A GRACEFUL END TO INFLATION[1)]

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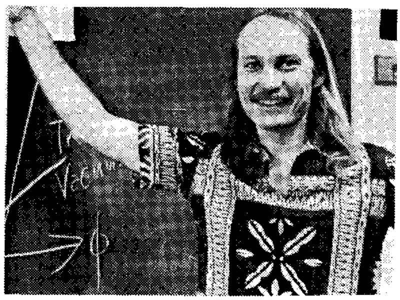


ABSTRACT

Guth has suggested that if the Universe underwent extreme supercooling during the phase transition associated with the spontaneous symmetry breaking (SSB) of the Grand Unified Theory (GUT), then a number of apparent 'cosmological puzzles' could be explained (the so-called 'inflationary Universe'). Among these puzzles are the isotropy, homogeneity, flatness/oldness, and monopole problems. The scenario in its original form while very attractive, suffered from the apparent lack of a 'graceful return' from the inflationary phase to a hot, Friedmann-Robertson-Walker cosmology. Recently, a new inflationary scenario has been suggested which appears to solve the 'graceful return' problem while retaining the other desirable features. I shall discuss the detailed evolution of the Higgs field responsible for SSB, and the temperature and scale factor of the Universe in this new scenario. Numerical calculations show that sufficient inflation can occur to solve the 'cosmological conundrums', and that the Universe smoothly reheats to a temperature of $O(10^{14}$ GeV), insuring that baryogenesis can proceed in the usual way. I will also give a very optimistic appraisal of 'the present minimal SU(5) model'. It now appears that we have an effective, low-energy ($\lesssim 10^{15}$ GeV) theory which accounts for our present understanding of particle physics, which has many beneficial consequences for cosmology, and which at present, does not suffer from any known 'fatal disease'.

I. INTRODUCTION

Although the hot big bang model[2] has proven to be a remarkably simple and reliable framework for understanding the evolution of the Universe - e.g., it nicely accounts for the universal expansion, the microwave background, and the large mass fraction of ^4He, there are several observational facts which to date it has failed to elucidate. These cosmological 'conundrums' include[3]: (1) the present high degree of isotropy (as evidenced by the 3K background) - isotropy is an unstable property of cosmological models[4]; (2a) the large-scale homogeneity (also evidenced by the 3K background) - at decoupling, the last epoch during which particle interactions could have homogenized the Universe, the present observable Universe was comprised of more than 10^7 causally-distinct regions; (2b) the small-scale inhomogeneity - the Universe is clearly very irregular on small scales (stars, galaxies, clusters, superclusters, etc.). Density fluctuations of the order of $\delta\rho/\rho \simeq 10^{-3\pm1}$ on a mass scale of the order of $10^{12\pm4} M_\odot$ are required at decoupling to insure that the present structure 'grows up' via the Jeans' instability[5]. The origin and precise nature of these perturbations is clearly a fundamental problem; (3) the oldness/flatness problem - the only timescale in the standard model is $t_{pl} \sim 10^{-43}$ s, and unless the initial 'KE' and 'PE' of the Universe had been equal to a high degree of precision at t_{pl}, corresponding to $|k|/R(t_{pl})^2 \lesssim 10^{-62}\ 8\pi G\rho/3$, the Universe would have very quickly ($t \sim$ few t_{pl}) recollapsed or become curvature - dominated. [In a curvature - dominated Universe $T^{-1} \sim R \sim t$, so that had the Universe become curvature - dominated at t_{pl}, today when $T \simeq 3K$ it should be $\sim 10^{-11}$ s old!]; (4) the baryon asymmetry of the Universe - although the laws of physics are very nearly matter - antimatter symmetric, the Universe appears to contain essentially no antimatter today[6]. In addition, the ratio of matter (baryons) to radiation (3K photons) has a rather curious value, $\eta \simeq (3\text{-}5) \times 10^{-10}$ (ref. 7). These two observations imply that at very early times ($t \lesssim 10^{-6}$ s, $T \gtrsim 1$ GeV) the Universe possessed only a very slight matter-antimatter imbalance, $O(10^{-10})$. Of course, Grand Unified Theories (GUT s) provide an attractive means of dynamically explaining the origin of this asymmetry. At a temperature of $O(10^{14}$ GeV) B, C, CP nonconserving interactions, predicted by GUT s, allow a symmetrical Universe to evolve a baryon asymmetry of the required magnitude.[8]

These problems are compounded when particle interactions which are described by spontaneously - broken gauge theories and GUT s in particular are incorporated into the model. During spontaneous symmetry breaking (SSB) the vacuum energy density changes by $O(T_c^4)$ ($T_c \simeq$ temperature of spontaneous symmetry restoration), resulting in an induced cosmological constant (if not other-wise adjusted to be zero) of order T_c^4 (ref. 9). Today, the cosmological term is known to be $\lesssim 4 \times 10^{-29}$ gcm$^{-3} \simeq 0(10^{-46}$ GeV^4). The simplest unified gauge models also predict a relic abundance of superheavy magnetic monopoles which is at least $0(10^{12})$ greater than the observational limits.[10)]

Guth[11)] has suggested that all of the above-mentioned puzzles might be explained if the phase transition associated with the SSB of the GUT is first order. During a first order phase transition the Universe can become 'trapped' in the symmetric phase even after the temperature drops below $T_c \sim 0(10^{14}$ GeV)- the critical temperature for this phase transition. While it is trapped, vacuum energy can dominate the total energy density, resulting in an exponential growth (de Sitter) phase. I will now very briefly review Guth's original scenario.

