

Nuclear Structure Using the Particles of High-Energy Physics. II

In a previous comment¹ it was stressed that, strictly speaking, the measurement of momentum distributions in nuclei does not constitute a model-free determination of nuclear correlations. The presence of high momentum components in the single-particle momentum distribution, such as those found² in the electron scattering from He⁴, makes their interpretation in terms of dynamical correlations among the nucleons very plausible, but a quantitative estimate of such correlations must rely on experiments which bear more directly on them.

It has been argued³ that the shape of the nuclear density at its center could be used to detect the effects of a repulsive-core type correlation among the nucleons. Indeed, the density at the nuclear center is rather sensitive to such short-range correlations, being depressed relative to the rest of the nucleus for repulsive-core interactions. Furthermore, as was pointed out by Ravenhall (quoted by Bethe and Elton⁴), the *combination* of experiments on high-energy electron scattering and μ -mesic x-rays can yield some information on a possible central depression in the nuclear density. This can be easily understood⁴ if we consider the Born amplitude for electron scattering at momentum transfer q , from a charge distribution $\rho(r)$:

$$f_B(q) = \frac{1}{q} \int \sin qr \rho(r) r dr.$$

For low values of q , such that $qR \ll 1$, where R is the nuclear radius, we can put $\sin qr \approx qr$ and see that electron scattering measures $\langle r^2 \rangle$. The same holds true for μ -mesic x-rays, and there is indeed a nice agreement between such data and the corresponding low-energy electron-scattering data.⁵ On the other hand, for high values of q , the oscillations of $\sin qr$ over regions in which $\rho(r)$ varies slowly, cancel the contributions of such regions to $f(q)$, and one is left with the bulk of the contribution coming from the region in which $\rho(r)$ has its steepest slope. The combination of experimental determinations of $\langle r^2 \rangle$ and the position of steepest slope

of $\rho(r)$ imposes a relation between the densities at the center and at the edges, and hence its power in determining "central depressions" in nuclear densities.

Yet, despite the feasibility of experimentally determining central depressions in the nuclear density and the possible connection of such a depression with repulsive-core type correlations, it has to be remembered that a nonlocal single-particle potential also leads to central depressions in the nuclear density (the "Perey⁶ Effect"). The presence of such depressions in nuclear density may then be indicative of non-local effects in the overall average potential rather than being a direct evidence for repulsive-core interactions.

The Pauli principle introduces well-known correlations among the nucleons in the nucleus. As a matter of fact these correlations, too, give rise to "high momentum" components: Without the Pauli correlations the maximum single-particle momentum would have been characterized by the nuclear *size*, being of order \hbar/R ; with the Pauli correlations the maximum momentum is characterized by the nuclear *density*, being of order \hbar/r_0 , where $R = r_0 A^{1/3}$, and A is the nuclear mass-number. However, the Pauli correlations are nondynamical in the sense that, in the absence of any other interaction among the nucleons, they are not capable of transferring momentum from one nucleon to another. Given the Pauli correlations *alone*, with no further dynamical correlations among the nucleons, an electron impinging on a nucleus can eject from it only one nucleon at a time, but not a cluster of nucleons. In this respect these correlations are similar to the long-range correlations introduced by the conservation of total angular momentum, which are responsible, among other things, for the angular correlations among successive radiations, but cannot give rise to a momentum transfer from one nucleon to another.

The interesting correlations among nucleons are those induced by their mutual interaction, over and above the Pauli correlations, and the detection of such correlations is a somewhat tricky problem. For an infinite nucleus, i.e. nuclear matter, the nuclear density uniquely determines the momentum distribution for free fermions, and any deviation from it can be ascribed to dynamical correlations beyond those induced by the Pauli principle at this density. This was the basis for the suggestion of Czyz and Gottfried⁷ that such correlations be measured through the study of inelastic electron scattering as a function of both momentum transfer and energy loss. However, it was shown later by Reiner⁸ that, at least in the independent pair approximation, the scattering of electrons from nuclear matter originates predominantly from *free* quasi-particles. The main features affecting this scattering have to do only

with the quasi-particle density, and the remaining effects of dynamical correlations between these quasi-particles are rather small.

