

## On neutrino properties and gravitational waves

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Multi-messenger astronomy can be a relevant tool for getting information about neutrino masses and their ordering using the measurement of the time lapses between the arrival of neutrinos and the other light messenger, i.e. the graviton, emitted in astrophysical catastrophes. We elucidate the experimental reach and challenges for planned neutrino detectors such as Hyper-Kamiokande as well as future several megaton detectors.

### 1 Introduction

The fascinating discovery by the LIGO collaboration<sup>2</sup> has opened up a new way of exploring the Universe. Gravitational waves (GW)s carry detailed information about astrophysical catastrophes and can provide a clear reference time for multi-messenger astronomy.

In particular it is interesting to understand how to use these events and the related observables as new tools to settle some open issues in particle physics.

We know that the Standard Model (SM) cannot be the ultimate theory of Nature since the neutrino sector and dark matter are not yet properly accounted for. In fact, the nature of the three light active neutrinos  $\nu_i$  ( $i = 1, 2, 3$ ) with definite mass  $m_i$  is unknown. To date, neutrinos can still be Dirac fermions if particle interactions conserve some additive lepton number, e.g. the total lepton charge  $L = L_e + L_\mu + L_\tau$ . However, if the total lepton charge is violated, they can have a Majorana nature<sup>3,?</sup>. The only feasible experiment, so far, that can unveil the nature of massive neutrinos is neutrinoless double beta,  $(\beta\beta)_{0\nu}$  decay (see e.g.<sup>5</sup> for a review).

Another pressing question to answer is how light are neutrinos. Oscillation experiments are not sensitive to their masses, therefore information about their tiny mass comes from cosmology where an upper bound on the sum of the active neutrinos  $\sum_i m_i < 0.23$  eV can be established<sup>6</sup>. More recently, more stringent limits have been obtained through the Lyman alpha forest power spectrum,  $\sum_i m_i < 0.12$  eV<sup>7</sup>. These constraints will be further tested independently by other experiments such as beta decay and neutrinoless double beta decay experiments. Future large scale structure surveys like the approved EUCLID<sup>8</sup>, will allow to constrain  $\sum_i m_i$  down to

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<sup>a</sup>Talk based on<sup>1</sup>.

0.01 eV when combined with Planck data.

The goal of this work is to investigate whether experiments, making use of GW detection in combination with the associated neutrino (and photon) counterparts, can make a dent in understanding the ordering of neutrino masses and/or constrain the absolute mass.<sup>b</sup>

## 2 Neutrino orderings: current status

Current available neutrino oscillation data<sup>10,7</sup> are compatible with two types of neutrino mass spectra. As is well known, depending on the sign of  $\Delta m_{31(32)}^2$ , which cannot be determined from the presently available neutrino oscillation data, two types of neutrino mass spectrum are possible:

i) *normal ordering (NO)*:  $m_1 < m_2 < m_3$ ,  $\Delta m_{31}^2 > 0$ ,  $\Delta m_{21}^2 > 0$ ,  $m_{2(3)} = (m_1^2 + \Delta m_{21(31)}^2)^{1/2}$ ;  
ii) *inverted ordering (IO)*:  $m_3 < m_1 < m_2$ ,  $\Delta m_{32}^2 < 0$ ,  $\Delta m_{21}^2 > 0$ ,  $m_2 = (m_3^2 + \Delta m_{32}^2)^{1/2}$  and  $m_1 = (m_3^2 + \Delta m_{32}^2 - \Delta m_{21}^2)^{1/2}$ . Depending on the value of the lightest neutrino mass,  $m_{min}$ , the neutrino mass spectrum can be ( $j = 1, 2, 3$ ):

a) *Normal Hierarchical (NH)*:  $m_1 \ll m_2 < m_3$ ,  $m_2 \cong (\Delta m_{21}^2)^{1/2} \cong 8.68 \times 10^{-3}$  eV,  $m_3 \cong (\Delta m_{31}^2)^{1/2} \cong 4.97 \times 10^{-2}$  eV or

b) *Inverted Hierarchical (IH)*:  $m_3 \ll m_1 < m_2$ , with  $m_{1,2} \cong |\Delta m_{32}^2|^{1/2} \cong 4.97 \times 10^{-2}$  eV or

c) *Quasi-Degenerate (QD)*:  $m_1 \cong m_2 \cong m_3 \cong m_0$ ,  $m_j^2 \gg |\Delta m_{31(32)}^2|$  and  $m_0 \simeq 0.10$  eV.

