RELATIVISTIC TRAJECTORIES OF CELESTIAL BODIES AS A TOOL TO CONSTRAIN F(R) THEORIES OF GRAVITY AND DARK MATTER CONCENTRATION NEAR THE GALACTIC CENTER

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Trajectories of test bodies may be used for a potential reconstruction, in particular, one can get constraints on a theory choice and/or on a choice of a specific model for a selected object. We mention an opportunity to put limits on alternative theories of gravity from planetary motions in our Solar system. We discuss constraints on DM concentration near the Galactic Center from apocenter shift data.

Shapes of celestial body trajectories may be used for a gravitational potential reconstruction similarly to nuclear physics where people reconstruct interaction potentials. In particular, we use planetary orbits in Solar system to put severe constraints on alternative theories of gravity such a $f(R)$ theory [1, 2, 3].

Advancements in infrared astronomy are allowing to test the scale of the mass profile at the center of our galaxy down to tens of AU. With the Keck and VLT telescopes, the proper motions of several stars orbiting the Galactic Center black hole have been monitored and almost entire orbits, as for example that of the S2 star, have been measured allowing an unprecedent description of the Galactic Center region [4, 5, 6]. Measurements of the amount of mass $M(< r)$ contained within a distance r from the Galactic Center are continuously improved as more precise data are collected. Observations extend down to the periastron distance ($\simeq 3 \times 10^{-4}$ pc) of the S16 star and they correspond to a value of the enclosed mass $\simeq 3.67 \times 10^6$ $M_\odot$ within $\simeq 3 \times 10^{-4}$ pc [4]. Here and in the following, we use the three component model for the central region of our galaxy based on estimates of enclosed mass proposed in [7]. This model is constituted by the central black hole, the central stellar cluster and the DM sphere (made of WIMPs), i.e.

$$M(< r) = M_{BH} + M_*(< r) + M_{DM}(< r) \ , \tag{1}$$

where M_{BH} is the mass of the central black hole Sagittarius A*. For the central stellar cluster, the empirical mass profile is

$$M_*(<r) = \begin{cases} M_* \left(\frac{r}{R_*}\right)^{1.6} , & r \leq R_* \\ \\ M_* \left(\frac{r}{R_*}\right)^{1.0} , & r > R_* \end{cases} \tag{2}$$

with a total stellar mass $M_* = 0.88 \times 10^6$ $M_\odot$ and a size $R_* = 0.3878$ pc. As far as the mass profile of the DM concentration is concerned, we have assumed a mass distribution of the form [7]

$$M_{DM}(<r) = \begin{cases} M_{DM} \left(\frac{r}{R_{DM}}\right)^{3-\alpha} , & r \leq R_{DM} \\ \\ M_{DM}, & r > R_{DM} \end{cases} \tag{3}$$

M_{DM} and R_{DM} being the total amount of DM in the form of WIMPs and the radius of the spherical mass distribution, respectively.

A likelihood analysis has allowed to estimate for the DM mass the value $M_{DM} \simeq 10^5$ $M_\odot$ while the DM sphere size results to be in the range $(10^{-4} - 1)$ pc. It is clear that present observations of stars around the Galactic Center do not exclude the existence of a DM sphere with mass $\simeq 4 \times 10^6 M_\odot$, well contained within the orbits of the known stars, if its radius R_{DM} is $\lesssim 2 \times 10^{-4}$ pc (the periastron distance of the S16 star in the more recent analysis [6]). However, if one considers a DM sphere with larger radius, the corresponding upper value for M_{DM} decreases (although it tends again to increase for extremely extended DM configurations with $R_{DM} \gg 10$ pc). In the following, we will assume for definiteness a DM mass $M_{DM} \sim 2 \times 10^5 M_\odot$, that is the upper value for the DM sphere in [7] within an acceptable confidence level in the range $(10^{-3}-10^{-2})$ pc for R_{DM}. As it will be clear in the following, we emphasize that even a such small value for the DM mass (that is about only 5% of the standard estimate $(3.67 \pm 0.19 \times 10^6)$ $M_\odot$ for the dark mass at the Galactic Center [6]) may give some observational signatures.

Evaluating the S2 apoastron shift [a] as a function of R_{DM}, one can further constrain the DM sphere radius since even now we can say that there is no evidence for negative apoastron shift for the S2 star orbit at the level of about 10 mas. In addition, since at present the precision of the S2 orbit reconstruction is about 1 mas, we can say that even without future upgrades of the observational facilities and simply monitoring the S2 orbit, it will be possible within about 15 years to get much more severe constraints on R_{DM}.

We study the motion of stars as a consequence of the gravitational potential $\Phi(r)$ due the mass profile given in Eq. (1). As usual, the gravitational potential can be evaluated as

$$\Phi(r) = -G \int_r^\infty \frac{M(r')}{r'^2}\, dr' \ . \tag{4}$$

According to GR, the motion of a test particle can be fully described by solving the geodesic equations. Under the assumption that the matter distribution is static and pressureless, the equations of motion in the PN-approximation become

$$\frac{d\mathbf{v}}{dt} \simeq -\nabla(\Phi_N + 2\Phi_N^2) + 4\mathbf{v}(\mathbf{v}\cdot\nabla)\Phi_N - v^2 \nabla\Phi_N \ . \tag{5}$$

We note that the PN-approximation is the first relativistic correction from which the apoastron advance phenomenon arises. In the case of the S2 star, the apoastron shift as seen from Earth

[a] We want to note that the periastron and apoastron shifts $\Delta\Phi$ as seen from the orbit center have the same value whereas they have different values as seen from Earth (see Eq. (7)). When we are comparing our results with orbit reconstruction from observations we refer to the apoastron shift as seen from Earth.

(from Eq. (7)) due to the presence of a central black hole is about 1 mas, therefore not directly detectable at present since the available precision in the apoastron shift is about 10 mas (but it will become about 1 mas in 10–15 years even without considering possible technological improvements). It is also evident that higher order relativistic corrections to the S2 apoastron shift are even smaller and therefore may be neglected at present, although they may become important in the future.

The Newtonian effect due to the existence of a sufficiently extended DM sphere around the black hole may cause an apoastron shift in the opposite direction with respect to the relativistic advance due to the black hole. Therefore, we have considered the two effects comparing only the leading terms.

For a spherically symmetric mass distribution (such as that described above) and for a gravitational potential given by Eq. (4), Eq. (5) may be rewritten in the form

$$\frac{d\mathbf{v}}{dt} \simeq -\frac{GM(r)}{r^3}\left[\left(1+\frac{4\Phi_N}{c^2}+\frac{v^2}{c^2}\right)\mathbf{r}-\frac{4\mathbf{v}(\mathbf{v}\cdot\mathbf{r})}{c^2}\right] , \tag{6}$$

$\mathbf{r}$ and $\mathbf{v}$ being the vector radius of the test particle with respect to the center of the stellar cluster and the velocity vector, respectively. Once the initial conditions for the star distance and velocity are given, the rosetta shaped orbit followed by a test particle can be found by numerically solving the set of ordinary differential equations in Eq. (6). As one can see, for selected parameters for DM and stellar cluster masses and radii the effect of the stellar cluster is almost negligible while the effect of the DM distribution is crucial since it enormously overcome the shift due to the black hole (for $R_{DM} = 10^{-3}$ pc). Moreover, as expected, its contribution is opposite in sign with respect to that of the black hole [8]. We note that the expected apoastron (or, equivalently, periastron) shifts (mas/revolution), $\Delta\Phi$ (as seen from the center) and the corresponding values $\Delta\phi_E^{\pm}$ as seen from Earth (at the distance $R_0 \simeq$ 8 kpc from the GC) are related by

$$\Delta\phi_E^{\pm} = \frac{d(1\pm e)}{R_0}\Delta\Phi, \tag{7}$$

where with the sign $\pm$ are indicated the shift angles of the apoastron (+) and periastron (–), respectively. The S2 star semi-major axis and eccentricity are $d = 919$ AU and $e = 0.87$ [6]. Taking into account that the present day precision for the apoastron shift measurements is of about 10 mas, one can say that the S2 apoastron shift cannot be larger than 10 mas. Therefore, any DM configuration that gives a total S2 apoastron shift larger than 10 mas (in the opposite direction due to the DM sphere) is excluded. The same analysis is done for two different values of the DM mass distribution slope, i.e. $\alpha = 1$ and $\alpha = 2$ [9, 10]. In any case, we have calculated the apoastron shift for the S2 star orbit assuming a total DM mass $M_{DM} \simeq 2\times 10^5$ $M_\odot$. As one can see, the upper limit of about 10 mas on the S2 apoastron shift may allow to conclude that DM radii in the range about $10^{-3} - 10^{-2}$ pc are excluded by present observations for DM mass distribution slopes. We notice that the results of the present analysis allows to further constrain the results [7], where it was concluded that if the DM sphere radius is in the range $10^{-3} - 1$ pc, configurations with DM mass up to $M_{DM} = 2\times 10^5$ $M_\odot$ are acceptable. The present analysis shows that DM configurations of the same mass are acceptable only for R_{DM} out the range between $10^{-3} - 10^{-2}$ pc, almost irrespectively of the α value.

We have considered the constraints that the upper limit (presently of about 10 mas) of the S2 apoastron shift may put on the DM configurations at the galactic center. When (in about 10–15 years, even without considering improvements in observational facilities) the precision of S2 apoastron shift will be about 1 mas (that is equal to the present accuracy in the S2 orbit reconstruction) our analysis will allow to further constrain the DM distribution parameters. In particular, the asymmetric shape of the curve in Fig. 1 implies that any improvement in the apoastron shift measurements will allow to extend the forbidden region especially for the

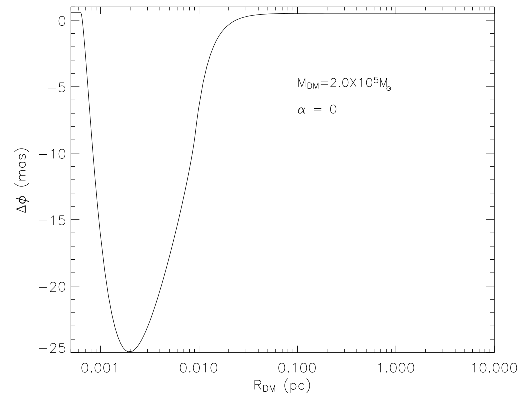

Figure 1: Apoastron shift as a function of the DM radius R_{DM} for $\alpha = 0$ and $M_{DM} \simeq 2 \times 10^5$ $M_{\odot}$. Taking into account present day precision for the apoastron shift measurements (about 10 mas) one can say that DM radii R_{DM} in the range $8 \times 10^{-4} - 10^{-2}$ pc are not acceptable.

upper limit for R_{DM}. Quantitatively, we have a similar behavior curves for other choices of slope parameters α for DM concentrations. In this context, future facilities for astrometric measurements at a level 10 μas of faint infrared stars will be extremely useful and they give an opportunity to put even more severe constraints on DM distribution. In addition, it is also expected to detect faint infrared stars or even hot spots orbiting the Galactic Center. In this case, consideration of higher order relativistic corrections for an adequate analysis of the stellar orbital motion have to be taken into account. In our considerations we adopted simple analytical expression and reliable values for R_{DM} and M_{DM} parameters following [7] just to illustrate the relevance of the apoastron shift phenomenon in constraining the DM mass distribution at the Galactic Center. If other models for the DM distributions are considered the qualitative aspects of the problem are preserved although, of course, quantitative results on apoastron shifts may be different.

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