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Gravitation in the surface tension model of spacetime

H A Perko[1]
[1]Office 14, 140 E. 4th Street, Loveland, CO, USA 80537

E-mail: hperko@kopparesearch.com

Abstract. A mechanical model of spacetime was introduced at a prior conference for describing perturbations of stress, strain, and displacement within a spacetime exhibiting surface tension. In the prior work, equations governing spacetime dynamics described by the model show some similarities to fundamental equations of quantum mechanics. Similarities were identified between the model and equations of Klein-Gordon, Schrödinger, Heisenberg, and Weyl. The introduction did not explain how gravitation arises within the model. In this talk, the model will be summarized, corrected, and extended for comparison with general relativity. An anisotropic elastic tensor is proposed as a constitutive relation between stress energy and curvature instead of the traditional Einstein constant. Such a relation permits spatial geometric terms in the mechanical model to resemble quantum mechanics while temporal terms and the overall structure of tensor equations remain consistent with general relativity. This work is in its infancy; next steps are to show how the anisotropic tensor affects cosmological predictions and to further explore if geometry and quantum mechanics can be related in more than just appearance.

1. Introduction

The focus of this research is to find a mechanism by which spacetime might curl, warp, or re-configure at small scales to provide a geometrical explanation for quantum mechanics while remaining consistent with gravity and general relativity. This research borrows techniques from physical chemistry of surfaces, mathematics of elasticity, and continuum mechanics. The research led to a proposed mechanical model of spacetime [1,2] with multi-dimensional surface tension (herein hypersurface tension). Equations governing energetic perturbations of stress, strain, and reconfiguration of spacetime with hypersurface tension were shown to resemble Schrödinger, Klein-Gordon, Heisenberg, and Weyl-like expressions. The previous work also provided several mathematical expressions for quantization and formation of particles from perturbations in space and time.

In this paper, the mechanical model discussed in [1] is extended to include further comparison with general relativity. It is shown that the model can provide geometrical equations resembling both quantum mechanics and general relativity by assuming a symmetric nondegenerate anisotropic constitutive relation between stress energy and curvature rather than the traditional scalar Einstein constant. The next step in development is to apply the model to describe physical systems and compare results with known experiments so that the model can be verified, falsified, or further refined.

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2. Hypersurface Geometry

It is well known in physics that every three-dimensional volume is a three-surface in four-dimensional space [3]. To aid in visualization of this geometry, an arbitrary three volume is shown by the cube in the center of Figure 1. Such a volume may be filled with points of matter and energy moving along different trajectories and velocities. Each point contained within the three volume corresponds to a point in each of three projected surfaces plotted with respect to two coordinates of space and one coordinate of time. The projected surfaces are coincident with the three volume and are sometimes referred to as time-like hypersurfaces or slices.

In the previous work [1], the geometry in Figure 1 was described as being associated with the "synchronous reference system" defined by Landau and Lifshitz [4]. The author [1] only intended to highlight the diagonal metric but unintentionally implied stationary conditions. The reference frame described in Figure 1 is simply that of any arbitrary observer. Each arbitrary observer records the position of matter and energy in their coordinate of time.

A coordinate frame with a diagonal metric is called a “principle frame" in continuum mechanics. When solving problems in continuum mechanics, it is often simplest to start from the principle frame, write equations of motion, allow the system to evolve, and then transform as necessary to find the stress energy and manifold configuration in other frames [5]. Thus, the choice of principle frame should not be considered preferential for any reason other than mathematical convenience. To the engineer, the principle frame suppresses shear strains. To the physicist, the principle frame suppresses the gravitational equivalent of magnetic fields [6].