The current cosmological bounds are strongly disfavouring the degenerate regime. However these results should be further tested independently for example by beta-decay and neutrino-less double beta decay experiments. The detection of GWs is a crucial test of general relativity and, as already discussed in the literature, it is also important to deduce other relevant physical properties. This new information can be derived when comparing, for example, their propagation velocity with those of photons and neutrinos coming both from the same astrophysical source. The observation of gravitational wave events accompanied by counterpart events, like neutrino detections, could improve our knowledge about the ordering and the masses of these tiny particles.

## 3 Ingredients

Let us consider a potential observation of an astrophysical catastrophe. Such events could be for instance, the merging of a neutron star binary or the core bounce of a core-collapsed supernova (SN). Using the same notation of<sup>12</sup>, we denote with  $T_g \equiv L/v_g$ ,  $T_{\nu_i} \equiv L/v_{\nu_i}$  and  $T_\gamma \equiv L/v_\gamma$  respectively the time of propagation of a GW, a given neutrino mass eigenstate and photons with group velocities  $v_g, v_{\nu_i}$ , and  $v_\gamma$ . Following Fig. 1 a GW is emitted at the time  $t_g^E$  from a source at distance  $L$  and detected on Earth at  $t_g$ . Similarly, we have emission and detection times for photons and neutrinos. The difference of the arrival times between the GWs and neutrinos,  $\tau_{obs} \equiv t_\nu - t_g$ , or the GW and a photon,  $\tau_{obs}^\gamma \equiv t_\gamma - t_g$ , are both observables, which can be positive or negative for an early or late arrival of a GW. Typically the emission times of the three signals (GW,  $\gamma$  and  $\nu$ ) do not coincide<sup>c</sup>. For instance in the supernova explosion SN1987A<sup>14</sup>, the neutrinos arrived approximately 2 – 3 hours before the associated photons.

Let us assume now that a neutrino is emitted at  $t_\nu^E = t_g^E + \tau_{int}^\nu$  and detected at time  $t_\nu$ . A relativistic mass eigenstate neutrino with mass  $m_i c^2 \ll E$  ( $i = 1, 2, 3$ ) propagates with a group

<sup>b</sup>The idea to test the absolute mass first appeared in<sup>9</sup>.

<sup>c</sup>In alternative theories of gravity the three particles under study — photons, gravitons and neutrinos— can couple to different effective metrics. In this case the Shapiro delay is not the same for the three signals<sup>13</sup>. In this work however we assume the same coupling to the metric for all the signals.

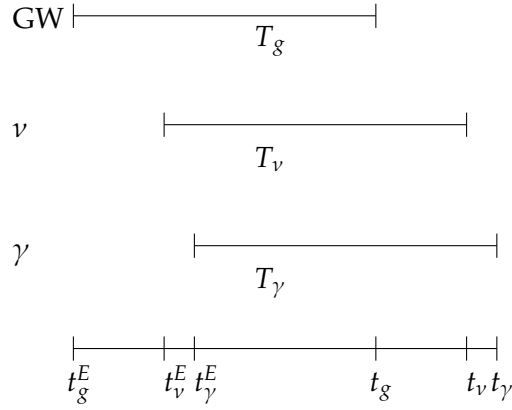


Figure 1 – GW, neutrino and photon propagation in time.

