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The progress of mini black holes: principles and analytical astronomical observation techniques

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Abstract. Mini-black hole (MBH) is a concept first proposed by Stephen Hawking in the 1970s. Normally, exploring MBHs will enhance the understanding of quantum theory and gravity theory as well as be helpful in predicting the configuration of the early universe. Based on information retrieval, this paper summarizes the progress of MBHs and takes three major aspects: background, models, practical methods for observations, and analysis. Specifically, the descriptive equations are derived, and different models are discussed separately. These results shed light on the prospective development of quantum field theorem, general relativity, and string theory.

Keywords: mini-black hole, evaporation, lifespan, gamma-ray burst, observation

1. Introduction

Mini-black hole (MBH) is a hypothetical model first developed in Ref. [1], which is a type of black hole with a much smaller mass (approximately 10^{11}kg) than the normal black holes detected by human beings in the current universe. Generally, the formation of a black hole is the gravitational collapse of a star. As the size of a star becomes smaller during the collapse, particles within the star become closer. According to the Pauli exclusion principle, those fermionic particles near the same location must have velocities with great differences, leading to a repulsion (also known as degeneracy pressure) against gravity. However, the speed of a particle cannot be greater than light speed, which means there is a maximum pressure due to the principle. As a consequence, a star must have sufficiently great mass to form a black hole. Another way for a black hole's formation is to accrete matter onto a white dwarf or a neutron star since the gravity increases when the white dwarf or neutron star gains mass. Each of them has a threshold value (a minimum) of mass for the final collapse to occur, known as Chandrasekhar limit and Oppenheimer limit, respectively (listed in Table 1).

Table 1. The minimum value of mass for the two kinds of stars to collapse and types of pressure overwhelmed during gravitational collapse.

Type of star[a]	The threshold value of mass/$M_{\odot}$	Origin of degeneracy pressure
White dwarf	1.46[b]	Electron
Neutron star	2.17[c]	Neutron

[a]Notes these values are for non-spinning stars. Those spinning stars have a bit greater threshold value of mass due to the centrifugal force in the rotational reference frame against gravity.

[b]The data is collected from Ref. [2]

[c]The data is collected from Ref. [3]

Based on the above data, there is no way to create an MBH with such a small mass in a present universe in theory. Hence, the only possibility for MBHs' existence is to create them at the early time of the universe. With great density perturbation, it provides an extremely high temperature and pressure to overcome the repulsion and compress the matter into such an enormous density during this period only. Therefore, investigating the MBH is very likely to help us find a 'boundary condition' at the beginning of spacetime. In other words, it will augment our knowledge about the initial state and configuration of our universe. Due to the radiation with tremendous power at the end of an MBH's life, it also provides a new possibility for the source of gamma-ray burst.

In general relativity, the evaporation of a black hole occurs in three stages: (1) 'balding', (2) spin-down, (3) Hawking evaporation [4]. Nevertheless, this model cannot be used to accurately describe an MBH when its mass becomes too small that the quantum effects have to be considered. Contemporarily, the Planck stage (fourth stage) will be reached universally accepted based on the idea of quantum gravity that the mass of a black hole approaches the Planck scale during evaporation [4]. Theoretically, MBHs formed in the early universe can still be detected by observing the background of gamma rays. However, they are extremely scarce that the gamma rays released are too weak to be detected experimentally when arriving in the solar system. Besides, other astronomical events also result in the emission of much stronger gamma rays, as exemplified by mergers of binary neutron stars or a neutron star and a black hole [5]. Those events become a source of interference factors for the detection of MBHs. Nevertheless, it is now generally believed that MBHs can be created, though they evaporate rapidly, during the hadronic collisions. Thus, detecting MBHs are notionally available on the Earth.

The rest of the paper is organized as follows. In Sec. 2, the significance and inherent correlation between models and formulae for MBHs that fit the observation well will be discussed. Subsequently, Sec. 3 presents some new methods for observation and analysis in recent progress. Finally, a summary is given, and the prospect of study for MBH is demonstrated in Sec. 4.