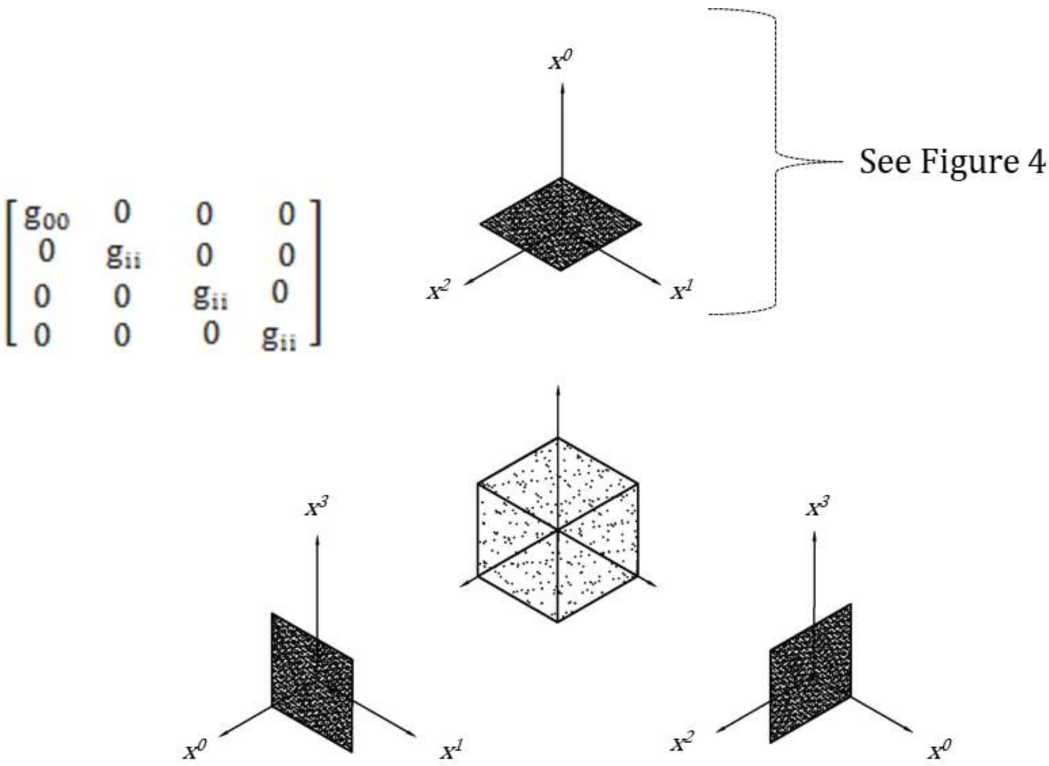

Figure 1. Hypersurface Projections in the Principle Frame

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3. Physical Chemistry of Hypersurfaces

The diagram shown in Fig. 1 illustrates how interfaces with time or hypersurfaces arise in four-dimensions leading to a "fabric of spacetime". Fields and forces are truncated for the arbitrary observer due to limitations imposed by the speed of light. It is argued here that such truncation must result in surface tension.

Ordinary surface tension is a familiar concept for most people from everyday interactions with liquids. Surface tension causes membrane-like behavior at an interface. It is important to note that this behavior does not imply a real physical membrane nor is the root cause a chemical process. Rather, surface tension and membrane behavior arise from truncated fields and bound multiplicity of quantum states when matter and energy are arranged at a surface for an observer. Physical chemists using Boltzmann statistics have proven that surface tension exists at any stable interface, regardless of material composition and phase. The only prerequisite for surface tension is a non-dispersive interface [7].

To better understand surface tension, consider the example distribution of particles arranged in a continuum in Figure 2a. Each set of circles represents probability contours describing the likely location of a particle. Particle field interactions lead to a statistically uniform distribution as a consequence of entropy. Creation of an interface, as shown by the dark line in Figure 2b, leads to an increase in local probability at the boundary. Surface tension is the mathematical result of probability density field truncation that exists at <u>any</u> stable interface.

More explicitly, let us define a quantum state, s_1, of existence above the boundary in Figure 2 and another state, s_2, of existence below the boundary. In Figure 2a, the two states have equal probability. In Figure 2b, the state $s_1$is no longer accessible and the probability of s_2 increases. The increase in probability of state $s_2$is associated with a decrease in multiplicity of states when moving from the system in Figure 2a to Figure 2b. Decrease in multiplicity means a decrease in entropy and requires work. The work done on the system is the energy of formation of the interface or *surface energy*.