II. OLD INFLATION

In the standard hot, big bang model of the Universe[2)] (the Friedmann - Robertson - Walker cosmology), the scale factor of the Universe, R(t), is governed by

$$(\dot{R}/R)^2 = (8\pi/3)\ \rho/m_{pl}^2 - k/R^2, \tag{1}$$

where $m_{pl} = 1.22 \times 10^{19}$ GeV is the planck mass, and $k = \pm 1, 0$ is the signature of the curvature. The energy density ρ includes matter, radiation, and vacuum energy. Because the vacuum energy contribution to the energy density is known to be small today, the undetermined (by microphysics) zero of vacuum energy is chosen to coincide with the T = 0, symmetry breaking minimum ($\phi=\sigma$) of the potential (see Fig. 1). Therefore, the value of the potential ('the vacuum energy') at the symmetric minimum ($\phi=0$) is $0(T_c^4)$.[12)] The contribution of radiation to the energy density is just

$$\rho_{rad} \simeq g_*(T)\ \frac{\pi^2}{30}\ T^4, \tag{2}$$

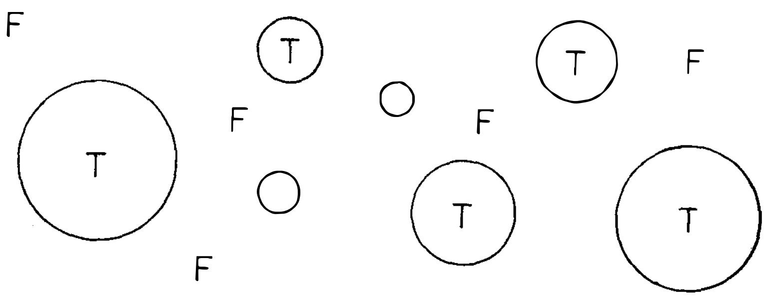
where $g_*(T)$ ($\equiv \Sigma\, g_B + 7/8\ g_F$) counts the number of degrees of freedom of all the relativistic species at temperature T (i.e., those with $m \ll T$). For temperatures $T \gtrsim T_c$, ρ_{rad} is larger than the vacuum energy density term, while for $T \lesssim T_c$ in the symmetric vacuum ($\phi=0$), the vacuum energy term is larger. [Of course, in the asymmetric minimum ($\phi=\sigma$) the vacuum energy term is zero].

In a first order phase transition a potential barrier exists between the two minima when $T \sim 0(T_c)$. Even though the asymmetric minimum maybe energetically favorable[12)], the transition, which must proceed via thermal and/or quantum tunnelling through the barrier, may take a while, and so as the Universe expands, it supercools (remains in the 'metastable' symmetric state below $T \simeq T_c$).

If the Universe supercools much below T_c, the vacuum energy term dominates the r.h.s. of (1), and the Universe begins an exponential growth (de Sitter) phase: $T^{-1} \sim R \sim \exp(t/t_{exp})$, $t_{exp} \simeq m_{pl}/T_c^2$.

The transition to the asymmetric vacuum (and the end of the de Sitter phase) occurs as bubbles of true, stable (asymmetric) vacuum nucleate and grow. The latent heat which is associated with the phase transition and is $O(T_c^4)$, is contained in the expanding bubble walls, and if released in bubble-wall collisions can result in the reheating of the Universe to $O(T_c)$.

If this reheating occurs, then the size and entropy of the Universe are thereby increased by a factor of $O[(R/R_o)^3]$, where R/R_o is the growth of the scale factor which occurs during the de Sitter phase. Guth[11)] argues that a growth factor $R/R_o \simeq O(10^{28})$, which corresponds to the phase transition requiring a time $\gtrsim O(65\ t_{exp})$ to complete itself, is sufficient to 'explain' the first 3 cosmological conundrums, and to dilute the monopole abundance to an acceptable level[13)], and if the Universe reheats to $T \simeq O(10^{14}$ GeV), baryogenesis can occur after the reheating in the usual way[8)]. Unfortunately, for models in which sufficient supercooling occurs (a temperature before reheating $\lesssim$ 0.1 K), the nucleation of bubbles is so slow that bubbles never collide. Basically, this is because their growth and creation rates can't keep up with the exponential expansion of the intervening regions of de Sitter Universe. Thus the bubble wall collisions which are crucial for the reheating and 'graceful' return to a radiation - dominated Universe essentially never occur. These problems have been discussed extensively by Guth[11)] and others[14)].



The 'swiss-cheese' Universe which results from the 'ungraceful' termination of a period of inflation. Empty bubbles of true vacuum are surrounded by the exponentially expanding false (metastable) vacuum phase.

III. NEW INFLATION

Recently, a new inflation scenario involving GUT s which undergo radiatively-induced SSB ('Coleman - Weinberg' SSB[15)]) has been proposed independently by

Linde[16] and by Albrecht and Steinhardt[17),18]. This scenario appears to preserve the desireable features of the original scenario while overcoming the troublesome features[19]. It was shown that in an SU(5) model with Coleman - Weinberg SSB, after the Universe supercools to a temperature of about 10^8 GeV, the barrier between the symmetric and asymmetric vacua becomes small, and that due to thermal fluctuations, the metastability limit of the transition is reached. The symmetric, or 'false vacuum' phase which had been metastable becomes unstable, and thermal fluctuations drive the Universe out of the symmetric phase and towards the asymmetric, or 'true vacuum' phase.[20]

In Coleman - Weinberg SSB there are no dimensionful coupling constants. The only parameter with dimensions of length that can affect the Universe when ϕ(the adjoint Higgs whose vacuum expectation value breaks SU(5)→SU(3)xSU(2)x U(1)) is small, is the inverse temperature, T^{-1}. Thus, the size of a typical fluctuation region should be $O(T_1^{-1}) \sim 10^{-22}$ cm, where $T_1 \simeq 10^8$ GeV is the metastability limit. The Coleman - Weinberg potential is very flat near $\phi=0$ (see Fig. 1), and once a region makes it over the barrier, the time required to evolve to the 'true vacuum', the asymmetric minimum at $\phi=\sigma\simeq 4.5\times 10^{14}$ GeV, is long compared to the expansion timescale $t_{exp} \simeq m_{pl}/T_c^2 \simeq 10^{-34}$ s. Until ϕ evolves to $\phi\simeq\sigma$, the vacuum energy density is $O(T_c^4)$ and dominates the energy density of the Universe so that the expansion is exponential. This accounts for the key feature of the new scenario: after a fluctuation region has overcome the potential barrier (and hence, is no longer trapped in the 'false vacuum'), and as it slowly (but inevitably) evolves towards the 'true vacuum', its size grows exponentially, until $\phi\simeq\sigma$ where the vacuum energy becomes essentially zero. If the growth factor is $O(10^{25})$, a single fluctuation region can become large enough to encompass the entire observable Universe, and as we shall discuss in §IV 'explain' the cosmological conundrums described in §I. This is in contrast to Guth's original scenario in which the exponential growth occurred while the Universe was still 'trapped' in the metastable vacuum. As we shall see, when ϕ 'races down' the steep part of the potential near $\phi \simeq \sigma$, the rapid variation of ϕ causes the 'vacuum energy' (which is at this point a combination of potential and coherent kinetic energy) to be converted into particles, reheating the Universe to $O(T_c) \sim 10^{14}$ GeV.