velocity:

$$\frac{v_i}{c} = 1 - \frac{m_i^2 c^4}{2E^2} + \mathcal{O}\left(\frac{m_i^4 c^8}{8E^4}\right), \quad (1)$$

where we assumed that the different species of neutrinos have been produced with a common energy value  $E$ . If a given neutrino is produced by a source at a distance  $L$ , the time-of-flight delay  $\Delta t_i$  with respect to a massless particle, emitted by the same source at the same time, is

$$\Delta t_i \cong \frac{m_i^2 c^4}{2E^2} \frac{L}{c} = 2.57 \left(\frac{m_i c^2}{\text{eV}}\right)^2 \left(\frac{E}{\text{MeV}}\right)^{-2} \frac{L}{50 \text{kpc}} \text{ s}. \quad (2)$$

Here we do not take into account cosmic expansion since we consider sources at low redshift,  $z \ll 0.1$ . This causes an error less than 5%. From the expression in (2) we observe that larger distances and small neutrino energies are needed in order to maximise the experimental sensitivity. For distances around 50 kpc (SN1987A) and an energy of 10 MeV, a neutrino with a mass of 0.07 eV (the upper current absolute mass scale inferred from the Planck collaboration <sup>6</sup>) would arrive  $\sim 10^{-4}$  s later than a massless particle. Similar to (2) we express the time delay between the arrival of two neutrino mass eigenstates as:

$$\Delta t_{\nu_i \nu_j} = \Delta t_i - \Delta t_j = \frac{\Delta m_{ij}^2 c^4}{2E^2} T_0 \quad \text{with} \quad T_0 = \frac{L}{c}, \quad (3)$$

with  $\Delta m_{ij}^2 = m_i^2 - m_j^2$  and to leading order in  $m^2 c^4 / E^2$ . We note, that in this limit the time intervals do not depend on the absolute neutrino mass scale, but solely on the square mass differences which are measured experimentally.

#### 4 Disentangling neutrino mass ordering

Using Eq. (3), we can observe that if the detector uncertainty is  $10^{-3}$  s we are able to disentangle the atmospheric (solar) squared mass differences with a signal coming from a distance larger than 0.8 (26) Mpc assuming neutrinos have an energy of about 10 MeV. These distances decrease if we lower the neutrino energy. Further this numbers scale accordingly using a better detector uncertainty.

This means, that for neutrinos with an average energy of 10 MeV, the delay time of the heaviest neutrino mass eigenstate with respect to the lightest is larger than  $10^{-3}$  s independently of the absolute neutrino mass scale and hierarchy, for distances larger than  $\sim 0.8$  Mpc. Therefore,

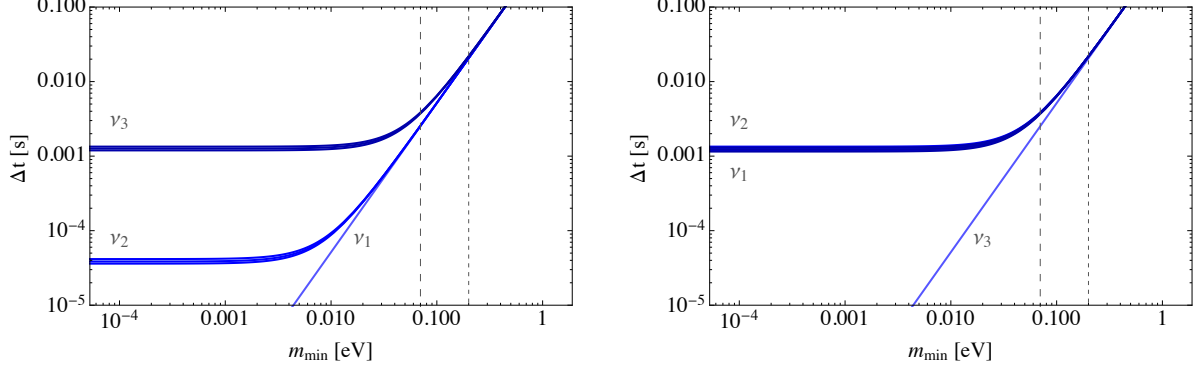


Figure 2 – The range of  $\Delta t_i$  ( $i = 1, 2, 3$ ), the time delay of neutrinos with respect to photons, vs the lightest of the neutrino masses,  $m_{min}$ , for a distance of 1 Mpc and 10 MeV. We show the results for NO and IO (left and right panels) considering a the  $3\sigma$  uncertainty in the oscillations parameters given in<sup>10</sup>. The dashed and dotted vertical lines correspond to the Planck limit on the sum of neutrinos masses and the perspective upper limits from the KATRIN experiment (more details in the text).