The equilibrium density of nuclear matter depends on some specific features of the nuclear force. In this sense the effectiveness of the Pauli correlations, which depends on the nuclear density, reflects also the dynamics of the system. A search for additional dynamical correlations must take this point into account, especially in finite nuclei. There, what we consider to be the "Pauli correlations", depends on the assumed average single-particle potential, since this potential determines the "dynamically uncorrelated", zeroth order, wave function. If the average nuclear density has a depression around the center, it may be due to repulsive-core type correlations among the nucleons, but its effects, say on electron scattering, can be sometimes "buried" in the Pauli correlations of a suitably chosen shell-model wave function derived from a "wine-bottle" potential.

It becomes apparent from the comments made above that for the establishment of dynamical correlations which go beyond those expressible in terms of the Pauli correlations one should design the experiment in a special manner. Ideally the probe should interact only with one nucleon, transferring to it a certain momentum, and one should then look for the sharing of this momentum between two, or more, nucleons. Typical reactions of this type are (γ, pn) , (γ, d) , $(e, e'2p)$ etc., but little experimental information is available at this stage. The theoretical studies of such reactions are also far from being complete. For short-range dynamical correlations one is interested in the interaction of high-energy photons or electrons with complex nuclei; these may leave the nucleus in a highly excited state whose lifetime is comparable to the "ejection time" of the nucleons; the interaction of the residual nucleus with the knocked-out nucleons during their ejection time should be therefore well understood in order to have a complete description of the process.

An alternative approach to the problem is that of comparing different processes in the same nucleus. The simplest case is that of He^4 which, in a sense, has no Pauli correlations because all its four particles are different from each other. One can then consider the elastic electron scattering as a direct determination of the single-particle density $\rho_1(r)$, and in a naive approach take the four-particle density to be

$$\rho_4(r_1, r_2, r_3, r_4) = \prod_{i=1}^4 \rho_1(r_i).$$

With such an ansatz for the four-particle density one can then proceed⁹ to study high-energy multiple proton scattering using the Glauber

approximation, and ascribing discrepancies between "theory" and experiment to irreducible correlations in $\rho_4(r_1, r_2, r_3, r_4)$ which make it impossible to express ρ_4 as a product of four single-particle densities. Such an analysis, however, suffers at this stage from well-known complexities of handling the center-of-mass motion and, more seriously, from the various limitations on the validity of the Glauber approximation.

It seems that further studies are required, both experimental and theoretical, to be able to pin down the exact nature of short-range dynamical correlations in finite nuclei. On the theoretical side one would want to have a better estimate of the range of validity of multiple scattering and various high-energy reaction theories; experimentally one would want then more versatile results with such probes and in such regions of energies and momenta for which the theory is reliable. That such studies are worthwhile is evident from the fact that they touch on some of the most fundamental aspects of nuclear structure. The Hartree-Fock approximation, for instance, can provide us the answer to the question as to whether the *free* nucleon-nucleon interaction is also the dominant factor determining the structure of complex nuclei. But the convergence of the Hartree-Fock method can be ascertained only to the extent that the effects of the dynamical correlations, beyond the Pauli correlations, can be properly estimated. An experimental determination of the extent to which such correlations exist in nuclei is thus of great importance.

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References

1. A. de-Shalit, *Comments Nuclear Particle Phys.* **3**, 42 (1969).
2. R. F. Frosch *et al.*, *Phys. Rev.* **160**, 874 (1967).
3. C. Ciofi degli Atti, preprint ISS 68/31, and to be published in the *Physical Review*.
4. H. A. Bethe and L. R. B. Elton, *Phys. Rev. Letters* **20**, 745 (1968).
5. See, for instance, D. B. Isabelle in the *Proceedings of the Second International Conference on High Energy Physics and Nuclear Structure, Rehovot, 1967*, edited by G. Alexander (North Holland Publishing Co., Amsterdam, 1967), p. 126.
6. F. G. Perey, *Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962*, edited by E. Clementel and C. Villi (Gordon and Breach, New York, 1963).
7. W. Czyz and K. Gottfried, *Ann. Phys. (N.Y.)* **21**, 47 (1963).
8. A. S. Reiner, *Phys. Rev.* **138**, B389 (1965).
9. W. Czyz and L. Lesniak, *Phys. Letters* **25B**, 319 (1967).