Constrain on Brans-Dicke Cosmology from view point of Cosmic age

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Abstract

Recent observational bounds taken into consideration, I investigated cosmology of Brans-Dicke gravity. Brans-Dicke scalar field at high $\omega_{\rm BD}$ behaves as stiff fluid, And it makes cosmic age short. But, cosmic age is constrained by observation of globular clusters. This paper shows constraint on Brans-Dicke cosmology from view point of cosmic age.

1 introduction

g Is the constants of nature truly constantHh This is one of the radical questions for nature. Especially, varying gravitational constant G is interesting from viewpoint of cosmology, because it decides strength of the gravity which is dominant in macro scale.

2 Brans-Dicke gravity theory

In BD theories, the coupling of a massless scalar field to the Ricci scalar provides a natural framework of realizing the time-variation of the gravitational constant via the dynamics of the scalar field. In the Brans-Dicke theory a constant coupling parameter $\omega_{\rm BD}$ is introduced. In the limit $\omega_{\rm BD}$, the gravitational constant cannot change and Einstein gravity is recovered. The issue that historically influenced the pioneers of the scalar-tensor theory strongly is Mach's principal. It is often said that Brans-Dicke theory, but not general relativity, satisfies Mach's principle.

$$S_{\rm BD} = \int d^4x\sqrt{-g}\left[\frac{c^4}{16\pi G}[\phi R - \omega_{\rm BD}\frac{(\nabla\phi)^2}{\phi}] + \mathcal{L}_{\rm matter}\right] \tag{1}$$

$$G_{\mu\nu} = \frac{8\pi G}{c^4\phi}T^{\rm matter}_{\mu\nu} + T^{(\phi)}_{\mu\nu} \tag{2}$$

$$T^{(\phi)}_{\mu\nu} = \frac{\omega_{\rm BD}}{\phi^2}\left[\phi_{;\mu}\phi_{;\nu} - \frac{1}{2}g_{\mu\nu}\phi_{;\alpha}\phi^{\alpha}_{;}\right] + \frac{1}{\phi}\left[\phi_{;\mu}\phi_{;\nu} - g_{\mu\nu}\Box\phi\right] \tag{3}$$

$$\Box\phi = \frac{8\pi G}{c^4(3+2\omega_{\rm BD})}T^{\mu\rm matter}_{\mu} \tag{4}$$

3 Constraint of BD theory

But, idea of varying G and non-Einstein gravity theory are limited by variety of astronomical measurements in recent years . Varing G is constrained by lunar laser ranging experiment. the newest limit is this value: $(\dot{G}/G)|_{\rm now} = (4\pm 9)\times 10^{-13}$/yr.(William et al.2004)[4]

The value of $\omega_{\rm BD}$ consistent with experiment has risen with time. In 2003 evidence - derived from the Cassini-Huygens experiment- shows that the value of $\omega_{\rm BD}$ must exceed 40000. this value is measured by Shapiro's delay.(Bertotti et al.2003)[5];(Berti et al.2005) [6]

Recent observational bounds taken into consideration, I investigated Brans-Dicke cosmology.

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4 Basic Equation of BD cosmology

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3\phi}\,\rho - \frac{kc^2}{a^2} - \frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{\omega_{\rm BD}}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{5}$$

$$\ddot{\phi} = -3\frac{\dot{a}}{a}\dot{\phi} + \frac{-\rho c^2 + 3P}{c^2(3+2\omega_{\rm BD})} \tag{6}$$

These are Friedman equation of BD theory. Considering equation of state and indeed following parameters.

