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Review

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Recent Measurements of Decay Asymmetry Parameter and *CP* Asymmetry for Charmed Baryon Decays at Belle

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Abstract: We review the recent results from the Belle experiment on the measurements of branching fractions and decay asymmetry parameters (α) for the hadronic weak decays of charmed baryons, including $\Lambda_c^+ \to \mathbf{B} + P$: $\Lambda_c^+ \to (\Lambda, \Sigma^0)\, h^+$ ($h = \pi, K$) and $\Lambda_c^+ \to \Sigma^+(\pi^0, \eta, \eta')$; $\Xi_c^0 \to \mathbf{B} + P$: $\Xi_c^0 \to \Xi^-\pi^+$; $\Xi_c^0 \to \mathbf{B} + V$: $\Xi_c^0 \to \Lambda\bar{K}^{*0}$, $\Xi_c^0 \to \Sigma^0\bar{K}^{*0}$, and $\Xi_c^0 \to \Sigma^+K^{*-}$. In addition, we present an overview of the searches for *CP* violation via the α-induced *CP* asymmetry for $\Lambda_c^+ \to (\Lambda, \Sigma^0)h^+$ and $\Xi_c^0 \to \Xi^-\pi^+$. Finally, we discuss the promising decay modes of Ω_c^0 which can be measured in the near future and are indispensable in searching for *CP* violation in the charm sector.

Keywords: charmed baryon; weak decay; asymmetry parameter; *CP* violation; Belle experiment

1. Introduction

The study of the hadronic weak decays of baryons can provide an excellent understanding of the baryon decay dynamics and the matter–antimatter asymmetry [1,2]. However, both experimental and theoretical progress in the research field has been slow for a long time [3]. More than two decades ago, a general formulation of the topological-diagram scheme for the nonleptonic weak decays of baryons had been proposed [4]. In the early 1990s, theoretical research in this field reached its peak, and then gradually faded out of people's vision. Up to now, unlike the heavy-flavored meson decays described well by quantum chromodynamics, the complicated decay mechanism of heavy-flavored baryon decays is hard to be described well by any theoretical models [5]. Experimentally, studies of charmed baryon decays are more challenging than those of charmed mesons due to lower production rates. Nevertheless, in the past two decades, as the collision data sets increase, many new excited charmed baryon states have been discovered by BaBar, Belle, CLEO, and LHCb [3]. It is even more encouraging that the absolute branching fractions of the reference modes of charmed baryons Λ_c^+ and $\Xi_c^{+,0}$ have been measured in the Belle experiment [6–8]. These decay branching fractions are useful, since most of the other hadronic weak decay branching fractions of SU (3) anti-triplet Λ_c^+ and $\Xi_c^{+,0}$ charmed baryons are measured relative to them. Based on the results of the absolute branching fractions of $\Lambda_c^+ \to pK^-\pi^+$ and $\Xi_c^0 \to \Xi^-\pi^+$, the measurements of Λ_c^+ and Ξ_c^0 decay modes were reported recently.

Charge–parity (*CP*) violation is one of the key factors required to explain the matter–antimatter asymmetry of the universe [2]. The single complex phase in the Cabibbo–Kobayashi–Maskawa matrix provides a source of *CP* violation (CPV) in the Standard Model (SM). CPV investigation is also being carried out in the lepton sector in addition to the quark sector. However, this is not large enough to explain the observed matter–antimatter asymmetry. Baryogenesis, the process by which the baryon–antibaryon asymmetry of the

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universe developed, is directly related to baryon CPV [9,10]. The direct *CP* asymmetry, taking $\Lambda_c^+ \to f$ decays as an example, is defined as

$$\mathcal{A}_{CP}^{\text{dir}} = \frac{\Gamma(\Lambda_c^+ \to f) - \Gamma(\bar{\Lambda}_c^- \to \bar{f})}{\Gamma(\Lambda_c^+ \to f) + \Gamma(\bar{\Lambda}_c^- \to \bar{f})}. \quad (1)$$

Here, the partial decay widths are denoted by $\Gamma(\Lambda_c^+ \to f)$ and $\Gamma(\bar{\Lambda}_c^- \to \bar{f})$, while f and $\bar{f}$ are the final states for Λ_c^+ and $\bar{\Lambda}_c^-$, respectively. To date, CPV has been observed in the open-flavored meson sector (i.e., K, D, and B mesons) but not yet established in the baryon sector. Therefore, searching for CPV in the charmed baryon sector is one of the most important research directions in particle physics at experiments, although only a few theoretical papers have reported phenomenology studies for CPV in charmed baryon decays [11–14]. Theoretical CPV predictions in two-body decays are more straightforward than in multi-body decays, which are complicated due to plentiful intermediate processes. Furthermore, it is recognizable in both theories and experiments that CPV can manifest as an asymmetry of the decay asymmetry parameters in the two-body charmed baryon decays and its charge conjugation. The decay asymmetry parameter α was introduced by Lee and Yang to study the PV and parity-conserving (PC) amplitudes in weak hyperon decays [15]. For Λ_c^+ weakly decaying into a baryon with spin 1/2 and positive parity plus a meson with spin 0 and negative parity, the formula for the decay asymmetry parameter (α) is

$$\alpha = \frac{2 \cdot Re(S^* P)}{(|S|^2 + |P|^2)}, \quad (2)$$

where S and P are the PV S-wave and PC P-wave amplitudes in the decay, respectively. Since α is *CP*-odd, the α-induced *CP* asymmetry for Λ_c^+ decays is

$$\mathcal{A}_{CP}^{\alpha} \equiv \frac{\alpha_{\Lambda_c^+} + \alpha_{\bar{\Lambda}_c^-}}{\alpha_{\Lambda_c^+} - \alpha_{\bar{\Lambda}_c^-}}, \quad (3)$$

where $\alpha_{\Lambda_c^+}$ and $\alpha_{\bar{\Lambda}_c^-}$ are the decay asymmetry parameters for Λ_c^+ and $\bar{\Lambda}_c^-$, respectively. As mentioned above, the two-body decays of charmed baryons are sensitive to the *CP* asymmetry due to the charm quark, but theoretical and experimental studies are scarce. Most of the model predictions were reported nearly 30 years ago, and most of the experimental measurements need to be updated [5]. According to Equations (1) and (3), the precise measurements of the decay branching fractions and the decay asymmetry parameters can be used to search for CPV. Meanwhile, since CPV in charm decays is predicted in the SM to be very small [16–18], an observation of CPV in charm decays larger than 10^{-3} will indicate new physics beyond the SM [19,20].

Recently, Belle reported a number of experimental results on the decay asymmetry parameters and searching for CPV through different charmed baryon decays [21–24]. Different theoretical models predicted a variety of values for the decay asymmetry parameters of the charmed baryons. Therefore, this review paper is mainly to summarize the latest measurements and compare them with different theoretical predictions. Meanwhile, more measurements of charmed baryon decays are reminded, which may provide valuable insights for researchers in the future.

This paper is organized as follows: Section 2 briefly describes the Belle detector. Sections 3 and 4 review the recent results of the decay asymmetry parameters and *CP* asymmetry in the Λ_c^+ and Ξ_c^0 decays, respectively. The promising decay modes of Ω_c^0 are discussed in Section 5. Finally, the summary and prospects are presented in Section 6.

2. Belle Experiment

The Belle experiment ran at the KEKB energy-asymmetric collider [25,26]. As the sole detector operating at KEKB, Belle detector [27,28] is a large-solid-angle magnetic spectrometer consisting of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter (ECL) comprising CsI(Tl) crystals located inside a superconducting solenoid coil providing a 1.5 T magnetic field. The iron flux-return of the magnet is instrumented to detect K_L^0 mesons and to identify muons (KLM). A detailed description of the detector is given in Ref. [27]. These subdetectors make Belle be a detector with advanced performances on momentum and vertex resolution, particle identification, etc. The operation of KEKB was started in 1998 and stopped in 2010. Therefore, Belle accumulated its final data set more than ten years ago. This data set provides us a large charm sample to study charm physics by both B decays and the $e^+e^- \to c\bar{c}$ process. In this paper, we review some recent charmed baryon results at Belle. Now, KEKB has been upgraded to SuperKEKB [29] with higher performance. The data sample to be accumulated in Belle II [30] will be 50 times that of Belle. With the huge sample, we can measure more branching fractions and decay asymmetry parameters of charmed baryons with significantly improved accuracy in the future.

3. Studies of Λ_c^+ Decays

Measuring decay parameters of Λ_c^+ plays an important role in understanding the entire field of charmed baryons. Since the discovery of Λ_c^+, many phenomenological models have reported the decay branching fractions of its hadronic weak decays including the flavor symmetry model [31], factorization model [32], pole model (Pole) [33], and current algebra (CA) [34]. In 2014, the model independent absolute branching fraction of $\Lambda_c^+ \to pK^-\pi^+$ was measured to be $(6.84 \pm 0.24^{+0.21}_{-0.27})\%$ by Belle [6]. Subsequently, the value of $\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$ was measured to be $(5.84 \pm 0.27 \pm 0.23)\%$ based on a e^+e^- collision data sample of 567 pb^{-1} at $\sqrt{s} = 4.599$ GeV at BESIII [35]. The average result of $\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$ from Belle and BESIII measurements is $(6.28 \pm 0.32)\%$ [36]. Based on this Λ_c^+ absolute branching fraction, in recent years, Belle has studied a series of Λ_c^+ decay modes [37–43].