I will now focus on the evolution of a single fluctuation region, from the time it overcomes the barrier until it 'comes to rest' in the 'true vacuum' ($\phi=\sigma$). The physical quantities of interest are: ϕ, R, and T, the temperature of the region, which is determined by the radiation energy density, $\rho_r \simeq g_*(\pi^2/30)T^4$. The vacuum expectation of the adjoint Higgs is

$$\Phi = \begin{pmatrix} 1 & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ & 0 & & -3/2 & \\ & & & & -3/2 \end{pmatrix} \phi \ , \tag{3}$$

which is responsible for the breaking of SU(5) → SU(3) x SU(2) x U(1). The effective Higgs potential at finite temperature is given by

$$V_T(\phi) = B\phi^4 \left\{ \ln(\phi/\sigma)^2 - \tfrac{1}{2} + \tfrac{1}{2}(\sigma/\phi)^4 \right\} + 18T^4/\pi^2 \int_0^\infty x^2 dx \ln\left\{1 - \exp[-(x^2 + 25\, g^2\phi^2/T^2)^{\frac{1}{2}}]\right\} \ , \tag{4}$$

where $B = (5625/1024\pi^2)\, g^4$, g is the gauge coupling constant, and $\sigma = 4.5 \times 10^{14}$ GeV. $V_T(\phi)$ is shown in Fig. 1. Once a fluctuation region has overcome the barrier, the finite temperature corrections (the last term in (4)) to the effective potential are unimportant, and for purposes of calculating the subsequent evolution can be ignored.

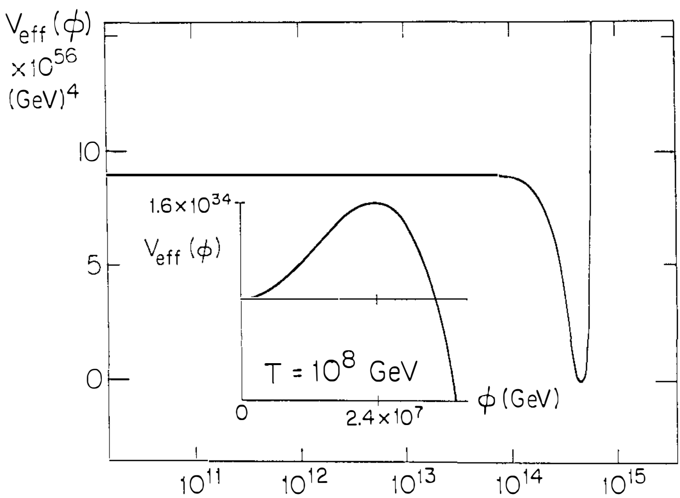


Fig. 1 - The effective potential at T = 0 ('the vacuum energy') as a function of ϕ, cf. equation (4). The insert shows the barrier which exists between the metastable vacuum (ϕ = 0) and the true vacuum at T = 10^8 GeV. Note, the scale for ϕ is linear in the insert, and 1.6×10^{34} GeV^4 is the height of the barrier above the potential at ϕ = 0.

The time evolution of ϕ and ρ_r are determined by the vanishing of the total divergence of the stress energy tensor of the adjoint Higgs field Φ and the radiation fields:

$$\frac{d}{dt}\,[(15/4)\dot{\phi}^2 + V(\phi)] = -3\frac{\dot{R}}{R}\,(\frac{15}{4}\,\dot{\phi}^2) - \delta\,, \tag{5}$$

$$\frac{d}{dt}\quad \rho_r = -4\,\frac{\dot{R}}{R}\,\rho_r + \delta\,, \tag{6}$$

where ϕ is assumed to be constant within a fluctuation region, and δ represents the energy density per unit time which is drained from the Higgs field due to the radiation of particles. The factor of 15/2 which multiples $\frac{1}{2}\dot{\phi}^2$ arises from taking the trace of $\frac{1}{2}\,\dot{\Phi}^2$. The $(\dot{R}/R)$ terms on the r.h.s. of equations (5) and (6) are the usual energy loss terms which result from the expansion of the Universe.

A term like δ is expected because all the quantum particle fields which obtain a mass from the vacuum expectation value ϕ are coupled to a time-varying classical field - the vacuum expectation value ϕ. It is difficult to calculate the precise form of such a term from first principles. However, it should depend upon ϕ and $\dot{\phi}$, and the most general, dimensionally - correct term involving just those two quantities is

$$\delta = a\ g^2\ \phi^{5-2d}\ \dot{\phi}^d\quad , \tag{7}$$

where a is a dimensionless constant, and the factor of g^2 has been included since δ is likely to depend upon the coupling strength. A variety of values for d and a have been considered, and fortunately the numerical results are extremely insensitive to both. In what follows, I shall take d = 2 since that allows me to elucidate the numerical results by solving the equation for ϕ approximately in two regimes. With $\delta = a\,g^2\dot{\phi}^2\phi$, equations (5) and (6) become,

$$\ddot{\phi} + \dot{\phi}\,[3(\dot{R}/R) + (2a/15)g^2\phi] + (2/15)V'(\phi) = 0\ , \tag{5a}$$

$$\dot{\rho}_r + 4(\dot{R}/R)\rho_r - ag^2\dot{\phi}^2\phi\ = 0, \tag{6a}$$

where prime denotes $d/d\phi$. Equations (5a) and (6a) must be supplemented by the evolution equation for R, equation (1):

$$(\dot{R}/R)^2 = 8\pi(\rho_r + V(\phi) + \tfrac{1}{2}\dot{\phi}^2)/3m_{pl}{}^2 - k/R^2. \tag{8}$$

In addition, the gauge coupling constant evolves with ϕ; following Sher

$$g^2(\phi) = 12\pi^2/(10\ \ln\ \phi^2/\Lambda^2)\ , \tag{9}$$

where $\Lambda \simeq 2.8 \times 10^5$ GeV, so that $\alpha_{GUT} \equiv g^2(\sigma)/4\pi \simeq 1/45$.

Equations (5a), (6a), (8), and (9) form a set of four, coupled ordinary differential equations which can be integrated to obtain the time evolution of ϕ, R, and T. The initial value of ϕ (i.e., the value of ϕ in the fluctuation region just after it penetrates the barrier) should be of order of the metastability temperature $T_1 \simeq 10^8$ GeV. Therefore, these equations have been integrated subject to the following initial data: $\phi(0)=\rho_r^{1/4} = \beta\, 10^8$ GeV, $R(0) = R_o$, and $\dot{\phi}(0) = 0$. The evolution of ϕ and $T \simeq \rho_r^{1/4}$ are shown in Fig. 2 for a=1. and $\beta = 3$.

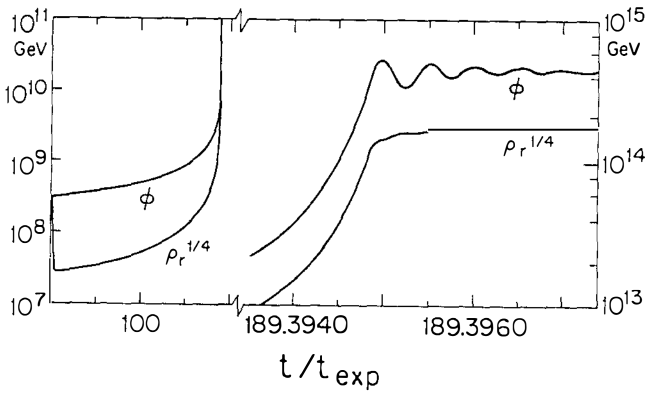


Fig. 2 - The time evolution of ϕ and $\rho_r^{1/4} \simeq T$ for $\phi(0) = 3 \times 10^8$ GeV ($\beta = 3$) and a=1. Time is measured in units of $t_{exp} \simeq (7.2 \times 10^9\ \mathrm{GeV})^{-1} \simeq 0.91\mathrm{x}10^{-34}$ s. Note the drastic change in timescale near $t = 189\ t_{exp}$.

I will now discuss the two interesting regimes in detail: (i) $\phi \simeq \phi(0) = \beta 10^8$GeV, owing to the flatness of $V(\phi)$ ϕ grows very slowly, and essentially all the growth in R occurs in this regime; (ii) $\phi \simeq \sigma$, ϕ changes very rapidly here ($\tau_\phi \sim \phi/\dot{\phi} << t_{exp}$), and the energy in the Higgs field ($\frac{1}{2}\dot{\phi}^2 + V$) is quickly converted to radiation, reheating the Universe to $O(T_c)$ and damping the motion of ϕ.

(i) $\phi \simeq \phi(0) << \sigma$: When $\phi \simeq \beta\ 10^8$ GeV the energy density of the Universe is dominated by $V(\phi) \simeq \frac{1}{2} B\sigma^4$ and $R = R_o \exp(t/t_{exp})$, where $t_{exp}^{-1} = (4\pi B/3)^{\frac{1}{2}}\ \sigma^2/m_{pl} \simeq 7.2 \times 10^9$ GeV $\simeq (10^{-34}\mathrm{s})^{-1}$. Equation (5a) can be linearized and for $\phi \simeq \phi(0)$, $\phi(t) \simeq \phi(0) \exp(\lambda t)$. For a $\lesssim$ 1700 $(1 - 8.5 \times 10^{-2} \ln\beta^2)\beta^{-1}$, the 'friction' due to the $3(\dot{R}/R)\ \dot{\phi}$ term regulates how fast ϕ grows, and

$$\lambda^{-1} \simeq 1817\ \beta^{-1.6}\ t_{exp}\ . \tag{10}$$

Since the potential steepens rapidly ($V'(\phi) \propto -\phi^3$), most of the time required for ϕ to reach $\phi \simeq \sigma$ elapses while $\phi \simeq \phi\ (0) << \sigma$, and the Universe is expanding exponentially. Thus, the 'inflation factor' R/R_o should be determined by λ: $\ln(R/R_o) \simeq O(10^3)\ \beta^{-1.6}$. The growth factor R/R_o is shown in Fig. 3 as a function of a for $\beta = 1, 3, 10$. As I will discuss in § IV, for $\beta \lesssim 7$ there is sufficient inflation, i.e., $R/R_o \gtrsim O(10^{25})$, to resolve the 'cosmological conundrums'. For large values of a, the radiation damping controls the growth of ϕ, and the growth factor R/R_o is even larger for a fixed value of β.

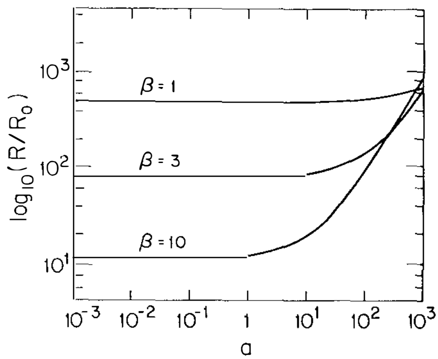


Fig. 3 - The growth factor of the fluctuation region, R/R_o, from $\phi = \phi(0) = \beta\, 10^8$ GeV, until $\phi=\sigma$ and the region has been reheated, as a function of a for β = 1, 3, 10. A growth factor log $(R/R_o) \gtrsim 25$ is needed to resolve the 'cosmological puzzles'.

(ii) $\phi \simeq \sigma$: In this regime ϕ oscillates around its value at the symmetry - breaking minimum, $\phi = \sigma$. The damping term due to the expansion of the Universe is negligible, and to a good approximation the equation for the evolution of ϕ is just that of a damped harmonic oscillator. The solution to (5a) in this approximation is,

$$\phi(t) - \sigma = \sigma \exp(-t/\tau_d) \cos(2\pi t/\tau_{osc}) \; . \tag{11}$$

For a $\lesssim$ 12 the oscillation period is: $\tau_{osc} \simeq 5 \times 10^{-4}\, t_{exp}$, and the damping timescale is: $\tau_d \simeq 2\, \tau_{osc}/a$. For a $\gtrsim$ 12 particle radiation drains energy so fast that the motion of ϕ is critically damped.

Since the oscillation period and damping time due to particle radiation are short compared to t_{exp}, the motion of ϕ is damped, and the vacuum energy converted to radiation in much less than an expansion time. Thus, the vacuum energy is efficiently changed into radiation, and is not redshifted away by the expansion (as one might have worried that it might be). The maximum value of $\rho_r^{1/4}$ is shown in Fig. 4 as a function of a for β = 3. If all the vacuum energy were converted into radiation, the maximum value of $\rho_r^{1/4}$ would be 1.73×10^{14} GeV. For $10^{-3} \leq a \leq 10^3$ the maximum value of $\rho_r^{1/4}$ is more than 60% of this. For small values of a, τ_d becomes comparable to t_{exp}, and thus during the time required to damp the motion of ϕ the scale factor R grows significantly, redshifting away the energy in the ϕ -field and in ρ_r. For large values of a, a significant fraction of the energy contained in the ϕ-field is changed into radiation while the Universe is still expanding exponentially ($\phi \lesssim \sigma$), and is redshifted away. Thus, for very large and very small values of a the maximum value of $\rho_r^{1/4}$ decreases.

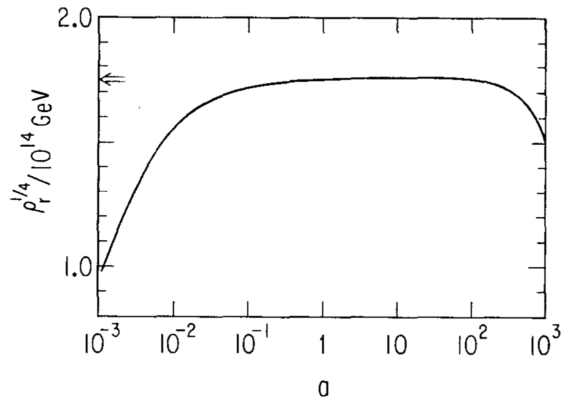


Fig. 4. - The maximum value to which $\rho_r^{1/4}$ rises after reheating, as a function of a for $\beta = 3$. The arrow indicates the value for 100% conversion of 'vacuum energy' to radiation. The reheating temperature $T \simeq \rho_r^{1/4} (30/\pi^2 g_*)^{1/4}$.

The complete evolution of $\rho_r^{1/4}$ is shown in Fig. 2. Initially, $\rho_r^{1/4}$ drops precipitously due to the exponential expansion. After a few t_{exp} the rate that energy is being 'pumped in' by ϕ and drained by the expansion reaches a balance, and $\rho_r^{1/4}$ stabilizes at a value: $\rho_r^{1/4} \simeq (a^{1/4} \beta^{3/2})$ 3 x 10^6 GeV. Then as ϕ and $\dot{\phi}$ increase dramatically, so does $\rho_r^{1/4}$. Because of the constant radiation of particles due to the time variation of ϕ, the Universe does not undergo extreme supercooling. This is in marked constrast to Guth's original scenario.

The particle species which are directly radiated due to the time-varying Higgs field are those particles which couple to the vacuum expectation value of ϕ, i.e., those species which acquire a mass during this stage of symmetry breaking. They include: the XY gauge bosons, the color triplet component of the $\underset{\sim}{5}$ of Higgs, and the adjoint Higgs Φ itself. Interactions among these species (decays, $2 \leftrightarrow 2$ scatterings, etc.) should populate the light particle species and lead to thermal distributions of particles. The temperature T and radiation energy density ρ_r are related by (2), so that

$$T \simeq (30/\pi^2 g_*)^{1/4} \rho_r^{1/4} . \tag{12}$$

Since $g_* \simeq 0(10^2)$, $T \simeq \rho_r^{1/4}/2$.

Before going on to discuss how the 'cosmological puzzles' described in § I are resolved in this scenario, I will briefly summarize the evolution of ϕ, R, and T. At a temperature of $0(10^8$ GeV), thermal fluctuations are sufficient to allow regions of the Universe to go from the metastable ('false') vacuum ($\phi = 0$) through the barrier and begin the inevitable 'downhill slide' to the true vacuum ($\phi = \sigma$). When the slide begins, the size of a typical fluctuation region is $\sim 0(T^{-1}) \simeq 10^{-22}$ cm and the (approximately) constant value of ϕ within the region is $T \simeq 0(10^8$ GeV). Because the Coleman - Weinberg potential is so flat, the early evolution of ϕ (which occurs while the energy density of the Universe is still dominated by vacuum energy) is very slow compared to t_{exp}. It is during this period, as ϕ grows from 0(T) to $0(\sigma)$, that the exponential growth of R takes place. When $\phi \simeq \sigma$ the potential becomes very steep, and the timescale for the evolution of ϕ ($\tau \sim \sigma^{-1}$) is short compared to t_{exp}. In much less than an expansion time, ϕ oscillates about $\phi = \sigma$, causing particles which couple to ϕ to be radiated. This particle radiation results in: the smooth reheating of the fluctuation region to $0(10^{14}$ GeV), and the damping of the motion of ϕ. The conversion of coherent Higgs field energy to radiation and the return to radiation - domination occurs very quickly ($\Delta t \ll t_{exp}$), so that essentially all the vacuum energy is converted to radiation, with little being redshifted away by the expansion.

IV. RESOLVING THE COSMOLOGICAL CONUNDRUMS

Precisely how does the scenario just described resolve the cosmological conundrums discussed in § I? First consider the <u>isotropy</u> and <u>large-scale homogeneity</u> puzzles. The large-scale structure of the Universe might be very irregular at the epoch when our fluctuation region forms. However, since its physical size, $T^{-1} \sim 0(10^{-22}$ cm) is smaller than the particle horizon at this epoch, $d_H \sim 10^{-17}$ cm, it is reasonable to expect that the temperature within the region is uniform and that the region forms coherently. During the de Sitter phase the physical size of the region grows by more than a factor of 10^{25}(if $\beta \lesssim 7$), to a size $\gtrsim 10^3$ cm. The present observable Universe ($d_H \sim 10^{28}$ cm), had a size of only ~ 1 cm when the temperature of the Universe was 10^{14} GeV (this is based upon the assumption of adiabaticity from the epoch of reheating until today, <u>i.e.</u>, RT = constant). Thus, 'our present Universe' is easily contained within one fluctuation region (if $R/R_o \gtrsim 10^{25}$). Although during the inflationary period the particle horizon grew to be $\gtrsim 10^3$ cm, (i.e., encompassed the entire fluctuation region) the distance over which a light signal could have traveled since reheating is given by the usual formula, $d_H \simeq n^{-1}\,ct$, where $n = \frac{1}{2}$ for a radiation - dominated Universe and $n = 2/3$ for a matter - dominated Universe. Thus, the region from which we could have received light signals since reheating is just the usual 'observable Universe' ($d \sim 10^{28}$ cm), while the whole fluctuation region which was once causally - coherent has a present size of $\gtrsim 10^{31}$ cm (see Fig. 5).

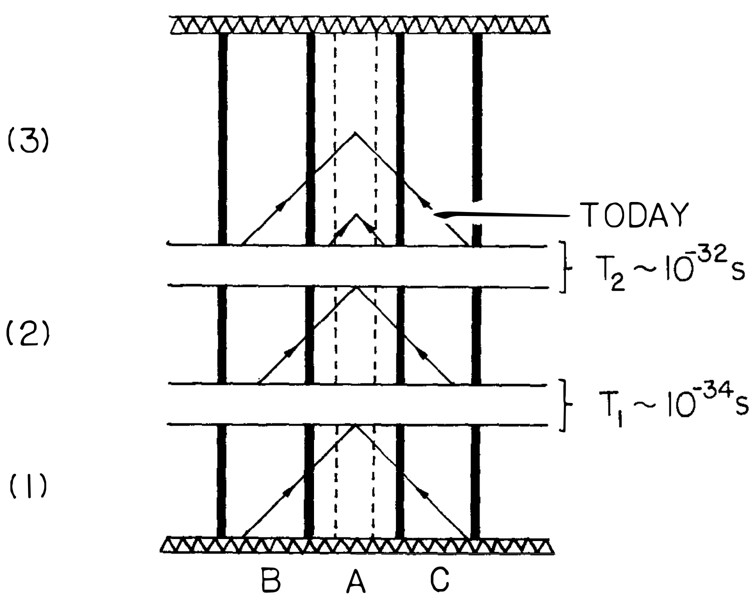
Had the period of inflationary growth not occurred, and had the Universe been able to evolve to its present state (i.e., not recollapsed first, etc.), then our present observable Universe would include many, many regions which in the inflationary scenario would have evolved as separate fluctuation regions. Without inflation the present observable Universe would encompass $\sim (1\text{ cm} / 10^{-22}\text{ cm})^3 \sim 10^{66}$ would be fluctuation regions! In this sense, the inflationary scenario is anti-inflationary!

If the Universe had been very irregular at the GUT epoch, then the subsequent evolution of fluctuation regions separated by distances greater than the horizon distance then, would be very different. However, the period of exponential growth 'pushed' these other regions outside of our present observable Universe. However, the de Sitter phase has only postponed the inevitable, eventually our particle horizon will grow to encompass other fluctuation regions. If the inflationary scenario is correct, then when this occurs the Universe will probably look highly irregular on the largest scales.[21a)]

During the exponential growth (de Sitter) phase, any growing mode perturbations of inhomogeneity or anisotropy are damped by a factor $\gtrsim (R/R_o)^2 \gtrsim 10^{50}$ (if $\beta < 7$).[22)] Of course, after the de Sitter phase ends, these perturbations will once again begin to grow - the ultimate irregularity having been only postponed. Hawking and Moss[18)] have likened the de Sitter phase, which effectively smooths out our local region, to a 'cosmic no hair theorem'.

What about the oldness/flatness puzzle? Before the de Sitter phase the ratio of the curvature term to the energy density term might have been O(1) or even greater if $k = -1$.[23)] After reheating, the energy density is once again $O(T_c^4)$, while R has grown by $\gtrsim 10^{25}$, so that k/R^2 has been reduced by a factor $(R/R_o)^2 \gtrsim 10^{50}$. Thus the value of the ratio $(k/R^2)/(8\pi G\rho/3)$ is 'naturally' set to a very small, $\lesssim O(10^{-50})$, number, which allows our Universe to evolve to the ripe old age of $\sim 10^{60}\ t_{pl}$ without becoming dominated by the curvature term. The ratio of the density of the Universe to the critical density ($\equiv 3H_o^2/8\pi G$), Ω, is related to the present value of $x = (k/R^2)/(8\pi G\rho/3)$, by

Fig. 5 - A (schematic) conformal diagram which illustrates the causal structure of the new inflationary Universe. The horizontal axis is space-like, the vertical axis is time-like, light rays propagate on 45^o lines, and comoving observers move on vertical, time-like paths. (1) The Universe begins as a k = +1 FRW model ('the teeth' represent the initial singularity). Region A will become the fluctuation region that we live in, and the broken lines denote the portion of A which becomes our present observable Universe. Regions B and C will become neighboring fluctuation regions. At $t_1 \simeq m_{pl}/T_c^2 \simeq 10^{-34}$ s the 'vacuum energy' begins to dominate ρ and the de Sitter phase commences. Just before it does, an observer within what will be our present observable Universe could have received information from all of A and parts of B and C. (2) The Universe is in a de Sitter phase until ϕ evolves from $\phi \simeq 0$ to $\phi \simeq \sigma$ and the Universe is reheated. This occurs at $t_2 \simeq (10^2 - 10^3)\ t_1 \simeq 10^{-32}$ s. During the de Sitter phase our hypothetical observer could have received information from all of A and parts of B and C. (3) The Universe again evolves like a k = +1 FRW model. By today (t ≃ 15 BY) our observer, since reheating, could have only received light signals from our present observable Universe. If the present size of our fluctuation region is 10^{31} cm, say, then not until t ≃ 10^{10} BY could she receive light signals from all of A, and start receiving information from B and C. If recollapse commences before this time, she will be able to 'see all of A' sooner than this. [This diagram is similar to Fig. 2 of ref. 21a.]



$$\Omega = 1/(1 - x) \tag{13}$$

Since $\rho \propto R^{-n-2}$, the ratio x grows as the Universe expands: $x = x_i (R/R_i)^n$; n = 2 (radiation - domination), n = 1 (matter - domination). Since reheating x has grown by a factor of order 10^{49}. Unless the exponential growth of R during the de Sitter phase was very close to $R/R_o \simeq 10^{25}$, the value of x today should still be << 1, and hence Ω should be equal to 1, to a high degree of precision.

Monopoles are topological defects associated with the orientation of the Higgs field. In the non-inflationary scenario it is argued that because of causality the orientation of the Higgs field can only be smooth on scales $\lesssim$ the horizon distance[10]. Thus, of the order of one monopole (topological defect) should result per horizon volume. This predicted relic abundance of monopoles exceeds the bound provided by the present mass density of the Universe by at least at factor of 10^{12}. In the inflationary scenario the present observable Universe lies within what was once one causally - coherent region in which the Higgs field could have had a uniform orientation (and indeed this is the lowest energy configuration). Thus, the only monopoles which should be present today are those which are produced by particle collisions. This number depends critically upon the temperature to which the Universe is reheated and the mass of the lightest monopole[24],

$$\frac{n_M}{n_\gamma} \simeq 3 \times 10^3 \, (m/T)^3 \exp(-2m/T), \tag{14}$$

here n_M and n_γ are respectively the monopole and photon number densities today. The quantity m/T in the new inflationary scenario could plausibly be in the range 40 to 80, resulting in a very large uncertainty in the predicted value of $n_M/n_\gamma \simeq 10^{-26} - 10^{-60}$. This corresponds to $\Omega_M \simeq 10^{-2} - 10^{-36}$.

The magnetic force on a 10^{16} GeV monopole within our galaxy is 0(100) times greater than the gravitational force on it, and thus it is unlikely that 10^{16} GeV monopoles would cluster with our galaxy[24b]. Assuming that they do not cluster with galaxies, and that their velocity relative to galaxies is $0(10^{-3}c)$ - due to the peculiar velocities of galaxies through the Universe, then the flux of monopoles is expected to be: $F \lesssim 10^{-17} cm^{-2} sr^{-1} s^{-1}$ (note, $10^{-12} cm^{-2} sr^{-1} s^{-1}$ = 1 event per m^2 - πsr - yr). The survival of galactic magnetic fields and the mass density of the Universe preclude a flux (of 10^{16}GeV, unclustered monopoles) greater than $10^{-15} cm^{-2} sr^{-1} s^{-1}$ [24a,24b]. The prediction for the new inflationary scenario easily fall below these limits.

Recently, Cabrera[24c)] has reported evidence for a single candidate monopole event, which naïvely corresponds to a 'flux' of $10^{-9} cm^{-2} sr^{-1} s^{-1}$. If this 'flux' is indicative of the average galactic flux, it exceeds the bounds based upon the survival of the galactic field by at least a factor of 10^3 (ref. 24). These limits do not preclude local sources (e.g., the sun); a monopole to nucleon ratio in the sun as low as 10^{-22} might 'explain' Cabrera's flux.[25)] If this were to represent a universal monopole to nucleon ratio of 10^{-22}, then it would correspond to an average monopole to photon ratio of $O(10^{-32})$ - which could be produced by particle collisions after reheating.

In addition to resolving the domain problem associated with the Higgs field (i.e., monopoles), the potentially - catastrophic problem of domain walls which arises when discrete symmetries are spontaneously broken[26)], is avoided since our observable Universe lies within one domain. The discrete symmetries which might be spontaneously broken include: C, CP, the reflection symmetry $\phi \rightarrow -\phi$ which the Higgs potential often has, and Z(N) - a discrete symmetry associated with the Peccei - Quinn (PQ) symmetry. The PQ symmetry is often imposed to avoid the 'strong CP problem'.[27)] Again, the problem is only postponed, since these walls (if they exist) will eventually enter our observable Universe.

In the standard scenario for producing the baryon asymmetry[8)], all the important processes happen at temperatures $T < M$, where $M \sim O(10^{14}$ GeV) is the mass of the superheavy boson whose out-of-equilibrium decays produce a net baryon number. Since the Universe is reheated to a temperature of $O(10^{14}$ GeV) baryogenesis should proceed in the usual way. The details and the final asymmetry produced may be slightly different since the superheavy bosons responsible for producing the baryon asymmetry may be initially under- or over-abundant, depending upon precisely which particle species are produced by the time - variation of ϕ. However, this should only change the quantitative aspects of baryogenesis and not the qualitative fact that a baryon asymmetry of the desired magnitude can be produced in the usual way.[28)] If the C, CP violation necessary for baryogenesis is spontaneous rather than intrinsic, then as mentioned above the usual problem of domain walls is avoided, and the problem of small matter and antimatter domains in a Universe which is symmetric overall does not occur since the observable Universe should be contained within one such C, CP domain.

The reheating of the Universe to a temperature of $O(10^{14}$ GeV) is crucial for the subsequent evolution to a Universe which locally (i.e., within the present horizon) resembles ours. In fact, as Guth has emphasized[11)], it is the reheating which prompted the name, 'inflationary Universe'. I will briefly elaborate. Taking the size of the fluctuation region which our Universe is within to be $O(10^{-22}cm)$ when the metastability limit is reached ($T \sim 10^8$ GeV), there are only O(100) particles within that region when the de Sitter phase begins, and in the absence of particle radiation due to the time-variation of the Higgs field, the same number when $\phi = \sigma$ and the region has grown to a size $\gtrsim 10^3$cm. [I have taken the number density of particles to be: $n \simeq O(10^2)\ T^3$.] After the vacuum energy is converted into radiation and the temperature rises once again to $O(10^{14}$ GeV), the number of particles within the region increases to $O(10^{92})$! [For comparison there are $O(10^{86})$ photons within the present observable Universe.] The origin of the term 'inflation' is now manifest: the essential feature of the inflationary scenario is 'the minting of new coins of the realm', i.e., particles. It is amusing to note that a minimum of 92 digit inflation is required to resolve the cosmological conundrums!

Finally, there is the issue of the inhomogeneities which are necessary for the ultimate formation of the structure which is so conspicuous in the Universe today. From the discussion above it should be clear that any pre-inflationary inhomogeneities that were present would be drastically reduced. In addition, the reheating process proceeds very smoothly, and in the approximation that ϕ is spatially - constant within a fluctuation region, the region is uniformly reheated. Of course, some inhomogeneity in ϕ is expected even though the fluctuation of least action is one in which ϕ is constant throughout. It may be that such variations in the initial value of ϕ within a fluctuation region can lead to a suitable spectrum of density perturbations. Or, it is possible that the necessary density perturbations are produced in a phase transition which occurs after the inflationary phase.[29)] Thus far, the 'new inflationary scenario' has not shed any light on the origin of the density fluctuations, and in fact, precludes the possibility that the primordial perturbations were produced during the quantum gravity epoch.

V. THE MINIMAL SU(5) MODEL: A GRAND UNIFIED THEORY?

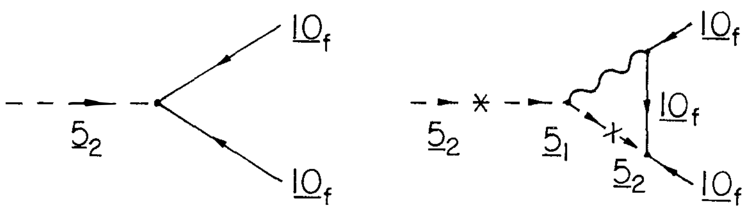
In this concluding section I will begin with some rather optimistic remarks about the current state of affairs in cosmology and particle physics, and finish with some sobering comments about 'the new inflation'.

Consider the following 'minimal SU(5) model': three (or four, if necessary) families of quarks and leptons, an adjoint 24 of Higgs, 2 vector 5 s of Higgs, and a Coleman - Weinberg radiatively - induced scalar potential(for the purpose of inflation). This is the minimal Higgs structure which allows the Peccei - Quinn symmetry to be incorporated into the model[30)]. Such a symmetry is an attractive way of solving the 'strong CP problem'. Without this symmetry or some other mechanism, there is nothing to prevent the neutron electric dipole moment from being a factor of $O(10^8)$ larger than the present upper limit. One of the Higgs $\underline{5}$'s, say $\underline{5}_1$, couples only to $\underline{10}_f \times \underline{10}_f$ giving masses to the up-like quarks, and the other, $\underline{5}_2$, couples only to the $\overline{\underline{5}}_f \times \underline{10}_f$ giving masses to the down-like quarks and the charged leptons ($\bar{d}_L$, e^-_L, ν_L are put in the $\overline{\underline{5}}_f$ and u_L, d_L, $\bar{u}_L$, e^+_L are put in the $\underline{10}_f$).

When viewed as an effective, low-energy theory (i.e., $E \lesssim 10^{15}$ GeV), this model is rather successful, and at present without fatal disease. Among other things this model unifies the very low energy ($E \lesssim 1$ TeV) gauge group $SU(3) \times SU(2) \times U(1)$, 'explains' charge quantization, predicts $\sin^2\theta_w$ with accuracy comparable to or slightly better than the experimental and theoretical uncertainties, predicts a proton lifetime of $O(10^{30\pm1}$ yrs) which is consistent with the observed stability (thus far) of the proton, and which is accessible to the current round of proton decay experiments, makes a successful prediction of the bottom quark to tau lepton mass ratio, and does not suffer from 'strong CP sickness'. Among its failures are its predictions of light fermion masses, and it does not have the capability of providing neutrino masses. Ellis and Gaillard[31] have pointed out that if it is only viewed as an effective theory, then it may also have additional nonrenormalizable terms in the Lagrangian which scale with inverse powers of the planck mass. They argue that since the grand scale is only 4 or so orders of magnitude lower than the planck scale, there may be residual effects associated with this scale which are not negligible. Although the effects of these terms in general should be small, they could quite plausibly make significant contributions to the light fermion (including neutrino) masses. In fact, they would be large enough to bring the light fermion mass ratio predictions into accord with experiment, and provide neutrino masses of $O(10^{-5}$ eV).[32)] The minimal model described above, while a potentially viable and thus far very successful effective theory, sheds no light on the gauge hierarchy problem (i.e., the difficulty of having at least two very different scales of symmetry breaking: $\sim 10^{14}$ GeV and ~ 1000 GeV), the reason why there are 3 (or more) generations of fermions, or the origin of family mixing.

This minimal model has a number of very attractive consequences for cosmology. As I have discussed at length in § IV, within the context of new inflation, it has the potential to 'explain' the isotropy, homogeneity, and flatness/oldness of the Universe. It is also free of the domain wall and monopole problems. Of course, there is the original cosmological success of grand unification - the dynamical explanation of the baryon asymmetry of the

Universe. This is by no means a trivial success. In the original minimal SU(5) model (one $\underline{24}$ of Higgs, and one $\underline{5}$ of Higgs which coupled both to $\bar{\underline{5}}_f \times \underline{10}_f$ and $\underline{10}_f \times \underline{10}_f$) the calculated C, CP violation was far too small to account for the present value of the baryon to photon ratio[33]. Of course, the model also suffered from 'strong CP disease'. In the present minimal model, the necessary C, CP violation arises at the one-loop level in the decays of the color triplet component of the $\underline{5}$ of Higgs, and can easily be large enough to produce the desired baryon asymmetry[34].



The tree-graph and one-loop diagrams for the decay of the color-triplet component of $\underline{5}_2$. Their interference results in the C, CP violation necessary for the decays of $\underline{5}_2$ and $\bar{\underline{5}}_2$ bosons to produce a baryon asymmetry. The wavy line is a gauge boson and the x's are (complex) mass insertions.

In short, it is possible and without much difficulty) to paint a very rosy picture of the present state of affairs in cosmology and in particle physics.

In the excitement of the moment, it is also possible to overlook the loose thread(s) which could unravel the nearly-complete tapestry. There are of course the long standing problems of the gauge hierarchy, and of the number of families (why N, N $\geq$ 3?). The new inflationary scenario relies on the effective potential being Coleman - Weinberg like to a high degree of precision (i.e., very flat near $\phi = 0$, and very steep near $\phi = \sigma$). In a sense, this potential is perfectly suited (and with no room to spare!) to 'the new inflation'. It is easy to imagine induced terms in the potential which would spoil the scenario (e.g., an induced curvature term). The issue of curved-space effects has not been resolved yet. Thus far, the new scenario has not elucidated (and in fact has exacerbated) the problem of the origin of the density fluctuations necessary for galaxy formation. All inflationary scenarios take for granted that the vacuum energy density, which is formally infinite and known to have negligible influence on the evolution of the Universe at present, played a significant (in fact dominant) role much earlier. This in fact may be true. However, I do not think that it is unlikely that a fundamental understanding of the cosmological term, which will likely involve a deep connection between quantum field theory and gravity, may have some surprises in store for us. Perhaps, the vacuum energy term has always been negligible or zero! There is also the rather more mundane problem of the dark matter: Is it neutrinos of mass 10 - 100 eV? Is it gravitinos? Or, is it perhaps, baryons? Most of the problems which the new inflation 'resolves' involve initial data. Since there is an epoch about which our present understanding is very limited ('$t = 0 - 10^{-43}$ s', the quantum gravity epoch), there is always the possibility that the appropriate initial data are presented to us at t_{pl} due to quantum gravitational effects.

At the very least, 'the new inflation' is a very attractive scenario, and thus far the only inflationary scenario which evolves to a Universe which at least locally resembles our observable Universe. One set of necessary conditions for 'successful inflation' have been spelled out.

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