An underlying theorem of physical chemistry of surfaces is that surface energy is mathematically equivalent to surface tension [7]. Herein, surface energy and surface tension shall be denoted by the Greek letter Koppa, $Ϙ$.[1] To better understand the relationship between surface energy and surface tension, consider the simple mechanical model in Figure 3, which is adapted from [7]. If a window is opened in a boundary allowing access to quantum states above such that they can affect the probability of states below and the interface between states has surface energy, $Ϙ$, then the work done on the system is given by

$$Work = Ϙ dA$$

[1]In physical chemistry, the Greek letter gamma, γ, is typically used to represent surface tension. To avoid confusion with the Lorentz factor in physics, assignment of a new symbol was necessary. The archaic Greek letter Koppa, $Ϙ$, was chosen. Before being adopted and then omitted from the modern Greek alphabet, the letter Koppa, $Ϙ$, had its origins in ancient Hebrew. The ancestor Koppa symbol was drawn as the sun on the horizon and it signified light at the edge of time. Thus, Koppa seemed a fitting parameter to describe tension at the edge of time. In the Roman alphabet, Koppa corresponds to the letter Q.

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which can be rewritten as a force pulling on length L moving a distance dx,

$$Work = \text{Ϙ}Ldx$$

where $dA = Ldx$ represents the creation of additional surface area. In the second formulation, Ϙ appears as a tension per unit length or *surface tension*. Thus, surface tension and surface energy are mechanically equivalent concepts.

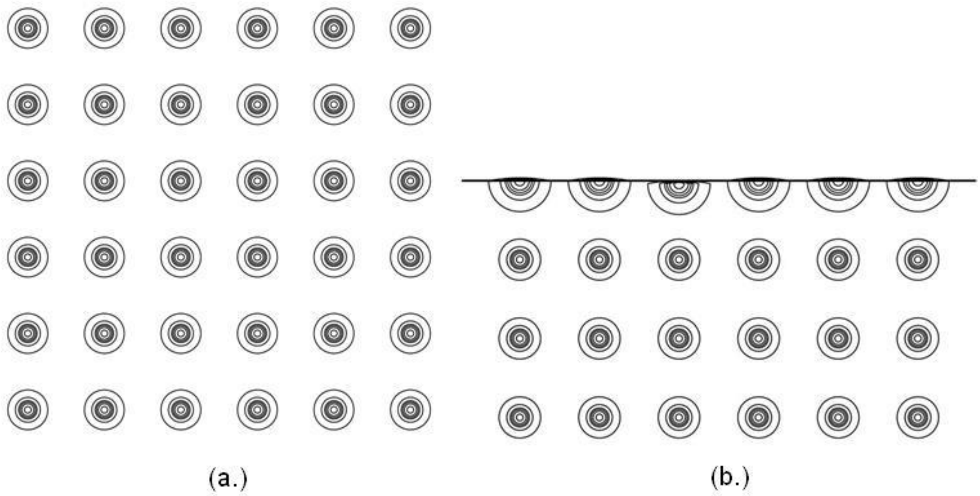

Figure 2. Probability Density of Interacting Particles (a. in continuum, b. with boundary)

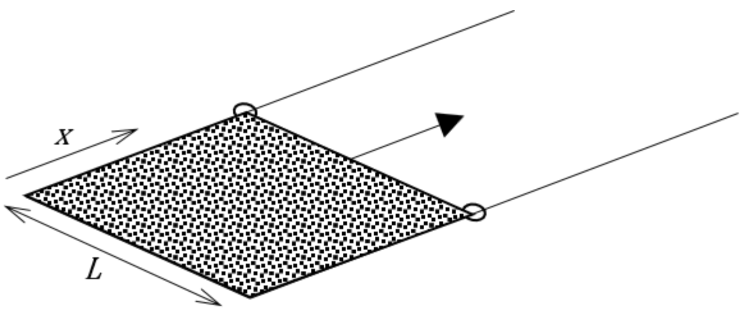

Figure 3. