phi)$ is the bulk potential and the brane potentials $\lambda_\alpha(\phi)$ satisfy the boundary conditions

$$\epsilon_\alpha \lambda_\alpha(\phi(y_\alpha)) = 6W(\phi(y_\alpha)), \quad \epsilon_\alpha \partial_\phi \lambda_\alpha(\phi(y_\alpha)) = 6\partial_\phi W(\phi(y_\alpha)) \tag{4}$$

where $\epsilon_\alpha = \pm 1$, depending on the Z_2 boundary conditions.

The model is defined by [6]

$$\begin{aligned} W(\phi) &= k(1+e^{\nu\phi}) \\ A(y) &= ky - \frac{1}{\nu^2}\log\left(1-\frac{ky}{ky_s}\right) \\ \phi(y) &= -\frac{1}{\nu}\log[\nu^2(ky_s - ky)] \end{aligned} \tag{5}$$

We will consider the case where the soft-wall singularity is "hidden" by a brane at $y_1 < y_s$. It may be considered as the case of a RS1 setup stabilized by the previous (super)potential at two branes located at $y = 0$ and $y = y_1$ where brane dynamics fixes the values of the field ϕ: $\lambda_0(\phi) \Rightarrow \phi = \phi_0$ @ UV rane band $\lambda_1(\phi) \Rightarrow \phi = \phi_1$ @ IR brane.

The interbrane distance y_1 as well as the location of the singularity at y_s and the warp factor $A(y_1)$ are related to the values of the field ϕ at the branes by the following expressions:

$$\begin{aligned} ky_1 &= \frac{1}{\nu^2}\left[e^{-\nu\phi_0} - e^{-\nu\phi_1}\right], \quad ky_s = \frac{1}{\nu^2}e^{-\nu\phi_0} \\ A(y_1) &= ky_1 + \frac{1}{\nu}(\phi_1 - \phi_0) \end{aligned} \tag{6}$$

which shows that the required large hierarchy can be naturally fixed with

$$|\nu\phi_0| \simeq a\,few, \quad \nu\phi_1 \geq 1 \tag{7}$$

Moreover the soft-wall is the limit $\phi_1 \to \infty$, $y_1 \to y_s$ [e.g. with a runaway potential $V_1 \sim e^{-\nu\phi}$].

3 The Higgs background

We will consider a 5D bulk Higgs as

$$H(x,y) = \frac{1}{\sqrt{2}}\, e^{ig_5\,\vec{\sigma}\vec{\chi}(x,y)} \begin{pmatrix} 0 \\ h(y) + \xi(x,y) \end{pmatrix} \tag{8}$$

and will assume that the dynamics of ϕ fixes y_1 so that the Higgs background does not perturb the radion fixing. We will then neglect the back-reaction of the Higgs background.

We will consider the potentials in the bulk and branes for the Higgs[a]

$$\begin{aligned} V(H) &= \left[a(a-4) - 4ae^{\nu\phi}\right]k^2|H|^2 \\ \lambda_0(H) &= M_0|H|^2 \\ \lambda_1(H) &= M_1|H|^2 + \gamma_1|H|^4 \end{aligned} \tag{9}$$

[a]For a different (non-tuned) class of bulk potentials see [7].

The EOM for the Higgs background yield

$$h(y) = \begin{cases} v_1 e^{k(4-a)(y-y_1)}, & a < 2 \\ v_1 e^{a(y-y_1)}, & a > 2 \end{cases} \tag{10}$$

while the boundary conditions yield

$$BC \Rightarrow \begin{cases} (4-a) = \gamma v_1^2 + M_1, & a < 2 \\ ak = \gamma v_1^2 + M_1, & a > 2 \end{cases} \tag{11}$$

The value of v_1 should be naturally of order k (to avoid a fine-tuning) and red-shifted to the TeV by the warp factor. If $v_1 \sim k$ is consistent with EWSB then the Higgs hierarchy is solved. Finally we will assume $y_s - y_1 \sim 1/k$ and consider the case $a \geq 2$ (sort of dual to walking technicolor in the RS1 case)

4 Electroweak symmetry breaking

We will illustrate the mechanism with an abelian example. The action is invariant under 5D gauge transformations $\alpha(x,y) = \alpha(x)f(y)$ and we will take the 5D gauge condition

$$\partial^\mu A_\mu - M_A^2 \chi + (e^{-2A} A_5)' = 0, \quad M_A(y) = g_5 h(y) e^{-A(y)} \tag{12}$$

The relevant degrees of freedom are the 5D gauge bosons $A_\mu(x,y)$, the Goldstone boson and the pseudoscalar defined as

$$G(x,y) = M_A^2 \chi - \left(e^{-2A} A_5\right)', \quad K(x,y) = \chi' - A_5 \tag{13}$$

The 4D theory is invariant under the $\alpha(x)$ gauge transformations and contains: as degrees of freedom

$$\begin{aligned} A_\mu(x,y) &= \frac{a_\mu(x) \cdot f(y)}{\sqrt{y_s}} \\ G(x,y) &= \frac{m_f\, G(x) \cdot f(y)}{\sqrt{y_s}} \\ K(x,y) &= \frac{K(x) \cdot \eta(y)}{\sqrt{y_s}} \end{aligned} \tag{14}$$

with profiles

$$\begin{aligned} m_f^2 f + (e^{-2A} f')' - M_A^2 f &= 0, \quad Neumann \\ m_\eta^2 \eta + \left[m_A^{-2} \left(e^{-2A} M_A^2 \eta \right)' \right]' - M_A^2 \eta &= 0, \quad Dirichlet \end{aligned} \tag{15}$$

We can find an approximation for the light gauge boson mode in the limit where the breaking is small and thus there is a light mode with almost constant profile

$$\begin{aligned} f_A(y) &= 1 - \delta_A + \delta f_A(y) \\ \delta f_A(y) &= \int_0^y dy'\, e^{2A(y')} \int_0^{y'} dy'' \left[M_A^2(y'') - m_{f_A^0}^2 \right] \\ \delta_A &= \frac{1}{y_1} \int_0^{y_1} dy\, \delta f_A(y) \end{aligned} \tag{16}$$

The light mode mass is then

$$m_{f_A^0}^2 = \frac{1}{y_1}\int_0^{y_1} M_A^2(y)dy \tag{17}$$

For the case of the Standard Model $SU(2)_L \times U(1)_Y$ of electroweak interactions the generalization of the previous formalism is straightforward and the 5D, y-dependent gauge boson masses are

$$M_W(y) = \frac{g_5}{2}h(y)e^{-A(y)} \ , \qquad M_Z(y) = \frac{1}{c_w}M_W(y) \ , \qquad m_\gamma(y) \equiv 0 \ . \tag{18}$$

5 Electroweak constraints

In our 5D model (for fixed values of the parameters ν, $y_1, \ldots$) we have the free parameters (g_5, g_5', v_1, a) which fix the physical spectrum of light mode masses. Once we have fixed the condition

$$m_{f_Z} = m_Z \tag{19}$$

then v_1 is fixed. For $A(y_1) = 35$, the values predicted for v_1 using the condition (17) are plotted in Fig. 1.

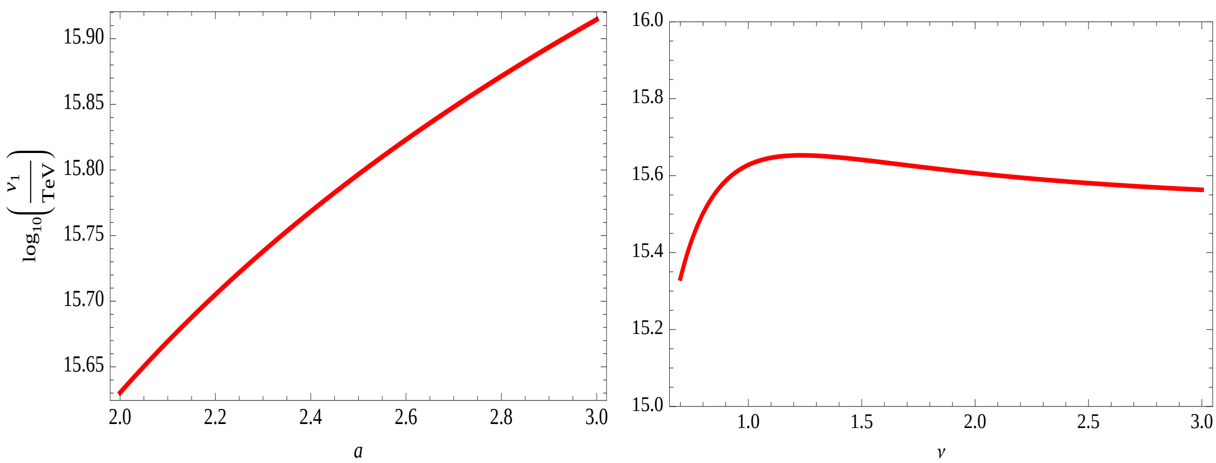


Figure 1: Left: value of v_1 as a function of a fpr $\nu = 1.5$. Right: value of v_1 as a function of ν for $a = 2$.

We can see from Fig. 1 that v_1 is of order the Planck scale for the considered range $2 < a < 3$ and for $\nu > 0.7$ and thus the hierarchy requirement is satisfied.

We will be assuming here (not necessarily an assumption) that fermions are localized on the UV brane in which case

$$g_V = g_V^{SM} f_V(0) \equiv g_V[1 - \delta_V(a, m_{KK})] \tag{20}$$

The latter changes the definition of the Fermi constant measured in the μ-decay and the Z widths which constrain the electroweak precision test EWPT parameters [8]: S, T and U.

Assuming the Standard Model $SU(2)_L \times U(1)_Y$ gauge fields propagating in the bulk they can be given the general expressions [7]:

$$\alpha T = s_W^2 \frac{m_Z^2}{k^2} k y_1 \int_0^{k y_1} \left(\Omega(y) - \frac{y}{y_1}\right)^2 e^{2A} \tag{21}$$

$$\alpha S = s_W^2 \frac{m_Z^2}{k^2} k y_1 \int_0^{k y_1} \left(1 - \frac{y}{y_1}\right)\left(\Omega(y) - \frac{y}{y_1}\right) e^{2A} \tag{22}$$

$$\alpha U = \mathcal{O}(\delta_Z^2) \simeq 0 \tag{23}$$

where

$$\Omega(y)=\frac{U(y)}{U(y_1)},\quad U'(y)=h^2(y)e^{-2A(y)}/v_0^2 \tag{24}$$

In the case of the Higgs profile given by Eq. (10) it is given for $y_1 \simeq y_s$ by

$$\Omega(y)\simeq\frac{\Gamma[1+\frac{2}{\nu^2},2(a-1)(y_s-y)]}{\Gamma(1+\frac{2}{\nu^2})} \tag{25}$$

The SM fit on the (S,T) plane, assuming $U=0$, for a reference Higgs mass $m_H^{ref}=117$ GeV, provides [9]

$$\begin{aligned} T &= \quad 0.02\pm 0.09 \\ S &= -0.04\pm 0.09 \end{aligned} \tag{26}$$

There is here a problem with the large and positive contribution (21) from the KK-modes which is enhanced by the large volume and put a lower bound on the mass of KK modes ~ 10 TeV. This problem can be alleviated a bit by considering a heavy Higgs boson which contributes to the S and T parameters as [10]

$$\Delta T=-\frac{3}{8\pi c_W^2}\log\frac{m_H}{m_H^{ref}},\quad \Delta S=\frac{1}{6\pi}\log\frac{m_H}{m_H^{ref}} \tag{27}$$

For instance for $m_H=500$ GeV the extra contribution is $\Delta T=-0.19$ and $\Delta S=0.06$.

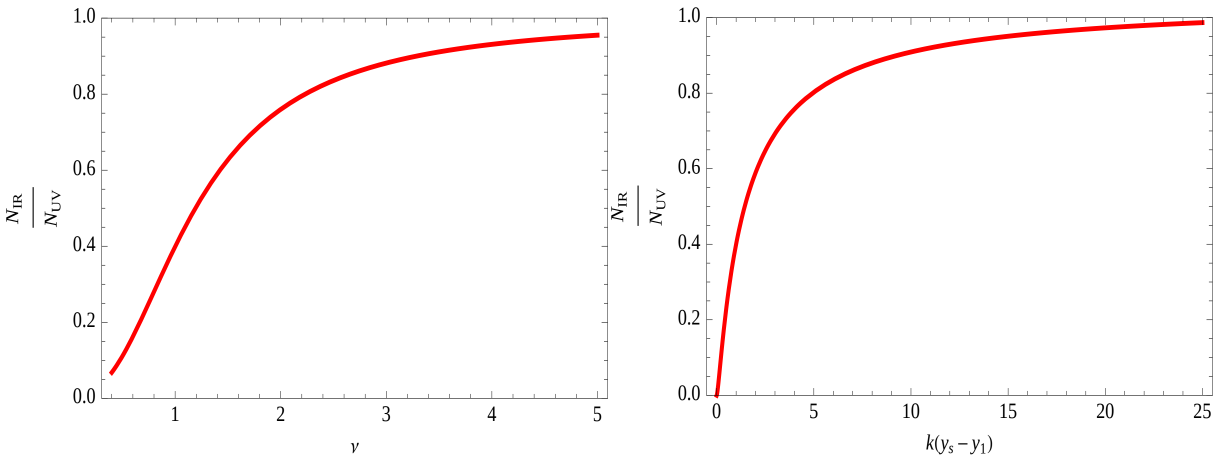


Figure 2: Left: N_{IR}/N_{UV} as a function of ν for $k(y_s-y_1)=1$ and $A(y_1)=35$. Right: N_{IR}/N_{UV} as a function of $k(y_s-y_1)$ for $\nu=1$ and $A(y_1)=35$.

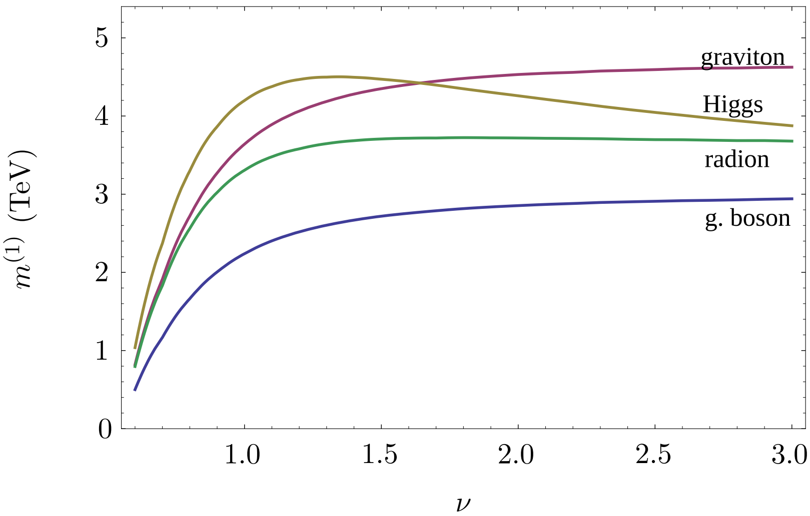
Another possibility is to introduce a custodial symmetry in the theory [11]. In this case the gauge symmetry in the bulk is enlarged to $SU(2)_L\times SU(2)_R\times U(1)_X$. The symmetry is broken to $SU(2)_L\times U(1)_Y$ by the boundary conditions at the UV brane while it is unbroken at the IR brane. In this case there is an additional contribution to the T parameter by which it becomes volume suppressed (subleading) instead of enhanced and the relevant parameter to be considered is the S parameter, which has the same general expression as in Eq. (22).

We expect that the bounds on KK masses will go down with ν as the number of degrees of freedom of the holographic theory at the IR brane N_{IR} also go down as a consequence of the corresponding departure from AdS behaviour. Notice that $\nu\to\infty$ corresponds to RS1.

A similar behavior is expected as a function of $k(y_s - y_1)$. In both cases the influence of the soft-wall singularity becomes stronger at the IR brane. This behavior is made explicit in the plots of Fig. 2 where we can see that the number of degrees of freedom becomes one order of magnitude smaller than for the RS1 case for $k(y_s - y_1) \simeq 1$ and/or $\nu \leq 1$.

Our numerical results for KK modes are summarized in the following figure

$$a = 2, \quad k(y_s - y_1) = 1, \quad A(y_1) = 35$$



where we plot, for the set of values of the parameters there indicated, the lower bounds on KK mode masses consistent with electroweak observables for models with custodial symmetry and the Higgs profile given by Eq. (10). We can see that for values of $\nu < 1$ one can go to masses for the KK gauge bosons ~ 1 TeV.

Acknowledgments

Work supported in part by the Spanish Consolider-Ingenio 2010 Programme CPAN (CSD2007-00042); and by CICYT, Spain, under contract FPA 2008-01430.

References

1. L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].
2. J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].
3. W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999) [arXiv:hep-ph/9907447].
4. A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006) [arXiv:hep-ph/0602229]; B. Batell and T. Gherghetta, Phys. Rev. D **78**, 026002 (2008) [arXiv:0801.4383 [hep-ph]]; A. Falkowski and M. Perez-Victoria, JHEP **0812**, 107 (2008) [arXiv:0806.1737 [hep-ph]].
5. O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, Phys. Rev. D **62** (2000) 046008.
6. J. A. Cabrer, G. von Gersdorff and M. Quirós, arXiv:0907.5361 [hep-ph].
7. J. A. Cabrer, G. von Gersdorff and M. Quirós, in preparation.
8. M. E. Peskin and T. Takeuchi, Phys. Rev. D **46** (1992) 381.
9. C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667** (2008) 1.
10. M. E. Peskin and J. D. Wells, Phys. Rev. D **64** (2001) 093003 [arXiv:hep-ph/0101342].
11. K. Agashe, A. Delgado, M. J. May and R. Sundrum, JHEP **0308** (2003) 050 [arXiv:hep-ph/0308036].