Table 1: (Left) Benchmark time lapses for  $\nu_1$ ,  $\nu_2$  and  $\nu_3$  respectively. We consider a distance of 10 kpc (1 Mpc) and a neutrino energy of  $E = 10$  MeV. (Right) The distance is set to 1 Mpc (10 Mpc) and the neutrino energy to  $E = 5$  MeV.

$m_{min}$ [eV]	$\Delta t_{\nu_i}$ [s]		$m_{min}$ [eV]	$\Delta t_{\nu_i}$ [s]	
	NO	IO		NO	IO
0	0	$1.23 \cdot 10^{-5}(10^{-3})$	0	0	$4.91 \cdot 10^{-3}(10^{-2})$
	$3.86 \cdot 10^{-7}(10^{-5})$	$1.26 \cdot 10^{-5}(10^{-3})$		$1.54 \cdot 10^{-4}(10^{-3})$	$5.06 \cdot 10^{-3}(10^{-2})$
	$1.26 \cdot 10^{-5}(10^{-3})$	0		$5.06 \cdot 10^{-3}(10^{-2})$	0
0.01	$5.14 \cdot 10^{-7}(10^{-5})$	$1.28 \cdot 10^{-5}(10^{-3})$	0.01	$2.06 \cdot 10^{-4}(10^{-3})$	$5.11 \cdot 10^{-3}(10^{-2})$
	$9.00 \cdot 10^{-7}(10^{-5})$	$1.32 \cdot 10^{-5}(10^{-3})$		$3.60 \cdot 10^{-4}(10^{-3})$	$5.27 \cdot 10^{-3}(10^{-2})$
	$1.32 \cdot 10^{-5}(10^{-3})$	$5.14 \cdot 10^{-7}(10^{-5})$		$5.27 \cdot 10^{-3}(10^{-2})$	$2.06 \cdot 10^{-4}(10^{-3})$

assuming an accuracy of  $10^{-3}$  s, the relevant sources are those at distances larger than 0.8 Mpc. With better time accuracy the distance decreases linearly. We show in Fig. 2 the time delay (for each mass eigenstate)  $\Delta t_i$  considering NO and IO (left and right panels respectively) as function of the lightest neutrino mass, setting the neutrino energy to 10 MeV and the distance of the source to 1 Mpc. The physically relevant arrival time differences between neutrino mass eigenstates  $\Delta t_{\nu_i \nu_j}$  can be readily determined from Fig. 2. We also report in the plot the future sensitivity on the absolute neutrino mass of the  $\beta$ -decay experiment KATRIN<sup>15</sup> which is expected to be around 0.2 eV and the constraints given by the Planck Collaboration on the sum of the light active neutrinos<sup>6</sup>  $\sum_i m_i \leq 0.23$  eV 95% CL. In Table 1 we produce relevant benchmark neutrino time lapses considering two different source-distances for different values of the lightest neutrino mass for 5 and 10 MeV neutrinos.

From Fig. 2 we observe that for the given distance and energy, the NO and IO spectra differ by having different time delay patterns. We note that for IO the delay between the two heaviest mass eigenstates is equivalent to the time lapse between the first two lighter mass eigenstates for NO. If we consider a conservative time accuracy of  $10^{-4}$  s for the next generation of detectors<sup>d</sup>, the time lapse differences between NO and IO will not be distinguishable.

However, in addition to the time information, also the ratio between the amplitudes of the different neutrinos reaching the detector can be measured. Since the distances considered here are very large, neutrinos will reach the detector incoherently such that the time integrated

<sup>d</sup>This accuracy is conservative compared with an estimate based on the uncertainty on the vertex reconstruction, which is about 3 m for Hyper-Kamiokande<sup>16</sup>. In order to obtain a global time, when comparing with other experiments, a higher uncertainty is expected.

arrival probability is:

$$P(v_\alpha \rightarrow v_\beta) = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2, \quad (4)$$

where  $\alpha$  and  $\beta$  are flavour eigenstates. In fact, this expression holds true whenever the time arrival differences among the three mass eigenstates is smaller than the detector time resolution. However, when  $\Delta t_{v_i v_j}$  is larger than the detector resolution, then each mass eigenstates  $v_i$  can be detected independently and will interact with the detector with probability<sup>e</sup>

$$P(v_\alpha \rightarrow v_\beta)_i = |U_{\alpha i}|^2 |U_{\beta i}|^2. \quad (5)$$

For simplicity, here we do not consider matter effects which could in principle take place in the propagation through the Earth itself. In Fig. 3 on the left panel we illustrate a possible pattern of neutrino detection following (5) using also the time-differences reported in Table 1. Depending on the source, its distance and the experimental time sensitivity the figure shows that, at least in principle, one can observe interesting time-patterns reflecting the neutrino ordering and mixing.

So far we discussed the basic setup and argued that neutrino detectors on Earth can help disentangle the neutrino ordering, when observing distant astrophysical catastrophes. It is time to move to additional precious information that we can gain when comparing time differences with respect to the other light messengers.

## 5 Absolute neutrino masses from time differences

The attractive idea to use a multi-signal approach was put forward in<sup>12</sup> where the authors translate a potential SN signal of GWs and neutrinos into limits on the speed of GWs and on the absolute neutrino mass scale. We define:

$$\Delta T_{v_i g} = T_{v_i} - T_g, \quad \text{that implies} \quad \Delta T_{v_i g} = \tau_{obs}^i - \tau_{int}^v, \quad (6)$$

where  $i$  denotes now the  $i$ -th neutrino mass eigenstate. The deviation from the speed of light for GWs and neutrinos reads:

$$\delta_g \equiv \frac{c - v_g}{c}, \quad \delta_{v_i} \equiv \frac{c - v_{v_i}}{c}, \quad \text{with} \quad \delta_{v_i} = \frac{m_i^2 c^4}{2E^2} + \mathcal{O}\left(\frac{m_i^4 c^8}{8E^4}\right). \quad (7)$$

From the definition in (6) follows:

$$\frac{\Delta T_{v_i g}}{T_0} = \frac{\delta_{v_i} - \delta_g}{(1 - \delta_g)(1 - \delta_{v_i})}, \quad (8)$$

where, as already defined earlier,  $T_0 = L/c$ . If in (8) we consider an uncertainty in the time of emission of neutrinos,  $\tau_{int}^v$ , in order to detect the GW and the neutrino signal, we must have  $|\Delta T_{v_i g}| > \tau_{int}^v$ , and using (8) to the first order in  $\delta_v$  and  $\delta_g$  one finds:

$$|\delta_{v_i} - \delta_g| T_0 \gtrsim \tau_{int}^v. \quad (9)$$

Using the inequality above and assuming  $\tau_{int}^v \sim 10$  ms (typical time for a SN burst) and an energy equal to the energy threshold of HK,  $E_\nu = 7$  MeV,  $\delta_g$  depends on the lightest neutrino mass for a reference distance. In principle, detectors with a lower energy resolutions, such as JUNO ( $E_\nu^{th} = 1.806$  MeV) could test lower values of  $m_{min}$  and could probe neutrino mass up to  $\sim 0.02$  eV for distances around 1 Mpc, which are at least an order of magnitude lower than present cosmological limits and the perspective upper limit from KATRIN.

<sup>e</sup>We work in the regime of incoherence. Defining  $\sigma_{xp}$  ( $\sigma_{xD}$ ) as the spatial width of the production (detection) neutrino wave packet, we work under the assumption that  $|(v_j - v_k)L/c| \gg \max(\sigma_{xp}, \sigma_{xD})$  being  $v_i$  and  $v_j$  the two group velocities of the two wave packets of neutrino mass eigenstates  $v_i$  and  $v_j$ .