- present cosmic expantion rate(Hubble constant):$\frac{\dot{a}}{a} = H_0$
- present radiation density:ρ_{r0}
- present dust matter densityρ_{m0}
- present dark energy density:$\rho_{\Lambda 0}$

then rewrite equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left(\frac{8\pi G}{3H_0^2\phi}\,(\frac{\rho_{r0}}{a^4} + \frac{\rho_{m0}}{a^3} + \rho_{\Lambda 0}) - \frac{kc^2}{H_0^2 a^2} + \frac{\omega_{\rm BD}}{6}(\frac{\dot{\phi}/\phi}{H_0})^2 - \frac{\dot{a}}{a}\frac{\dot{\phi}/\phi}{H_0^2}\right) \tag{7}$$

Defined that:

- present radiation density(normalized):$\frac{8\pi G\rho_{r0}}{3H_0^2} \equiv \Omega_{r0}$
- present dust matter density(normalized)$\frac{8\pi G\rho_{m0}}{3H_0^2} \equiv \Omega_{m0}$
- present dark energy density(normalized):$\frac{8\pi G\rho_{m0}}{3H_0^2} \equiv \Omega_{\Lambda 0}$
- normalized curvuture :$-\frac{kc^2}{H_0^2} \equiv \Omega_{k0}$

then,

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2(\frac{\Omega_{r0}}{\phi a^4} + \frac{\Omega_{m0}}{\phi a^3} + \frac{\Omega_{k0}}{a^2} + \frac{\Omega_{\Lambda 0}}{\phi} + \frac{\omega_{\rm BD}}{6}(\frac{\dot{\phi}/\phi}{H_0})^2 - \frac{\dot{a}}{a}\frac{\dot{\phi}/\phi}{H_0^2} \tag{8}$$

$$\ddot{\phi} = -3\frac{\dot{a}}{a}\dot{\phi} - \frac{H_0}{(3+2\omega_{\rm BD})}(\frac{\Omega_{m0}}{a^3} + 4\Omega_{\Lambda 0}) \tag{9}$$

These are normalized Friedman equation of BD theory. And,defined $\Omega_{\dot{\phi}}(t)$ conveniently:

$$\Omega_{\dot{\phi}}(t) = \frac{\omega_{\rm BD}}{6}(\frac{\dot{\phi}/\phi}{H_0})^2 - \frac{\dot{a}}{a}\frac{\dot{\phi}/\phi}{H_0^2} \tag{10}$$

5 BD scalar field behaves as stiff fluid

Brans-Dicke Scalar field at high omega behave as stiff fluid(w=1). This is un-normalized BD Friedman equation.Now,derivate vacuum solution.

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{\omega_{\rm BD}}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{11}$$

$$\ddot{\phi} = -3\frac{\dot{a}}{a}\dot{\phi} \tag{12}$$

and,following is the solution:

$$\phi \propto t^{\alpha}, a \propto t^{\frac{\alpha+1}{3}} \tag{13}$$

where,$\alpha = \frac{1}{1 \pm \sqrt{9+6\omega_{\rm BD}}}$. As compared to relation between equation of state and cosmic expansion solution:$a \propto t^{2/3(1+w)}$ then,we can discribe w parameter of BD scalar field as following.

$$w = \frac{2}{\alpha + 1} - 1 \tag{14}$$

If $\omega_{\rm BD} \longrightarrow$ very large,then $\alpha \longrightarrow 0$,$w \longrightarrow 1$.This result is construed the following.

General free(no potential,massless) scalar field's equaiton of states shows that Pressure = Energy density. Parameter of equation state is 1. Such imaginary matter is called stiff fluid. These solutions are known from old days(Mark S.Madsen 1985)[7](Mark S.Madsen 1988)[8]. Expressing BD scalar field and matter eqation again,

$$\Box\phi = \frac{8\pi G}{c^4(3 + 2\omega_{\rm BD})} T_{\mu}^{\mu \rm matter} \tag{15}$$

If $\omega_{\rm BD}$ is very large, coupling between matter and scalar field decoupled. then BD scalar field behaves as free scalar field.

6 BD cosmology from veiw point of cosmic age

Brans-Dicke scalar field behave as stiff fulid,And it make cosmic age short. But, cosmic age is constrained by observation of globular clusters. Lower limit of cosmic age is surmised thirteen giga years.(Krauss et al.2003)[9] It is desirable that BD scalar field's energy density is small enough. (You may think that if the nature adopt BD gravity, Varying G effect on development of star. But Recent observational bounds taken into consideration,varying of G is very small except only early universe.) I investigated BD cosmic solutions which allowed from recent observational bounds(:$\omega_{\rm BD} > 40000$). This picture show cosmic age of BD Λ CDM@model(Fig. 1).