In 2019, BESIII first reported four Λ_c^+ Cabibbo-favored (CF) decay asymmetry parameters in unpolarized e^+e^- collisions [44]. Two of these parameters are measured experimentally for the first time, and the other two have improved precision [44–47]. Belle detector has accumulated a huge amount of sample for $B\bar{B}$ pairs and $c\bar{c}$ pairs, which provide abundant charmed baryons. Based on the huge statistics of charmed baryons, researchers have enough reasons to believe that measuring more Λ_c^+ decay mode asymmetry parameters can also be achieved at Belle. Recently, Belle reported some research on Λ_c^+ two-body hadronic weak decay asymmetry parameters and CPV. The triplet charmed baryon ($\mathbf{B_c}$) often decays weakly into one octet baryon ($\mathbf{B}$) and a pseudoscalar (P) or vector (V) meson.

3.1. Branching Fractions and Decay Asymmetry Parameters in $\Lambda_c^+ \to \mathbf{B} + P$ Decays

The various model predictions of the branching fractions and decay asymmetry parameters of $\Lambda_c^+ \to \mathbf{B} + P$ decays are listed in Tables 1 and 2. Most of the model predictions, except for the columns under "Zou" and "Geng", were reported nearly 30 years ago. In addition, most of the experimental measurements need to be updated. Furthermore, there are only four experimental measurements for the decay asymmetry parameters of $\Lambda_c^+ \to \mathbf{B} + P$ decays in Tables 1 and 2.

Recently, Belle reported the measurements of branching fractions for Λ_c^+ singly Cabibbo-suppressed (SCS) decays $\Lambda_c^+ \to \Lambda K^+$ and $\Lambda_c^+ \to \Sigma K^+$ by taking $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda_c^+ \to \Sigma^0\pi^+$ as reference modes [21]. The branching fractions of signal modes are measured relative to those of the reference modes using

$$\frac{\mathcal{B}_{\rm sig}}{\mathcal{B}_{\rm ref}} = \frac{N_{\rm sig}/\varepsilon_{\rm sig}}{N_{\rm ref}/\varepsilon_{\rm ref}}, \tag{4}$$

where $N_{\rm sig}$ is the extracted signal yield by fitting the $M(\Lambda_c^+)$ distribution for the Λ_c^+ and $\bar{\Lambda}_c^-$ samples, and ε is the reconstruction efficiency, which is determined based on signal MC events. According to Equation (4), the branching fraction ratios are calculated and multiplying the values of the appropriate reference mode branching fractions which are listed in Table 1, the absolute branching fractions are measured to be $(0.066 \pm 0.004)\%$ and $(0.036 \pm 0.003)\%$ for the SCS decays $\Lambda_c^+ \to \Lambda K^+$ and $\Lambda_c^+ \to \Sigma^0 K^+$, respectively, which agree with the world average values [36] with the uncertainties' considerably improved precision.

Table 1. The branching fractions (%) (Up) and decay asymmetry parameters α (Low) of CF $\{\Lambda_c^+ \to \mathbf{B} + P\}$ decays in various approaches, including the covariant confined quark model (CCQM) [48,49], Pole [33,34,50,51], CA [33,52,53], and the SU(3) flavor symmetry (SU(3)) [54]. The last column gives the current experimental results.

Decay	Körner [48] CCQM	Xu [34] Pole	Cheng [33] CA/Pole	Ivanov [49] CCQM	Żenczykowski [50] Pole	Sharma [51] CA	Zou [53] CA	Geng [54] SU(3)	Exp [36]
$\Lambda_c^+ \to \Lambda\pi^+$	0.74 (Input)	1.62	1.46/0.88	0.79	0.52	1.12	1.30	1.30 ± 0.07	1.30 ± 0.07
$\Lambda_c^+ \to \Sigma^0\pi^+$	0.32	0.34	1.76/0.72	0.88	0.39	1.34	2.34	1.27 ± 0.06	1.29 ± 0.07
$\Lambda_c^+ \to \Sigma^+\pi^0$	0.32	0.34	1.76/0.72	0.88	0.39	1.34	2.34	1.27 ± 0.06	1.25 ± 0.10
$\Lambda_c^+ \to \Sigma^+\eta$	0.16	-	-	0.11	0.90	0.57	0.74	0.32 ± 0.13	0.44 ± 0.20
$\Lambda_c^+ \to \Sigma^+\eta'$	1.28	-	-	0.12	0.11	0.10	-	1.44 ± 0.56	1.5 ± 0.6
$\Lambda_c^+ \to \Xi^0 K^+$	0.26	0.10	-	0.31	0.34	0.13	0.73	0.56 ± 0.09	0.55 ± 0.07
$\Lambda_c^+ \to p\bar{K}^0$	2.10 (Input)	1.20	3.64/1.26	2.06	1.71	1.64	2.11	3.16 ± 0.16	3.18 ± 0.16
$\Lambda_c^+ \to \Lambda\pi^+$	-0.70	-0.67	$-0.99/-0.95$	-0.95	-0.99	-0.99	-0.93	-0.87 ± 0.10	-0.84 ± 0.09
$\Lambda_c^+ \to \Sigma^0\pi^+$	$+0.70$	$+0.92$	$-0.49/+0.78$	$+0.43$	$+0.39$	-0.31	-0.76	-0.35 ± 0.27	-0.73 ± 0.18
$\Lambda_c^+ \to \Sigma^+\pi^0$	$+0.70$	$+0.91$	$-0.49/+0.78$	$+0.43$	$+0.39$	-0.31	-0.76	-0.35 ± 0.27	-0.55 ± 0.11
$\Lambda_c^+ \to \Sigma^+\eta$	$+0.33$	-	-	$+0.55$	0.00	-0.91	-0.95	-0.40 ± 0.47	-
$\Lambda_c^+ \to \Sigma^+\eta'$	-0.45	-	-	-0.05	-0.91	$+0.78$	-	$+1.00^{+0.00}_{-0.17}$	-
$\Lambda_c^+ \to \Xi^0 K^+$	0	0	-	0	0	0	$+0.90$	$+0.94^{+0.06}_{-0.11}$	-
$\Lambda_c^+ \to p\bar{K}^0$	-1.0	$+0.51$	$-0.90/-0.49$	-0.97	-0.66	-0.99	-0.75	$-0.89^{+0.26}_{-0.11}$	$+0.2 \pm 0.5$

It is worth noting that the predictions of the branching fractions and the decay asymmetry parameters of $\Lambda_c^+ \to \Sigma^+\pi^0$ and $\Lambda_c^+ \to \Sigma^0\pi^+$ almost have the same values by considering the isospin symmetry [54]. However, the mass difference between up and down quarks, the electromagnetic interaction, or new physics can result in deviation from the expectation of isospin symmetry. For $\Lambda_c^+ \to \Sigma^+\pi^0$ decay mode, the decay asymmetry parameter has been measured to be -0.57 ± 0.12 for the first time in the Λ_c^+ baryon in unpolarized e^+e^- annihilations [44]. Now, the latest measurement of the decay asymmetry parameter of $\Lambda_c^+ \to \Sigma^+\pi^0$ is from the average of BESIII and the previous measurements [46] shown in Table 1 with large uncertainty. The isospin symmetry can be well tested by the precise measurements of $\mathcal{B}(\Lambda_c^+ \to \Sigma^+\pi^0)$ and $\mathcal{B}(\Lambda_c^+ \to \Sigma^0\pi^+)$, as well as corresponding decay asymmetry parameters [22]. Theoretically, the rate of $\Lambda_c^+ \to \Sigma^+\eta$ is larger than or comparable with that of$\Lambda_c^+ \to \Sigma^+\eta'$. However, the branching fractions of $\Lambda_c^+ \to \Sigma^+\eta$ and $\Lambda_c^+ \to \Sigma^+\eta'$ are measured to $(0.41 \pm 0.20)\%$ and $(1.34 \pm 0.57)\%$, respectively, at BESIII [55]. Only "Sharma" predicted the trend for $\Lambda_c^+ \to \Sigma^+\eta$ and $\Lambda_c^+ \to \Sigma^+\eta'$ among the early calculations in the 1990s by inspecting Table 1. Noting that the branching fractions of $\Lambda_c^+ \to \Sigma^+\eta$ and $\Lambda_c^+ \to \Sigma^+\eta'$ are measured with poor precision, and all the nonfactorizable diagrams contribute to these two decays [4]. Different models produce very different predictions for the branching fractions and decay asymmetry parameters in Table 1. Therefore, it is necessary to renew the above three CF two-body decays based on the full Belle data [22].

Table 2. The branching fractions (in unit of %) (Up) and decay asymmetry parameters α (Low) of SCS $\Lambda_c^+ \to \mathbf{B} + P$ decays in various approaches, including the Pole [56], CA [52,53], SU(3) [54,57–59], and the consideration of factorizable contributions (CFC) [60]. The last column gives the current experimental results.