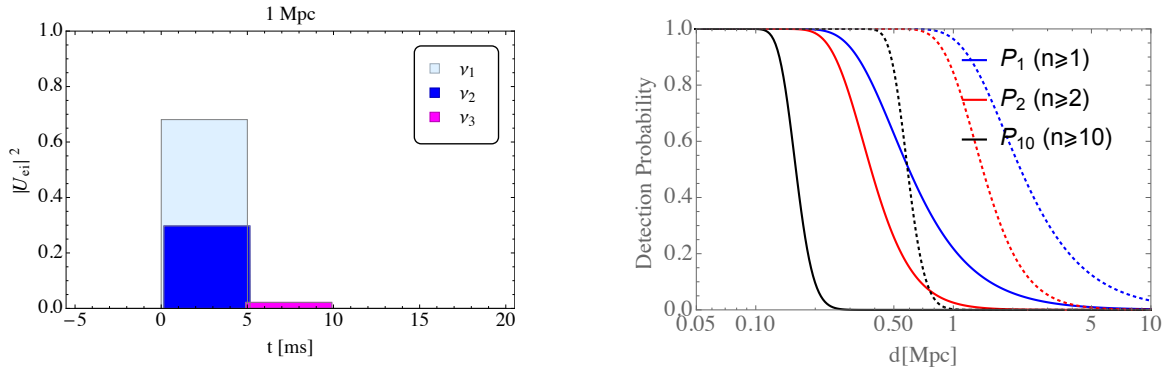


Figure 3 – (Left) Schematic representation of the square root of the probability given in (5) of detecting flavor state  $\nu_e$  if the source emits a short burst of  $\nu_e$  as a function of time. We set a distance of 1 Mpc and an energy of 5 MeV. For definiteness we assume NO and each bin corresponds to a fiducial collective time of 5 ms. (Right) Detection probability of neutrinos versus distance from the source to Hyper-Kamiokande (solid lines) and to an hypothetical future 5 Mton experiment (dotted lines)<sup>17</sup> using a 7 – 30 MeV energy range. Blue, red and black curves represent the detection probability resulting in requiring observation of at least one, two, and ten events per burst, respectively.

## 6 Concluding with a Preliminary Feasibility Study

So far we have been concerned with the theoretical setup, and since the framework presented here relies on distant sources, we will now perform a preliminary study of the actual experimental feasibility. In the following, we will not discuss the distribution and the expected number of various kinds of astrophysical events, but focus on the number of detected neutrinos assuming a specific source at a given distance. From the analysis above it is clear, that three parameters are vital to increase the time lapse between mass eigenstates: the distance from the source  $L$ , the energy of the emitted neutrino,  $E_\nu$ , and the absolute neutrino mass  $m_{min}$ . Conversely, the larger the distance is, the smaller is the rate. As a consequence, if the neutrino counterparts of events like GW150914 would be emitted by the source, it would be hard, if not impossible, to detect them on Earth.

As a benchmark investigation we will concentrate on the next generation of neutrino detection experiments such as 1 Mton Hyper-Kamiokande (HK) in Japan<sup>16</sup> that has already sparked interests in the astrophysical community. Astrophysical catastrophes like the merging of a neutron star black hole binary or the core bounce of a core-collapsed supernova are expected to produce a total neutrino output carrying an overall energy of circa  $10^{53}$  erg. For such an event one expects on Earth an integrated time flux per squared meter of about  $3 \times 10^{11} (d/\text{Mpc})^{-2} m^{-2}$ . Despite the fact that a large number of neutrinos will reach Earth because of their low cross section only a tiny fraction will be detected. Previous studies<sup>18</sup> indicate that HK can detect 1-2 neutrino events per year from supernovae in the range up to 10 Mpc.

However, our theoretical analysis made use only of the neutrinos emitted during the initial burst from the source which can be determined by integrating the following neutrino detection rate over the relevant time interval:

$$\frac{dN}{dt} = n_p \int_{E_e^{th}} dE_e \int_{E_\nu^{th}} dE_\nu \mathcal{F}(E_\nu, t) \sigma'(E_e, E_\nu) \epsilon, \quad (10)$$

where  $n_p$  is the number of protons in the target,  $E_{\nu,e}$  are respectively the (anti)neutrino and the (electron) positron energy of the event,  $\mathcal{F}(E_\nu, t)$  is the flux per unit time, area and energy and  $\epsilon$  is the detector efficiency. Finally  $\sigma'(E_e, E_\nu) = d\sigma/dE_e$  is the differential cross section of the process under study. We will assume the efficiency of the detector to be 100% for energies larger than the energy threshold of the detector,  $E_\nu > E_\nu^{th}$ .

Our estimates assume a typical energy in neutrinos emitted from astrophysical sources within the initial burst to be of the order of  $\sim 10^{51}$  erg as well as a mean neutrino energy  $\langle E_{\bar{\nu}_e} \rangle \sim 12$

MeV. From a SN at a distance  $d$ , HK (0.74 Mton,  $E_\nu^{th}=7$  MeV,  $E_e^{th}=4.5$  MeV) would expect the following number of detected neutrinos (indicated by  $\lambda_{ES}$ ) via neutrino-electron elastic scattering (ES) processes:

$$\lambda_{ES} = 1.8 \times 10^{-3} \left( \frac{d}{\text{Mpc}} \right)^{-2} \quad (11)$$

where the initial burst is primarily  $\nu_e$  from the neutronization process. Similarly, from a neutron star black hole (NS-BH) merger, where the burst consists mostly of  $\bar{\nu}_e$ , we get via inverse beta decay (IBD)<sup>19</sup> a number of neutrinos of

$$\lambda_{IBD} = 1.6 \times 10^{-1} \left( \frac{d}{\text{Mpc}} \right)^{-2}. \quad (12)$$

For such low rates it is useful to estimate the actual detection probability as function of the distance from the source. To assess this, we use the Poisson probability to detect  $n$  events as  $P_n = \lambda^n e^{-\lambda} / n!$  where  $\lambda$  is the expected number of events, given in eq. (11) or eq. (12). In Fig. 3 we show, as an illustrative example, the detection probability for IBD resulting from requiring at least one, two, and 10 events per burst, indicated respectively with blue, red and black curves. We use in our estimates the energy range 7 – 30 MeV. The plot shows the HK detection probability for  $\bar{\nu}_e$  for a NS-BH merger (solid line), as well as the one for a hypothetical 5 Mton detector (dashed line), e.g.<sup>17</sup>. We observe that even for  $\sim 1$  Mpc and a 7 – 30 MeV energy range one can still observe  $\sim 1$  event. These estimates show that it is possible to reach phenomenologically interesting neutrino mass differences from sources at  $\sim 1$  Mpc provided one can combine more than one Mton experiment. In order to compute the expected annual rate of detected neutrino events, one has to combine the above analysis with the annual rate of relevant astrophysical events. The annual rate of SNs is expected to be  $1/3 \text{ yr}^{-1}$  within 4 Mpc<sup>18</sup>, while the rate for NS-BH mergers is more uncertain with an expected rate of  $10^2 - 10^3 \text{ yr}^{-1}$  within 1 Gpc. This rate will in the future be constrained by LIGO<sup>20</sup>.

We stress that we have used conservative estimates, for example, in the total energy emitted with the neutrino burst. Another parameter that can be played with is the time resolution in neutrino detection that can, in the future, be expected to go below one millisecond. If this is the case it would allow sources as close as 100 kpc to become relevant for our analysis. In this case the neutrino flux increases by two orders of magnitude.

To conclude, we derived the theoretical and phenomenological conditions under which multi-messenger astronomy can disentangle or further constrain the neutrino mass ordering. We have also argued that it can provide salient information on the absolute neutrino masses. We added a preliminary feasibility study to substantiate and further motivate our theoretical analysis. We have seen that future experiments can be useful also in testing independently the cosmological bounds on neutrino absolute masses. However, this requires high resolution timing and a significant increase in the combined fiducial volume compared to the current Cherenkov water detectors.

Conversely one can use future results on neutrino properties to provide detailed information about astrophysical sources emitting simultaneously GWs, photons and neutrinos, and possibly lower uncertainties in the emitted multi-messenger signal from the source.

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