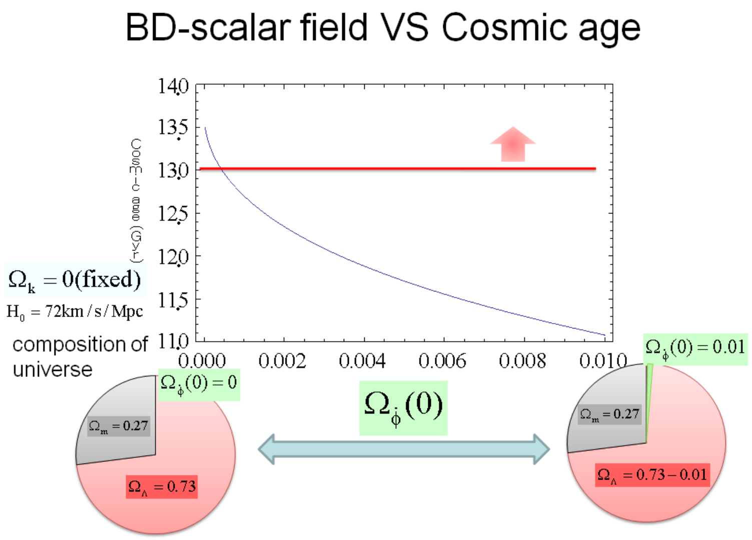

Figure 1: Relation between cosmic age and Scalar field density

If there were a little scalar field , cosmic age become short. And it become less than thirteen giga years.

Notice $\Omega_{\dot\phi}(0) = 0$ model. That mean $\dot{G}/G|_{\rm now}$ is zero exactly. And composition of universe is equivalent to standard model in GR. But Cosmic age is shorter 0.2 Gyr than standard cosmology. BD cosmology at high $\omega_{\rm BD}$ resemble cosmology with stiff fluid in GR. But The former does not quite correspond to the later.

7 Conclusion

BD scalar field's energy density must be less than 0.001.If $\omega_{\rm BD} > 40000$ then $\dot{G}/G|_{\rm now}$ is less than 10^{-14}. This is more strict limit value at one column than the lunar laser ranging experiment But this limit value suppose that varying G is caused by BD gravity, and this is model-depanding constraint value.

8 Supplement

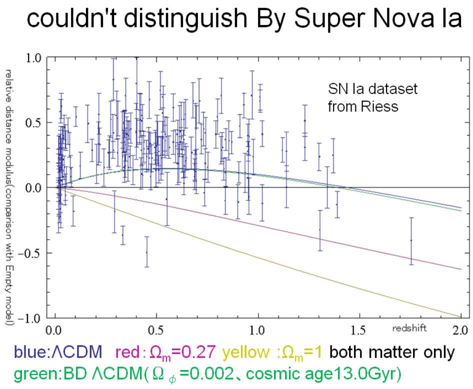

Figure 2: This is magnitude-redshift relation. blue line is ΛCDM model in GR. green line is BD ΛCDM model which is narrowly allowed from view point of cosmic age. we can't distinguish two model at low z region which is observed by type Ia supurnove.

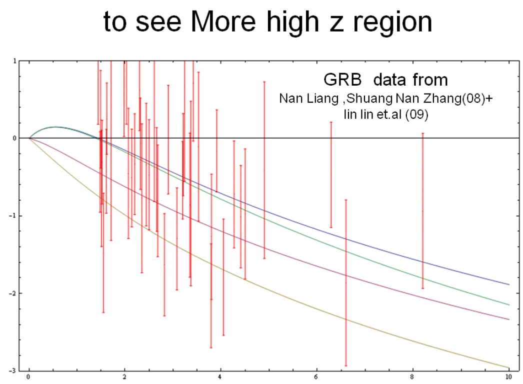

Figure 3: But at more high z region, we can distinguish two model. this sudden drop is peculiarity of BD model. high z region is observed by gammar lay burst. But ,GRB observation is not precise yet. Errer bar is too long to discuss.

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