Decay	Sharma [57] SU(3)	Uppal [52] CA	Chen [60] CFC	Lü [58] SU(3)	Geng [54] SU(3)	Zhao [59] SU(3)	Zou [53] CA	Cheng [56] Pole	Exp [36]
$\Lambda_c^+ \to \Lambda K^+$	0.02/0.14 [a]	0.12/0.09 [b]	0.018–0.039	-	0.065 ± 0.010	0.059 ± 0.017	0.107	0.106	0.061 ± 0.012
$\Lambda_c^+ \to \Sigma^0 K^+$	0.06/0.04	0.02/0.08	-	-	0.054 ± 0.007	0.055 ± 0.016	0.072	0.072	0.052 ± 0.008
$\Lambda_c^+ \to \Sigma^+ K^0$	0.12/0.09	0.04/0.08	-	-	0.109 ± 0.015	0.191 ± 0.048	0.144	0.144	-
$\Lambda_c^+ \to p\pi^0$	0.02	0.01/0.02	0.011–0.036	0.048	0.012 ± 0.012	$0.008^{+0.009}_{-0.008}$	0.013	0.008	<0.008
$\Lambda_c^+ \to p\eta$	0.02/1.7	0.03	-	-	0.124 ± 0.035	0.114 ± 0.035	0.128	0.124 ± 0.029	0.140 ± 0.011
$\Lambda_c^+ \to p\eta'$	0.06/0.06	0.004/0.02	-	-	0.245 ± 0.146	0.071 ± 0.014	-	-	-
$\Lambda_c^+ \to n\pi^+$	0.04	0.08/0.09	0.010–0.021	0.097	0.085 ± 0.020	0.077 ± 0.020	-	0.027	-
$\Lambda_c^+ \to \Lambda K^+$	+0.97/−0.54	−0.99	-	-	$+0.32 \pm 0.32$	-	−0.96	−0.96	-
$\Lambda_c^+ \to \Sigma^0 K^+$	−0.98/+0.68	−0.80	-	-	$-1.0^{+0.06}_{-0.00}$	-	−0.73	−0.73	-
$\Lambda_c^+ \to \Sigma^+ K^0$	−0.98/+0.68	−0.80	-	-	$-1.0^{+0.06}_{-0.00}$	-	−0.73	−0.74	-
$\Lambda_c^+ \to p\pi^0$	+0.05	+0.82/+0.85	-	-	-0.05 ± 0.72	-	−0.97	−0.95	-
$\Lambda_c^+ \to p\eta$	−0.03/−0.69	−1.00/−0.79	-	-	$-0.94^{+0.26}_{-0.06}$	-	−0.55	−0.56	-
$\Lambda_c^+ \to p\eta'$	−0.99/−0.97	+0.87	-	-	$+0.91^{+0.09}_{-0.21}$	-	-	-	-
$\Lambda_c^+ \to n\pi^+$	+0.05	−0.13/+0.67	-	-	$+0.12 \pm 0.19$	-	-	−0.90	-

[a] The values before and after the slashes are calculated under the assumptions of positive and negative *P*-wave amplitudes of $\Lambda_c^+ \to \Xi^0 K^+$, respectively. [b] The front of the slash represents without $|\Phi(0)|^2$ scale variation and the back represents with $|\Phi(0)|^2$ scale variation. The slashes in this column have the same effect.

When measuring the branching fractions of $\Lambda_c^+ \to \Sigma^+\eta$ and $\Lambda_c^+ \to \Sigma^+\eta'$, $\Lambda_c^+ \to \Sigma^+\pi^0$ mode is taken as the reference mode. The ratios of branching fractions are also obtained via Equation (4). Taking $\mathcal{B}(\Lambda_c^+ \to \Sigma^+\pi^0) = (1.25 \pm 0.10)\%$ [36], the absolute branching fractions of $\mathcal{B}(\Lambda_c^+ \to \Sigma^+\eta)$ and $\mathcal{B}(\Lambda_c^+ \to \Sigma^+\eta')$ are measured to be $(0.31 \pm 0.04)\%$ and $(0.42 \pm 0.08)\%$, respectively, with much improved precision compared with the current world averages [36]. However, the central value of the branching fraction of $\Lambda_c^+ \to \Sigma^+\eta'$ is still larger than that of $\Lambda_c^+ \to \Sigma^+\eta$. For more accurate measurements to constrain the theoretical models, it is necessary to update the measurements with a huge statistical sample in the future.

In the text above, we briefly introduce the decay asymmetry parameter α, which was first proposed by Lee and Yang [15]. To date, model calculations of α in Λ_c^+ two-body decays are quite uncertain, which are listed in Tables 1 and 2. Furthermore, to date, none of the decay asymmetry parameters of charmed baryon SCS decays has been measured experimentally. A measurement of α in Λ_c^+ decays is a necessary input for various dynamical modes. Based on the huge statistic sample, Belle reported the decay asymmetry parameters $\alpha_{\Lambda_c^+}$ of two-body SCS decay modes $\Lambda_c^+ \to \Lambda K^+$ and $\Lambda_c^+ \to \Sigma^0 K^+$, and two-body CF decay modes $\Lambda_c^+ \to \Sigma^+\eta$ and $\Lambda_c^+ \to \Sigma^+\eta'$ for the first time, and updated those of two-body CF modes $\Lambda_c^+ \to \Lambda\pi^+$, $\Lambda_c^+ \to \Sigma^0\pi^+$, and $\Lambda_c^+ \to \Sigma^+\pi^0$ [21,22]. For $\Lambda_c^+ \to \Lambda h^+$ (where h denotes π or K, unless otherwise stated) and $\Lambda_c^+ \to \Sigma^+ n$ (where n denotes π^0, η, or η' unless otherwise stated), the differential decay rate related to the α parameters and the helicity angle as [61]

$$\frac{dN}{d\cos\theta_i} \propto 1 + \alpha_{\Lambda_c^+}\alpha_- \cos\theta_i, \tag{5}$$

where $\alpha_{\Lambda_c^+}$ is the decay asymmetry parameter of $\Lambda_c^+ \to \Lambda h^+$ ($\Lambda_c^+ \to \Sigma^+ n$), θ_i is the angle between the vectors of the proton momentum and the inverseΛ_c^+ momentum in the $\Lambda(\Sigma^+)$ rest frame, α_- denotes the most precise measurement of the decay asymmetry parameter of $\Lambda \to p\pi^-$ in [62] ($\Sigma^+ \to p\pi^0$ in [36]), and dN is the number of signal events in each $\cos\theta_i$ bin.

For $\Lambda_c^+ \to \Sigma^0 h^+$ decays, considering $\alpha(\Sigma^0 \to \gamma\Lambda)$ is zero due to PC for an electromagnetic decay [21], and the differential decay rate related to the α parameters and helicity angles is given by

$$\frac{dN}{d\cos\theta_{\Sigma^0} d\cos\theta_\Lambda} \propto 1 - \alpha_{\Lambda_c^+}\alpha_- \cos\theta_{\Sigma^0}\cos\theta_\Lambda, \quad (6)$$

where θ_Λ (θ_{Σ^0}) is the angle between the vectors of the proton (Λ) momentum and the inverse the Σ^0 (Λ_c^+) momentum in the Λ (Σ^0) rest frame and dN is the value of signal yields in each $[\cos\theta_{\Sigma^0}, \cos\theta_\Lambda]$ bin. More details of the schematic of the helicity are shown in Ref. [21]. The efficiency-corrected yields dependent on the cosine of helicity angle are fitted with Equations (5) and (6) to extract asymmetry parameters, and the fit results are shown in Figure 1 for $\Lambda_c^+ \to \Lambda h^+$ and Figure 2 for $\Lambda_c^+ \to \Sigma^0 h^+$.

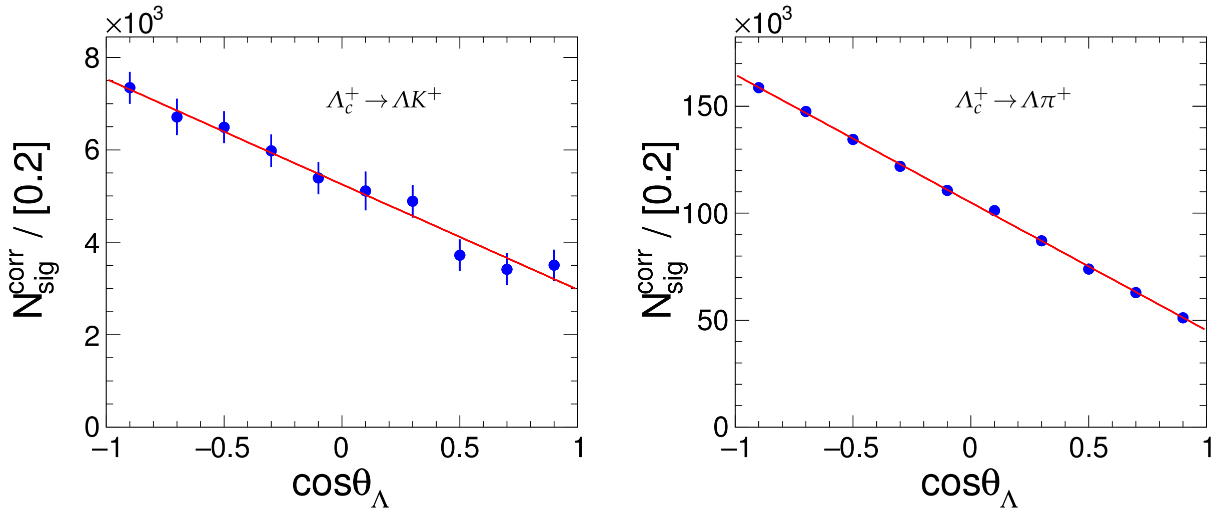

Figure 1. The $\cos\theta_\Lambda$ distributions of $\Lambda_c^+ \to \Lambda K^+$ and $\Lambda_c^+ \to \Lambda\pi^+$ and their conjugated decays after efficiency corrections. The red curves show the fitted results. The figure is quoted from Ref. [21].

Finally, the fitted slopes $\alpha_{\Lambda_c^+}\alpha_-$ are extracted [21]. Combining the average value of the most precise α_- from BESIII [62], the decay asymmetry parameters $\alpha_{\Lambda_c^+}$ are measured to be $-0.585 \pm 0.049 \pm 0.018$, $-0.55 \pm 0.18 \pm 0.09$, $-0.755 \pm 0.005 \pm 0.003$, and $-0.469 \pm 0.016 \pm 0.008$, for $\Lambda_c^+ \to \Lambda K^+$, $\Lambda_c^+ \to \Sigma^0 K^+$, $\Lambda_c^+ \to \Lambda\pi^+$, and $\Lambda_c^+ \to \Sigma^0\pi^+$, respectively. The measured values of α for $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda_c^+ \to \Sigma^0\pi^+$ are consistent with the current world average values with greatly improved precision.

For $\Lambda_c^+ \to \Sigma^+ n$, the final efficiency-corrected $\cos\theta_{\Sigma^+}$ distributions for $\Lambda_c^+ \to \Sigma^+ n$ are shown in Figure 3 with fitted results by Equation (5). Using $\alpha_- = -0.983 \pm 0.013$ [36], the decay asymmetry parameters of $\Lambda_c^+ \to \Sigma^+\pi^0$, $\Lambda_c^+ \to \Sigma^+\eta$, and $\Lambda_c^+ \to \Sigma^+\eta'$ are calculated to be -0.48 ± 0.03, -0.99 ± 0.06, and -0.46 ± 0.07, respectively. For $\Lambda_c^+ \to \Sigma^+\pi^0$, Belle updated a measurement of $\alpha_{\Sigma^+\pi^0}$ value with a significant precision improvement. The result agrees with the measurement of the above $\alpha_{\Sigma^0\pi^+}$, which meets the expectation from the isospin symmetry [54]. For $\Lambda_c^+ \to \Sigma^+\eta$ and $\Lambda_c^+ \to \Sigma^+\eta'$, the decay asymmetry parameters are measured for the first time. It will also improve the knowledge of contributions from PV and PC amplitudes to two-body charmed decays and the dynamical properties of Λ_c^+ decays.

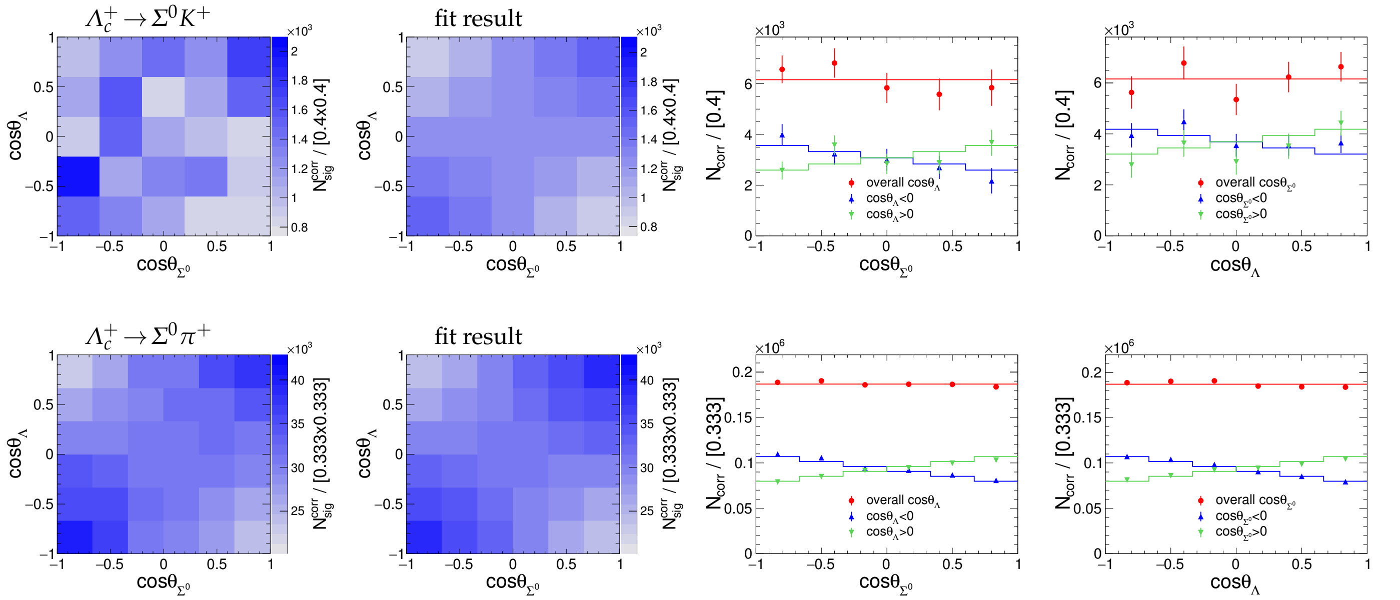

Figure 2. The first column shows the $[\cos\theta_{\Sigma^0}, \cos\theta_{\Lambda}]$ distributions of $\Lambda_c^+ \to \Sigma^0 K^+$ and $\Lambda_c^+ \to \Sigma^0 \pi^+$ and their conjugated decays after efficiency correction; the second column shows the fitted results of the first column for $\Lambda_c^+ \to \Sigma^0 K^+$ and $\Lambda_c^+ \to \Sigma^0 \pi^+$, respectively. The third column shows the projections of the $\cos\theta_{\Sigma^0}$ distributions (point with error) and the fit results (histograms) in overall (red) or negative (blue) or positive (green) $\cos\theta_{\Lambda}$ region, and vice versa in the fourth column. The figure is quoted from Ref. [21].

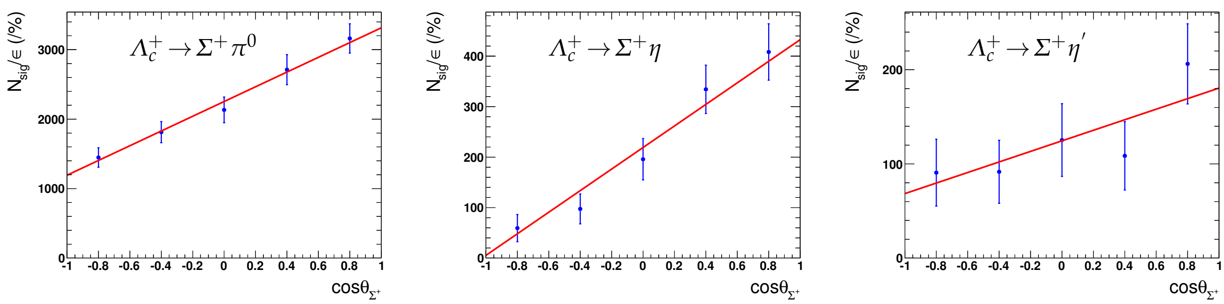

Figure 3. The maximum likelihood fits to the efficiency-corrected $\cos\theta_{\Sigma^+}$ distributions of data to extract $\alpha_{\Lambda_c^+}\alpha_-$ for $\Lambda_c^+ \to \Sigma^+ \pi^0$, $\Lambda_c^+ \to \Sigma^+ \eta$, and $\Lambda_c^+ \to \Sigma^+ \eta'$, respectively. The data are indicated by points with error bars, and the best fits are demonstrated with the solid lines. The figure is quoted from Ref. [22].

3.2. Search for CPV in $\Lambda_c^+ \to \mathbf{B} + P$ Decays

The SCS decays of charmed hadrons provide an ideal laboratory for studying CPV as they are a unique window into the physics of decay dynamics in the charm sector. The sole observation of CPV in the charm sector was finished by LHCb in the SCS charmed meson decays, $D^0 \to h^+ h^-$ [63]. Experimentally, *CP* asymmetry measurements in SCS charmed baryon decays are more challenging than in charmed meson decays and relatively unexplored. Not only that, direct *CP* asymmetry measurements for two-body SCS decays of charmed baryons provide useful constraints on theoretical predictions for CPV in the charmed baryon sector. Searches for direct CPV in SCS charmed baryon decays were made by LHCb [64,65]. However, no searches for the direct CPV were performed in two in two-body SCS decays of charmed baryons to date. Equation (1) shows the experimental method to measure direct *CP* asymmetry in charmed baryon decays; however, the values of $\Gamma(\Lambda_c^+ \to f)$ and $\Gamma(\overline{\Lambda}_c^- \to \bar{f})$ can not be directly measured in experiments. There is a novel method to calculate the $\Lambda_c^+ \to \Lambda K^+$ and $\Lambda_c^+ \to \Sigma^0 K^+$ direct *CP* asymmetry in Ref. [21]. Based on the method, Belle reported the direct *CP* asymmetries of the Λ_c^+ SCS decays

$\Lambda_c^+ \to \Lambda K^+$ and $\Lambda_c^+ \to \Sigma^0 K^+$ by taking the CF decays $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda_c^+ \to \Sigma^0\pi^+$ as the reference modes [21], respectively. First, the two flavors Λ_c^+ and $\overline{\Lambda}_c^-$ are divided into two sub-samples to determine the signal $N_{\rm sig}$ at the reconstruction level. The raw asymmetry of the decays of $\Lambda_c^+ \to f$ and $\overline{\Lambda}_c^- \to \bar{f}$ is defined with $N_{\rm sig}$ as follows:

$$\mathcal{A}_{\rm raw} = \frac{N_{\rm sig}(\Lambda_c^+ \to f) - N_{\rm sig}(\overline{\Lambda}_c^- \to \bar{f})}{N_{\rm sig}(\Lambda_c^+ \to f) + N_{\rm sig}(\overline{\Lambda}_c^- \to \bar{f})}, \quad (7)$$

Here, taking $e^+e^- \to c\bar{c} \to [\Lambda_c^+ \to \Lambda K^+] + X$ as an example, several sources contribute to the raw *CP* asymmetry, which is given by

$$\mathcal{A}_{\rm raw} = \mathcal{A}_{CP}^{\Lambda_c^+ \to \Lambda K^+} + \mathcal{A}_{CP}^{\Lambda \to p\pi^-} + \mathcal{A}_{\varepsilon}^{\Lambda} + \mathcal{A}_{\varepsilon}^{K^+} + \mathcal{A}_{\rm FB}^{\Lambda_c^+}, \quad (8)$$

where all terms are small (at the order of 10^{-2} or smaller). Here, $\mathcal{A}_{CP}^{\Lambda_c^+ \to \Lambda K^+}$ ($\mathcal{A}_{CP}^{\Lambda \to p\pi^-}$) is the direct *CP* asymmetry associated with Λ_c^+ (Λ), which can be also written as $\mathcal{A}_{CP}^{\rm dir}(\Lambda_c^+ \to \Lambda K^+)$ ($\mathcal{A}_{CP}^{\rm dir}(\Lambda \to p\pi^-)$). $\mathcal{A}_{\varepsilon}^{\Lambda}$ ($\mathcal{A}_{\varepsilon}^{K^+}$) is the detection asymmetry resulting from differences in the reconstruction efficiency between Λ (K^+) and its anti-particle $\overline{\Lambda}$ (K^-), and $\mathcal{A}_{FB}^{\Lambda_c^+}$ arises from the forward-backward asymmetry (FBA) of Λ_c^+ production due to $\gamma - Z^0$ in interference and higher-order QED effects in $e^+e^- \to c\bar{c}$ collisions [66,67]. The FBA is an odd function in $\cos\theta^*$, where θ^* is the Λ_c^+ production polar angle in the e^+e^- center-of-mass frame, but due to asymmetric acceptance, small residual asymmetry remains after integrating over $\cos\theta^*$. Considering the reference mode $\Lambda_c^+ \to \Lambda\pi^+$ has nearly the same Λ kinematic distributions as the mode $\Lambda_c^+ \to \Lambda K^+$, the differences between them are mainly related to $\mathcal{A}_{CP}^{\rm dir}(\Lambda_c^+ \to \Lambda h^+)$ and $\mathcal{A}_{\varepsilon}^{h^+}$. Here, $\mathcal{A}_{\varepsilon}^{h^+}$ depends on the cosine of the polar angle and transverse momentum of the h^+ tracks in the laboratory frame and were determined at Belle [68,69]. The charged track detection asymmetry $\mathcal{A}_{\varepsilon}^{h^+}$ was removed using a weighting method on Λ_c^+ and $\overline{\Lambda}_c^-$ sample. Now, the differences of the corrected raw asymmetries are

$$\mathcal{A}_{\rm raw}^{\rm corr}(\Lambda_c^+ \to \Lambda K^+) - \mathcal{A}_{\rm raw}^{\rm corr}(\Lambda_c^+ \to \Lambda\pi^+) = \mathcal{A}_{CP}^{\rm dir}(\Lambda_c^+ \to \Lambda K^+) - \mathcal{A}_{CP}^{\rm dir}(\Lambda_c^+ \to \Lambda\pi^+). \quad (9)$$

Since *CP* is well conserved in CF charm decays not involving a K_S^0 or K_L in the final state in the SM, the direct *CP* asymmetry of the reference mode can be set to zero. Then, an unbinned fit with maximum likelihood is used for the $M_{\Lambda_c^+}$ distributions of the weighted Λ_c^+ and $\overline{\Lambda}_c^-$ samples simultaneously to measure the corrected raw asymmetry differences. The fitted $\mathcal{A}_{\rm raw}^{\rm corr}$ values are $(+3.66 \pm 2.59)\%$, $(+1.55 \pm 0.30)\%$, $(+8.60 \pm 5.34)\%$, and $(+6.11 \pm 0.40)\%$ for $\Lambda_c^+ \to \Lambda K^+$, $\Lambda_c^+ \to \Lambda\pi^+$, $\Lambda_c^+ \to \Sigma K^+$, and $\Lambda_c^+ \to \Sigma\pi^+$, respectively. Finally, using Equation (9), Belle reported the direct *CP* asymmetries of SCS decay mode $\Lambda_c^+ \to \Lambda K^+$ and $\Lambda_c^+ \to \Sigma K^+$ are $(+2.1 \pm 2.6)\%$ and $(+2.5 \pm 5.4)\%$, respectively. This is the first direct *CP* asymmetry measurement for SCS two-body decays of charmed baryons, and no evidence of CPV is found. Meanwhile, this paper also tried to measure the values of the α-induced *CP* asymmetries for $\Lambda_c^+ \to \Lambda h^+$ and $\Lambda_c^+ \to \Sigma^0 h^+$ and search Λ-hyperon CPV in the CF Λ_c^+ decays [21]. As α-induced *CP* asymmetry describes in Equation (3), the total α-induced *CP* asymmetries for $\Lambda_c^+ \to \Lambda\pi^+$, $\Lambda \to p\pi^-$ and $\Lambda_c^+ \to \Sigma^0\pi^+$, $\Sigma^0 \to \Lambda\gamma$, $\Lambda \to p\pi^-$ decay chains are determined with the following formula:

$$\mathcal{A}_{CP}^{\alpha}(\text{total}) \equiv \frac{\alpha_{\Lambda_c^+}\alpha_- - \alpha_{\overline{\Lambda}_c^-}\alpha_+}{\alpha_{\Lambda_c^+}\alpha_- + \alpha_{\overline{\Lambda}_c^-}\alpha_+}, \quad (10)$$

where the decay asymmetry parameters of $\Lambda \to p\pi^-$ and $\overline{\Lambda} \to \overline{p}\pi^+$ are demonstrated by α_- and α_+, respectively [36]. Under the assumption that $\alpha_{\Lambda_c^+} = -\alpha_{\overline{\Lambda}_c^-}$ for CF Λ_c^+ decays, which is the expectation of in the SM, $\mathcal{A}_{CP}^{\alpha}(\text{total}) = \mathcal{A}_{CP}^{\alpha}(\Lambda \to p\pi^-)$. The helicity angle distributions for the Λ_c^+ and $\overline{\Lambda}_c^-$ samples are fitted separately to measure $\alpha_{\Lambda_c^+}$ and $\alpha_{\overline{\Lambda}_c^-}$ with the same method described above. The results are listed in Table 3.

Table 3. The fitted slopes $\alpha_{\Lambda_c^\pm}\alpha_\mp$ for the Λ_c^+ and $\bar{\Lambda}_c^-$ samples, and the decay asymmetry parameters $\alpha_{\Lambda_c^+}$ and $\alpha_{\bar{\Lambda}_c^-}$ for individual Λ_c^+ and $\bar{\Lambda}_c^-$ samples using the most precise $\alpha_\mp$ from BESIII [62], and the corresponding α-induced *CP* asymmetry $\mathcal{A}_{CP}^{\alpha}$, comparing with current world averages (W.A.).

Decay	$\alpha_{\Lambda_c^+}\alpha_-$	$\alpha_{\bar{\Lambda}_c^-}\alpha_+$	$\alpha_{\Lambda_c^+}$	$\alpha_{\bar{\Lambda}_c^-}$	$\mathcal{A}_{CP}^{\alpha}$	W.A. $\mathcal{A}_{CP}^{\alpha}$
$\Lambda_c^+\to\Lambda\pi^+$	-0.418 ± 0.053	-0.442 ± 0.053	-0.566 ± 0.076	-0.592 ± 0.106	-0.023 ± 0.116	-
$\Lambda_c^+\to\Lambda K^+$	-0.582 ± 0.006	-0.565 ± 0.006	-0.784 ± 0.010	$+0.754\pm0.020$	$+0.020\pm0.015$	-0.07 ± 0.22
$\Lambda_c^+\to\Sigma^0\pi^+$	$+0.43\pm0.18$	-0.37 ± 0.21	-0.58 ± 0.26	-0.49 ± 0.31	$+0.08\pm0.38$	-
$\Lambda_c^+\to\Sigma^0 K^+$	-0.340 ± 0.016	-0.358 ± 0.017	-0.452 ± 0.032	$+0.473\pm0.042$	-0.023 ± 0.045	-

Using the average value of the most precise α_- and α_+ from BESIII [62], the α-induced *CP* asymmetry for $\Lambda_c^+\to\Lambda K^+$, $\Lambda_c^+\to\Sigma^0K^+$, and $\Lambda_c^+\to\Sigma^0\pi^+$ were measured for the first time. The results of $\Lambda_c^+\to\Lambda\pi^+$ are consistent with previous results, but with lower uncertainty. According to Equation (10), the α-induced *CP* asymmetry of $\Lambda\to p\pi^-$ is measured to be $+0.0169\pm0.0073\pm0.0120$ in $\Lambda_c^+\to\Lambda\pi^+$ and $0.026\pm0.034\pm0.030$ in $\Lambda_c^+\to\Sigma^0\pi^+$. Finally, the average value of $\mathcal{A}_{CP}^{\alpha}(\Lambda\to p\pi^-)$ is calculated to be $+0.013\pm0.007\pm0.011$. This is the first measurement of hyperon CPV in CF charm decays. No evidence of Λ-hyperon CPV is found. It is worth noting that the method used to measure $\mathcal{A}_{CP}^{\alpha}(\Lambda\to p\pi^-)$ can be applied to other hyperon decays, such as $\Xi_c^{+,0}\to\Xi^{0,-}\pi^+$. In the future, this method is promising for precise measurements of hyperon CPV at Belle II and LHCb.

4. Studies of Ξ_c^0 Decays

Based on SU(3) and dynamical models, theoretical predictions for the Ξ_c^0 weak decays are reported. Nevertheless, predictions from various models are very different. Therefore, it is necessary to study Ξ_c^0 experimentally. Using a data set collected at the 10.58 GeV energy point, Belle measured the absolute branching fraction of $B^-\to\bar{\Lambda}_c^-\Xi_c^0$ and the product branching fraction $\mathcal{B}(B^-\to\bar{\Lambda}_c^-\Xi_c^0)\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)$, and first reported the value of $\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)$ [3,7]. Based on this Ξ_c^0 absolute branching fraction, to date, many branching fractions of Ξ_c^0 decay modes have been published [23,24,70–72]. In addition, two of them also measured the decay asymmetry parameters and *CP* asymmetries.

4.1. $\Xi_c^0\to\mathbf{B}+P$ Decays

A number of theoretical models are used to study the CF $\Xi_c^0\to\mathbf{B}+P$ decays, and the corresponding values of the branching fractions and decay asymmetry parameters are calculated and listed in Table 4. Note that the values in columns under "Zou" and "Geng" are calculated by using modern CA and SU (3) approaches, respectively. However, there are few experimental measurements of branching fractions and decay asymmetry parameters for CF $\Xi_c^0\to\mathbf{B}+P$ decays in Table 4 (last column). For the experimental decay asymmetry parameter of $\Xi_c^0\to\Xi^-\pi^+$, the value is dominated by the new Belle measurement [23]. In 2021, the decay asymmetry parameter of $\Xi_c^0\to\Xi^-\pi^+$ has been measured by using the data sample from the Belle detector [23]. Taking $\Xi^-\to\Lambda\pi^-$ as the secondary decay mode of Ξ_c^0 decay, the differential decay rate related to the α parameters and helicity angles is given by

$$\frac{dN}{d\cos\theta_{\Xi^-}}\propto 1+\alpha_{\Xi^-\pi^+}\alpha_{\Xi^-}\cos\theta_{\Xi^-}, \tag{11}$$

where θ_{Ξ^-} is the angle between the vectors of Λ momentum and the inverse Ξ_c^0 momentum in the Ξ^- rest frame [73], dN is the value of signal events in each $\cos\theta_{\Xi^-}$ bin, α_{Ξ^-} is the decay asymmetry parameter of the Ξ^- [74], and $\alpha_{\Xi^-\pi^+}$ is the decay asymmetry parameter of $\Xi_c^0\to\Xi^-\pi^+$.

Table 4. The branching fractions (%) (Up) and decay asymmetry parameters (Low) of CF $\Xi_c^0 \to \mathbf{B} + P$ decays in various approaches, including the CCQM [48,49], Pole [33,34,50,51], CA [33,53], and SU(3) [54]. The last column gives the current experimental results.

Decay	Körner [48] CCQM	Xu [34] Pole	Cheng [33] CA/Pole	Ivanov [49] CCQM	Sharma [51] Pole	Żenczykowski [50] Pole	Zou [53] CA	Geng [54] SU(3)	Exp [36]
$\Xi_c^0 \to \Xi^- \pi^+$	1.42	2.37	1.13/1.71	1.60	2.46	0.61	6.47	2.21 ± 0.14	1.43 ± 0.32
$\Xi_c^0 \to \Lambda \bar{K}^0$	0.17	0.50	1.36/0.37	0.55	0.54	0.35	1.33	1.05 ± 0.06	0.60 ± 0.16
$\Xi_c^0 \to \Sigma^0 \bar{K}^0$	1.61	0.14	0.03/0.18	0.26	0.07	0.11	0.04	0.08 ± 0.08	-
$\Xi_c^0 \to \Sigma^+ K^-$	0.17	0.17	-	0.35	0.12	0.36	0.78	0.59 ± 0.11	-
$\Xi_c^0 \to \Xi^0 \pi^0$	0.05	0.77	1.71/0.38	0.05	0.87	0.69	1.82	0.76 ± 0.10	-
$\Xi_c^0 \to \Xi^0 \eta$	0.32	-	-	0.37	0.09	0.01	2.67	1.03 ± 0.20	-
$\Xi_c^0 \to \Xi^0 \eta'$	1.16	-	-	0.41	0.14	0.09	-	0.91 ± 0.41	-
$\Xi_c^0 \to \Xi^- \pi^+$	-0.38	-0.38	$-0.47/-0.99$	-0.84	-0.97	-0.79	-0.95	$-0.98^{+0.07}_{-0.02}$	-0.60 ± 0.04
$\Xi_c^0 \to \Lambda \bar{K}^0$	-0.76	$+1.00$	$-0.88/-0.73$	-0.75	-0.79	-0.29	-0.86	-0.68 ± 0.28	-
$\Xi_c^0 \to \Sigma^0 \bar{K}^0$	-0.96	-0.99	$+0.85/-0.59$	-0.55	$+0.48$	-0.50	-0.94	-0.07 ± 0.90	-
$\Xi_c^0 \to \Sigma^+ K^-$	0	0	-	0	0.00	0.00	$+0.98$	$+0.81 \pm 0.16$	-
$\Xi_c^0 \to \Xi^0 \pi^0$	$+0.92$	-0.92	$-0.78/-0.54$	$+0.94$	-0.80	$+0.21$	-0.77	$-1.00^{+0.07}_{-0.00}$	-
$\Xi_c^0 \to \Xi^0 \eta$	-0.92	-	-	-1.0	-0.37	-0.04	$+0.30$	$+0.93^{+0.07}_{-0.19}$	-
$\Xi_c^0 \to \Xi^0 \eta'$	-0.38	-	-	-0.32	$+0.56$	-1.00	-	$+0.98^{+0.02}_{-0.27}$	-

Then, $\Xi_c^0 \to \Xi^- \pi^+$ and its charge-conjugate decay mode $\bar{\Xi}_c^0 \to \bar{\Xi}^+ \pi^-$ are processed to extract decay parameters of $\alpha_{\Xi^-\pi^+}$ and $\alpha_{\bar{\Xi}^+\pi^-}$, respectively. Finally, the efficiency-corrected $\cos\theta_{\Xi^-}$ and $\cos\theta_{\bar{\Xi}^+}$ distributions for $\Xi_c^0 \to \Xi^- \pi^+$ and $\{\bar{\Xi}_c^0 \to \bar{\Xi}^+ \pi^-\}$ decays are shown in Figure 4 with fitted results by Equation (11), respectively. Finally, the values of $\alpha_{\Xi^-\pi^+}$ and $\alpha_{\bar{\Xi}^+\pi^-}$ are measured to be -0.64 ± 0.05 and 0.61 ± 0.05 by using the measurements of α_{Ξ^-} and $\alpha_{\bar{\Xi}^+}$ [74], respectively. The corresponding average absolute value of the decay asymmetry for two-body CF charmed baryon decay $\Xi_c^0 \to \Xi^- \pi^+$ is 0.63 ± 0.03 with a great precision comparing the result $-0.56 \pm 0.39^{+0.10}_{-0.09}$ from CLEO [75]. Using Equation (3), the *CP*-asymmetry parameter $\mathcal{A}_{CP} = (\alpha_{\Xi^-\pi^+} + \alpha_{\bar{\Xi}^+\pi^-})/(\alpha_{\Xi^-\pi^+} - \alpha_{\bar{\Xi}^+\pi^-})$ is measured to be 0.024 ± 0.054. The measured $\mathcal{A}_{CP}$ favors no CPV . However, it is clear that different models give very different predictions for branching fractions and decay asymmetry parameters. Therefore, more accurate measurements of CF $\Xi_c^0 \to \mathbf{B} + P$ decays are important to test the rightness of the model.

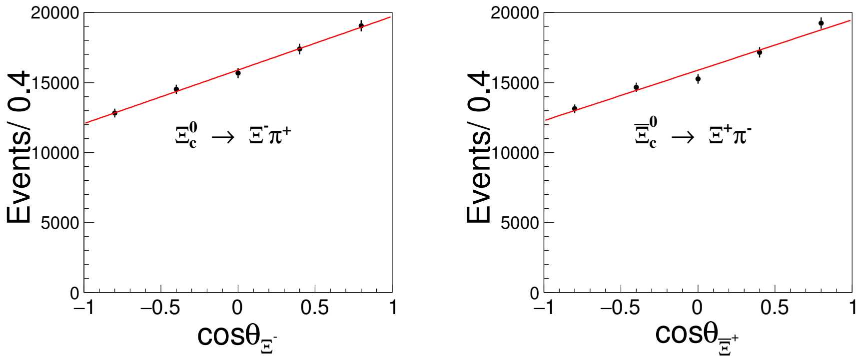

Figure 4. Fit to the efficiency-corrected $\cos\theta_{\Xi^-}$ and $\cos\theta_{\bar{\Xi}^+}$ distributions of data to extract $\alpha_{\Xi^-\pi^+}\alpha_{\Xi^-}$ and $\alpha_{\bar{\Xi}^+\pi^-}\alpha_{\bar{\Xi}^+}$, respectively. The black points with error bars indicate data and the best fits are shown in the solid red line. The figure is quoted from Ref. [23].