2. Analytical models for MBHs

2.1. Four stages of evaporation

As is described in the previous section, there are four stages in an MBH's lifespan. In the first stage (balding), the black hole loses its multipole momenta and quantum numbers via the emission of gauge bosons until it reaches the Kerr solution for a spinning black hole. The second stage (spin-down) gets rid of the residual angular momentum and becomes a Schwarzschild black hole. Further numerical simulations indicate that 75% of angular momentum and about 25% of mass are radiated away [6]. In this case, more than half of the mass is emitted after the black hole has reached a non-rotating configuration [7]. In the third stage (Hawking radiation), it decays via emission of black-body radiation, also known as Hawking radiation [8], with a characteristic Hawking temperature-dependent only on its mass:

$$T_H = \frac{hc^3}{16\pi^2 k_B G M} \tag{1}$$

where h is Planck's constant, c is the speed of light in vacuum, k_B is Boltzmann constant, G is gravitational constant, and M is mass of the black hole. Details of the fourth stage (Planck stage) are completely unknown since it involves quantum gravity. Plenty of scholars have proposed some speculations about this stage, but none of them is possible, given our lack of theories about quantum gravity.

2.2. Model estimation of MBH's lifespan

Among the four stages, the third stage, Hawking radiation, is considered as the dominant stage. Thus, many papers have been devoted to investigating black hole evaporation in higher dimensions, resulting in a better understanding of the third stage [9]. Additionally, black hole evaporation occurs primarily in three spatial dimensions [4]. According to Refs. [10, 11], the spin of an MBH is expected to be negligible [12]. Therefore, one shall regard an MBH as a Schwarzschild (non-spinning) black hole and can use semiclassical approximation to describe MBH. Considering Stefan-Boltzmann law in three dimensions:

$$P = A\sigma T_H^4. \tag{2}$$

where P is the black-body radiation power, A is the surface area of MBH, T_H is its Hawking temperature, σ is Stefan-Boltzmann constant, it can be further expressed into

$$\sigma = \frac{2\pi^5 k_B^4}{15\ \ {}^2h^3}. \tag{3}$$

By treating the MBH as a sphere, one can express the surface area of a black hole in terms of its Schwarzschild radius: $A = 4\pi r_S^2$, where r_S is related to the present mass of the black hole by

$$r_S = \frac{2G}{c^2}. \tag{4}$$

Substitute (1) (3) (4) into (2), the power radiated by a Schwarzschild black hole can be expressed in terms of the present mass of the black hole:

$$P = \frac{hc^6}{30720\ \ {}^2G^2M^2}. \tag{5}$$

For simplicity, set $\mu = \frac{hc^4}{1024\ \ {}^2G^2M_0^2}$ one obtains

$$P = \frac{c^2}{3}\mu(\frac{M}{M_0})^{-2} \tag{6}$$

According to Stephen Hawking's statement, if the tidal force around a black hole is large enough, the transient virtual particle pairs, in which one of the partners has positive energy and the other one has negative energy, produced by fluctuation of the field just beyond the event horizon will be pulled apart before annihilation occurs. Thus, a flow of negative energy particles into the black hole diminishes its mass, and particles with positive energy will be released to conserve energy as [1].

As a result, the power will be related to the rate of decrease in mass of the black hole by mass-energy equivalence formula:

$$P = -\frac{dM}{dt}c^2 = \frac{c^2}{3}\mu(\frac{M}{M_0})^{-2}. \tag{7}$$

Solving the differential equation with $M = M_0$ at $t = 0$, where M_0 is the initial mass of the black hole, one derives the mass of a black hole in terms of time after the creation:

$$M = \sqrt[3]{-\mu M_0^2 t + M_0^3}. \tag{8}$$

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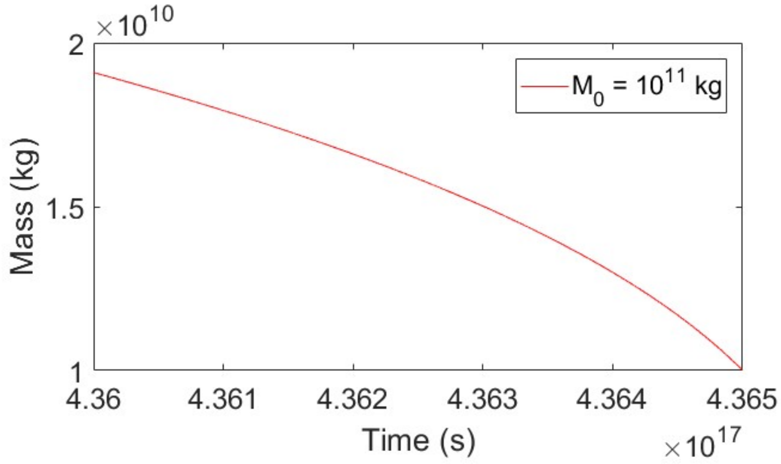


Figure 1. The mass-time relationship of a black hole with an approximate lifespan is similar to the age of the universe. Notice that near the end of its life (the zero-point on the graph), the approximation is no longer valid because the black hole has reached Planck mass, which means the last short section of the vertical part should be ignored.

Since the Planck mass is extremely small, one can approximately express the lifespan of a black hole in terms of M_0 by taking $M = 0$ and $t = \tau$:

$$\tau = \frac{M_0}{\mu}. \tag{9}$$

On this basis, a conclusion can be drawn in line with Ref. [13] that an MBH with an initial mass of about $10^{11}kg$ would have a lifespan as same as the age of our universe by setting $\tau = 4.358 \times 10^{17}s$ and find M_0 (see Figure 1). It is also noticeable that this model is only possible for MBHs with a rather high temperature because those with enormous mass have much smaller Hawking temperature than the temperature of the microwave radiation that fits the universe (about $2.7K$). As the black hole is not a perfect black body, it can still absorb extra radiation from the universe. Hence, these huge black holes will absorb more than emitting and gain mass until their temperature exceeds the cosmological temperature. Even if it is the case, the following evaporation process will still last much longer than the universe's age. Moreover, another brilliant method is to estimate the lifespan of a Schwarzschild black hole using Stefan's law of radiation in three dimensions according to [4].

2.3. New source for gamma ray burst (GRB)

In Ref. [14], David B. Cline pointed out that the abundance of MBHs is not inconsistent with the fluctuation density in the early universe, especially near the time of the quark-gluon phase transition. Therefore, it is still worthwhile searching for evidence for their existence. Following Wien's law, the wavelength of the radiation of an MBH reaches the peak at a certain value that depends only on its Hawking temperature:

$$\lambda_{max} T_H = b. \tag{10}$$

where b is Wien's constant. By substituting Eqs. (1) and (10) together, one can easily conclude qualitatively that the wavelength becomes fairly small when the MBH becomes very tiny (but still much larger than Planck scale), resulting in gamma radiation with almost infinite energy. Quantitatively, if one sets $\lambda_{max} = 10^{-10}m$ and calculates corresponding values of T_H and M_0, the threshold value of mass for a black hole to emit gamma rays is much larger than Planck mass. Additionally, the MBH will emit radiation with enormous power if considering Eq. (6) quantitatively when reaching the last part of Hawking stage but still being far away from the Planck stage.

It is now generally believed that the GRBs are classified into three types according to their bulk properties: duration, fluency, and spectrum. The methods for this taxonomy had been explained statistically based on data analysis in Ref. [15]. Here, the duration of evaporation (explosion) nearly at the end of the MBH with a lifespan similar to the age of our universe will be considered mathematically. By substituting Eq. (8) into Eq. (7), one derives the power in terms of time:

$$P = \frac{\mu c^2}{3}\left(-\frac{\mu t}{M_0} + 1\right)^{-\frac{2}{3}}. \tag{11}$$