4.2. $\Xi_c^0 \to \mathbf{B} + V$ Decays

The values of the branching fractions and decay asymmetry parameters for $\Xi_c^0 \to \Lambda\bar{K}^{*0}$, $\Xi_c^0 \to \Sigma^0\bar{K}^{*0}$, and $\Xi_c^0 \to \Sigma^+K^{*-}$ are studied in a number of theoretical models [24], as listed in Table 5. The differences in the values of decay asymmetry parameters and $\mathcal{B}(\Xi_c^0 \to \Lambda\bar{K}^{*0})$, $\mathcal{B}(\Xi_c^0 \to \Sigma^0\bar{K}^{*0})$ are huge in one model in Table 5. Therefore, it is necessary to study the branching fractions and decay asymmetry parameters of the charmed baryons experimentally. In 2021, Belle first studied and published the processes of $\Xi_c^0 \to \Lambda\bar{K}^{*0}$, $\Xi_c^0 \to \Sigma^0\bar{K}^{*0}$, and $\Xi_c^0 \to \Sigma^+K^{*-}$ [3,24].

Table 5. The Branching fractions (%) (Up) and decay asymmetry parameters (Low) for the two-body CF $\Xi_c^0 \to \mathbf{B} + V$ decays in various approaches. Including the CCQM [48], Pole [50], and SU(3) [76,77]. The current world average of $\tau(\Xi_c^0)$ [36] is used for all of models. The last column gives the current experimental results.

Decay	Körner [48] CCQM	Żenczykowski [50] Pole	Hsiao [76] SU(3)	Geng [77] SU(3)	Exp [36]
$\Xi_c^0 \to \Lambda\bar{K}^{*0}$	1.52	1.15	0.46 ± 0.21	1.37 ± 0.26	0.33 ± 0.11
$\Xi_c^0 \to \Sigma^0\bar{K}^{*0}$	0.84	0.77	0.27 ± 0.22	0.42 ± 0.23	1.24 ± 0.37
$\Xi_c^0 \to \Sigma^+K^{*-}$	0.53	0.37	0.93 ± 0.29	0.24 ± 0.17	0.61 ± 0.21
$\Xi_c^0 \to \Xi^0\rho^0$	2.33	1.22	1.4 ± 0.4	0.88 ± 0.22	-
$\Xi_c^0 \to \Xi^0\omega$	3.16	0.15	$0.1^{+0.86}_{-0.1}$	2.78 ± 0.45	-
$\Xi_c^0 \to \Xi^0\phi$	0.24	0.10	$0.015^{+0.071}_{-0.015}$	0.14 ± 0.13	-
$\Xi_c^0 \to \Xi^-\rho^+$	16.78	1.50	0.86 ± 0.12	8.98 ± 0.55	-
$\Xi_c^0 \to \Lambda\bar{K}^{*0}$	+0.58	+0.49	-	-0.67 ± 0.24	$+0.15 \pm 0.22$
$\Xi_c^0 \to \Sigma^0\bar{K}^{*0}$	−0.87	+0.25	-	-0.42 ± 0.62	-
$\Xi_c^0 \to \Sigma^+K^{*-}$	−0.60	+0.51	-	$-0.76^{+0.64}_{-0.24}$	-0.52 ± 0.30
$\Xi_c^0 \to \Xi^0\rho^0$	−0.33	+0.15	-	-0.26 ± 0.32	-
$\Xi_c^0 \to \Xi^0\omega$	+1.09	+0.09	-	-0.71 ± 0.12	-
$\Xi_c^0 \to \Xi^0\phi$	+17.67	−0.08	-	$+0.61 \pm 0.27$	-
$\Xi_c^0 \to \Xi^-\rho^+$	+4.36	+0.87	-	-0.94 ± 0.01	-

Using the absolute branching fraction of $\Xi_c^0 \to \Xi^-\pi^+$ [7], the values of $\mathcal{B}(\Xi_c^0 \to \Lambda\bar{K}^{*0})$, $\mathcal{B}(\Xi_c^0 \to \Sigma^0\bar{K}^{*0})$ have been measured and listed in the last column of Table 5. Belle also reported the decay asymmetry parameters of $\Xi_c^0 \to \Lambda\bar{K}^{*0}$, $\Xi_c^0 \to \Sigma^0\bar{K}^{*0}$, and $\Xi_c^0 \to \Sigma^+K^{*-}$. Taking $\Xi_c^0 \to \Lambda\bar{K}^{*0}$ decay mode as an example, the differential decay rate related to the α parameters and helicity angles is given by

$$\frac{dN}{d\cos\theta_\Lambda} \propto 1 + \alpha_{\Xi_c^0 \to \Lambda\bar{K}^{*0}}\alpha_{\Lambda \to p\pi^-}\cos\theta_\Lambda, \quad (12)$$

where the asymmetry parameters of $\Xi_c^0 \to \Lambda\bar{K}^{*0}$ and $\Lambda \to p\pi^-$ are demonstrated by $\alpha_{\Xi_c^0 \to \Lambda\bar{K}^{*0}}$ and $\alpha_{\Lambda \to p\pi^-}$ [24], respectively. θ_Λ is the angle between the vectors of the proton momentum and the inverse Ξ_c^0 momentum in the Λ rest frame [24]. Similarly, the method is also used to measure the decay asymmetry parameters of $\Xi_c^0 \to \Sigma^0\bar{K}^{*0}$ and $\Xi_c^0 \to \Sigma^+K^{*-}$. Meanwhile, since the conservation of parity for an electromagnetic decay, $\alpha_{\Sigma^0 \to \Lambda\gamma}$ should be zero. Therefore, $\alpha_{\Xi_c^0 \to \Sigma^0\bar{K}^{*0}}$ can not be extracted under one-dimensional angular distribution because the slope factor $\alpha_{\Xi_c^0 \to \Sigma^0\bar{K}^{*0}}\alpha_{\Sigma^0 \to \Lambda\gamma}$ is zero. The final efficiency-corrected $\cos\theta_\Lambda$, $\cos\theta_{\Sigma^0}$, and $\cos\theta_{\Sigma^+}$ distributions for $\Xi_c^0 \to \Lambda\bar{K}^{*0}$, $\Xi_c^0 \to \Sigma^0\bar{K}^{*0}$, and $\Xi_c^0 \to \Sigma^+K^{*-}$ decays are shown in Figure 5, respectively. The blue points with error bars indicate data, and the solid black line shows the best fits.

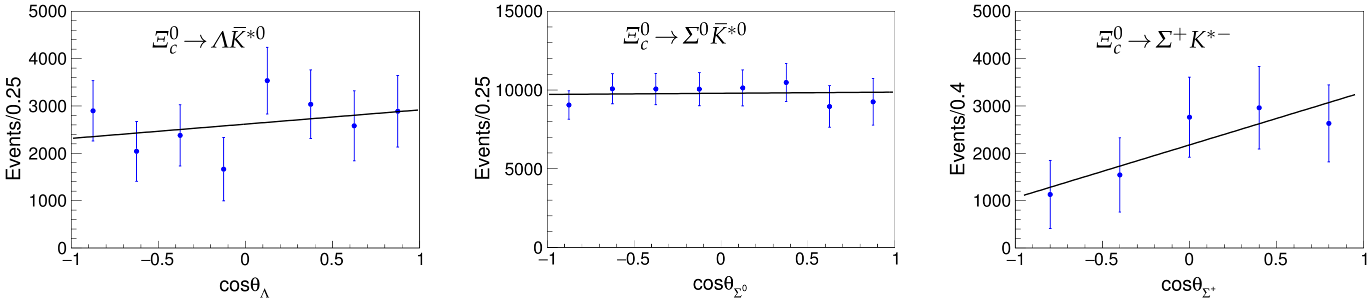

Figure 5. Fit to the efficiency-corrected $\cos\theta_\Lambda$, $\cos\theta_{\Sigma^0}$, and $\cos\theta_{\Sigma^+}$ distributions of data to extract $\alpha_{\Xi_c^0\to\Lambda\bar{K}^{*0}}\alpha_{\Lambda\to p\pi^-}$, $\alpha_{\Xi_c^0\to\Sigma^0\bar{K}^{*0}}\alpha_{\Sigma^0\to\Lambda\gamma}$, and $\alpha_{\Xi_c^0\to\Sigma^+K^{*-}}\alpha_{\Sigma^+\to p\pi^0}$, respectively. The figure is quoted and adapted from Ref. [24].

The fitted slope for $\Xi_c^0 \to \Sigma^0\bar{K}^{*0}$ in Figure 5 is consistent with zero, which shows no bias. Finally, taking the values of $\alpha_{\Lambda\to p\pi^-}$ and $\alpha_{\Sigma^+\to p\pi^0}$ from PDG [36], the decay asymmetry parameters for $\alpha_{\Xi_c^0\to\Lambda\bar{K}^{*0}}$ and $\alpha_{\Xi_c^0\to\Sigma^+K^{*-}}$ are measured and listed in Table 5 last column, for the first time. In fact, for $\Xi_c^0 \to \Sigma^0\bar{K}^{*0}$ decay mode, the new method to measure the value of $\alpha_{\Xi_c^0\to\Sigma^0\bar{K}^{*0}}$ has been mentioned above in Equation (6). To date, only two decay asymmetry parameters of $\Xi_c^0 \to \mathbf{B} + V$ decay modes were measured at Belle, which is far from enough to constrain for theoretical models. It is desired to measure more decay asymmetry parameters of $\Xi_c^0 \to \mathbf{B} + V$ decay modes in the future.