According to Kouveliotou's suggestion in 1993 [15], the bimodality in the burst duration variable T_{90} (time taken for 90% of the flux to arrive) is noticeable, proposing the presence of two distinct types of gamma ray bursts separated by a threshold value of $T_{90} \cong 2s$. Hence, determining whether the MBH evaporation causes the gamma ray bursts is to calculate the time taken for an MBH to lose 90% of its present (remaining) mass via Hawking radiation. By taking integration from t_0 to $t_0 + T_{90}$, and equating it with 90% of the remaining mass-energy of the MBH, one can derive an expression of T_{90} in terms of the present mass of the MBH, independent on t_0.

$$\int_{t_0}^{t_0+T_{90}} \frac{\mu c^2}{3}\left(-\frac{\mu t}{M_0} + 1\right)^{-\frac{2}{3}} dt = 0.9Mc^2 \tag{12}$$

After reaching the final equation, substitute (8) into the terms for elimination, one can obtain:

$$T_{90} = \frac{0.999M^3}{\mu M_0^2} \tag{13}$$

If setting $M = 10^{-6}M_0$, which is a significantly larger mass than the Planck mass, T_{90} at the end of its life can be computed. The numerical value is about $0.4354s$ which is smaller than the threshold value of $2s$. This result indicates that the evaporation of an MBH during the last part of its life could be responsible for the short gamma ray burst taking place in the universe.

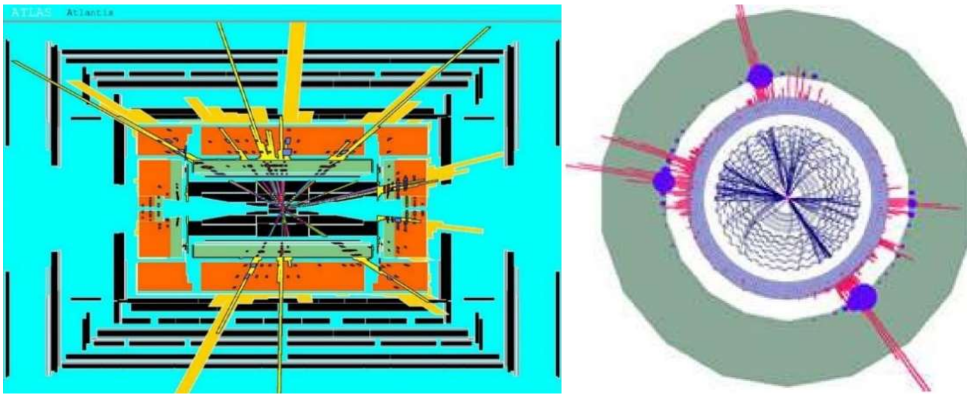
3. Observational and analytical techniques in practice

According to the analysis above, the detection of MBHs can be conducted by observing the background of gamma rays. The truth is physicists do have found such a background: there mustn't be more than 300 MBHs per cubic light-year, which may lead to a conclusion that only if the early universe was homogeneous can the fact that there are not a considerable number of MBHs be demonstrated. Nowadays, many papers have analyzed the experiment and observation for MBHs on the Earth. The most important source for MBH's creation is the Large Hadron Collider (LHC) in CERN. In order to study the properties of an MBH in the collider, the simulation tools are of great importance [4]. In Refs. [22] and [23], two Monte Carlo generators capable of doing this: TRUENOIR and CHARYBDIS, are discussed. Both generators have been successfully interfaced with the LHC detector simulation packages. Figure 2(a) and (b) show simulations of a black hole event in the ATLAS and CMS detectors, respectively [4].

However, since most of the black holes produced at the LHC would carry an electric charge and baryon number, it is vital to have a complementary way of producing a completely neutral black hole of fixed mass. An e^+e^- machine with sufficient energy would offer such possibility. The only possible way is two-beam acceleration that reaches energy of 3-5 TeV.[]

Nowadays, the detections of gravitational waves due to the black hole mergers are feasible for LIGO/Virgo Collaboration, [12], which encourages physicists to comprehend a binary black hole population [8]. Considering the Bayesian inference, one possibility for the formation is offered by the

MBH scenario. The statistical model proposed in [12] provides strong evidence for the existence of populations of black hole mergers of this origin (illustrated in Figure 3 [19]). Additionally, a hierarchical Bayesian analysis on the GWTC-2 catalogue has been performed recently in [15], which discarded those confusing events that challenge our understanding but still provided evidence for the existence of MBHs.