5. Ω_c^0 Decays

The Ω_c^0 is comprised of a charm quark and two strange quarks and is the heaviest among the ground-state charmed baryons [36,78]. Because of the lower production cross section and the complicated final states, the information of the Ω_c^0 hadronic weak decays is less than that of the other charmed baryon decays or the heavy mesons. However, with the accumulation of experimental data, in recent years, the results of Ω_c^0 are also reported. In 2017, the five new narrow states of Ω_c^0 were reported at LHCb [78,79]. Later in the same year, Belle confirmed four of these five narrow states [80]. In 2018, a surprising measurement of the Ω_c^0 lifetime was reported in the LHCb experiment [81]. The value in PDG at that time is four times smaller than the LHCb result, which stimulates more related measurements on the lifetimes of other charmed baryons. These new resonances are studied by various theoretical models, including the pentaquark picture. In addition, progress has also been made on the ground state [78].

In the early 1990s, four papers predicted the values of the branching fractions and decay asymmetry parameters of Ω_c^0 decays based on various models [48,49,52,82]. Recently, the studies of the Ω_c^0 hadronic weak decays are given in Refs. [78,83]. For CF $\Omega_c^0 \to \mathbf{B}(1/2^+) + P(V)$ decay modes, where $1/2^+$ denotes the spin quantum number, only $\Omega_c^0 \to \Xi^0\bar{K}^0(\bar{K}^{*0})$ satisfied. For the unique $\Omega_c^0 \to \Xi^0\bar{K}^0$ mode, the decay width is from both large nonfactorizable contributions and destructive interference between the S- and P-wave amplitudes. Meanwhile, the values of $\mathcal{B}(\Omega_c^0 \to \Xi^0\bar{K}^0)$ and the decay asymmetry parameter are calculated theoretically to be 3.8% and 0.50, respectively. Therefore, it is very promising to measure them in the near future. As for the $\Omega_c^0 \to \Xi^0\bar{K}^{*0}$ mode attracted in past studies [33,48,49], it is also important to discover it experimentally. In addition, the decay partial widths of $\Omega_c^0 \to \Xi^0\bar{K}^{*0}$ is half of $\Omega_c^0 \to \Xi^0\bar{K}^0$ mode while the decay asymmetry parameter is quite small [48]. For CF $\Omega_c^0 \to \mathbf{B}(3/2^+) + P(V)$ decay modes, $\Omega_c^0 \to \Omega^-\pi^+$, $\Omega^-\rho^+$ and $\Omega_c^0 \to \Xi^{*0}\bar{K}^0$, $\Xi^{*0}\bar{K}^{*0}$, the decay rates and decay asymmetry parameters are predicted in Ref. [84] without the values of parameters $a_{1,2}$ [3]. Only factorizable contribution and external W-emission are donated to $\Omega_c^0 \to \Omega^-\pi^+$ mode, whereas $\Omega_c^0 \to \Xi^{*0}\bar{K}^0$ via internal W-emission diagram. Experimental measurements of them can provide parameters for the constraint model and determine the unknown parameters $a_{1,2}$. For SCS $\Omega_c^0 \to \mathbf{B}(1/2^+) + P(V)$ decay modes, the branching fractions are predicted with a large value (except $\Omega_c^0 \to \Sigma^0\bar{K}^0$) while the decay asymmetry parameter is tiny [78]. Recently, Belle reported the branching ratio between the SCS $\Omega_c^0 \to \Xi^-\pi^+$ mode and the reference mode $\Omega_c^0 \to \Omega^-\pi^+$ for the first time [85]. For Ω_c^0 DCS decay modes, the modes are mainly studied in [52,78] and have relatively smaller branching fractions. However, $\Omega_c^0 \to \Xi^+K^-, \Lambda\eta$, and $\Xi^0 K^0$ are predicted to have large branching fractions. Measurements of these modes are very promising at future particle experiments.

To date, the value of $\mathcal{B}(\Omega_c^0 \to \Omega^-\pi^+)$ has not been measured. Therefore, only the branching ratios of the fourteen Ω_c^0 decay modes relative to $\Omega_c^0 \to \Omega^-\pi^+$ have been measured in Ref. [36]. A number of their numerical contributions come from Belle's measurements in [86–88]. It is worth noting that 691 ± 29 signal events of $\Omega_c^0 \to \Omega^-\pi^+$ are extracted in the distribution of $\Omega^-\pi^+$ invariant mass corresponding to all data-sets at Belle. To date, a data sample of about 450 fb^{-1} was collected at Belle II. It means that nearly 1000 signal events will be reconstructed for $\Omega_c^0 \to \Omega^-\pi^+$ by combining Belle with Belle II data samples. Recently, the lifetime of Ω_c^0 at Belle II has been reported [89]. In the future, more Ω_c^0 decay modes will be measured by combining Belle with Belle II data samples.

6. Discussion and Conclusions

We have reviewed the recent results of charmed baryons at Belle. Many charmed baryon decay asymmetry parameters have been measured precisely, as shown in Table 6, which is crucial to test PV and various models in the charmed baryon sector.

Table 6. Recent results of the decay asymmetry parameter (α) for charmed baryons at Belle. The values in columns under "**W.A.**" are the averages taken from previous experimental results [36].

Decay	α	W.A.	Decay	α	W.A.
$\Lambda_c^+ \to \Lambda K^+$	-0.585 ± 0.052	-	$\Xi_c^0 \to \Xi^-\pi^+$	$+0.63 \pm 0.03$ [a]	-0.56 ± 0.39
$\Lambda_c^+ \to \Sigma^0 K^+$	-0.540 ± 0.201	-	$\Xi_c^0 \to \Lambda\bar{K}^{*0}$	$+0.15 \pm 0.22$	-
$\Lambda_c^+ \to \Sigma^0\pi^+$	-0.463 ± 0.018	-0.73 ± 0.18	$\Xi_c^0 \to \Sigma^+ K^{*-}$	-0.52 ± 0.30	-
$\Lambda_c^+ \to \Sigma^+\pi^0$	-0.480 ± 0.028	-0.55 ± 0.11			
$\Lambda_c^+ \to \Lambda\pi^+$	-0.755 ± 0.006	-0.84 ± 0.09			
$\Lambda_c^+ \to \Sigma^+\eta$	-0.990 ± 0.058	-			
$\Lambda_c^+ \to \Sigma^+\eta'$	-0.460 ± 0.067	-			

[a] The value is average of $\alpha_{\Xi_c^0 \to \Xi^-\pi^+}$ and $\alpha_{\bar{\Xi}_c^0 \to \bar{\Xi}^+\pi^-}$ from [23].

Many of the measurements listed in Table 6 agree well with the various model predictions within standard deviations. However, the predictions of $\Lambda_c^+ \to \Sigma^0\pi^+$ decay asymmetry parameter in most models do not agree very well with each other. Thus, the recent measurements of the decay asymmetry parameters of charmed baryons can provide vital inputs for the various dynamical models, improve knowledge on the contribution of S-wave PV and P-wave PC amplitudes in $\mathbf{B_c} \to \mathbf{B} + P$, and understand the decay dynamical properties of the charmed baryons. For the CF decay modes $\Lambda_c^+ \to \Sigma^+\pi^0$ and $\Lambda_c^+ \to \Sigma^0\pi^+$, the measurements of them agree with each other, as predicted by the isospin symmetry [54]. Second, the α-induced CP asymmetry values of $\Lambda_c^+ \to \Lambda\pi^+$, $\Lambda_c^+ \to \Sigma^0\pi^+$, and $\Xi_c^0 \to \Xi^-\pi^+$ are measured and consistent with zero. We also review a method of searching for Λ-hyperon CPV by assuming no CPV in Λ_c^+ CF decays $\Lambda_c^+ \to \Lambda\pi^+$, $\Lambda_c^+ \to \Sigma^0\pi^+$. To date, no evidence of Λ-hyperon CPV has been found. The CP asymmetries of Λ_c^+ SCS decay modes $\Lambda_c^+ \to \Lambda K^+$ and $\Lambda_c^+ \to \Sigma^0 K^+$ are measured for the first time by using Equations (1) and (3). In the end, we also review the current situations of theoretical and experimental research of Ω_c^0 decays. Several decay modes of Ω_c^0 are promising to study in the future.

In summary, most of the current experimental measurements with large relative uncertainties on charmed baryons at Belle, especially the decay asymmetry parameters and CPV, are not accurate enough as input parameters to effectively constrain theoretical models. The Belle II experiment, as the upgrade of the Belle experiment, will obtain in the future at least 50 times the data sample accumulated with the Belle detector. The 50 ab^{-1} Belle II data to be collected open more possibilities for measuring branching fractions and decay asymmetry parameters for more charmed baryon decays with improved precision.

It will also increase the possibilities of searching for CPV in charmed baryon decays. In the next few years, we expect that more experimental measurements will be published and used to constrain theoretical models. At the same time, the enthusiasm for theoretical research will be inspired by more and more results of charmed baryons at Belle.

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Abbreviations

The following abbreviations are used in this manuscript:

CP	Charge–Parity
CPV	*CP* Violation
SM	Standard Model
PV	Parity–Violation
PC	Parity-Conserving
CF	Cabibbo-Favored
SCS	singly Cabibbo-Suppressed

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