(a) (b)

Figure 2. (a) A black hole event generated with CHARYBDIS, seen by the ATLAS detector. (b) A black hole event was generated with TRUENOIR, seen by the CMS detector [4].

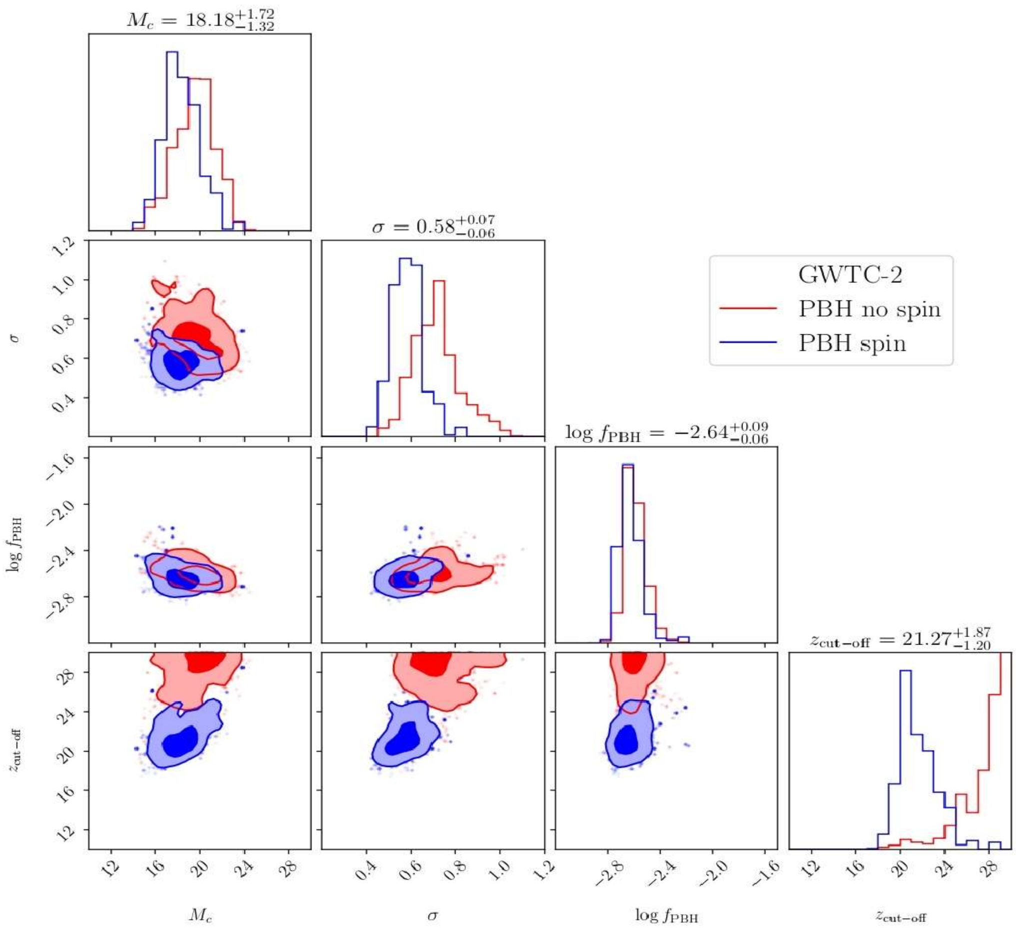


Figure 3. PBH population inference using the GWTC-2 catalogue. The marginalised values on top of the plots report the 90% confidence interval [19].

4. Conclusion

In summary, this paper introduces the progress of MBHs in principles and corresponding analytical techniques. Specifically, the background and current circumstances of MBH research are considered, followed by some analytical models with detailed estimation. Moreover, the most advanced methods for the detection and analysis of MBHs are discussed. Nonetheless, the MBH model is still a conjecture nowadays unless an innovatively effective observation way is discovered. These results offer a guideline for further development in astrophysics as well as cosmology.

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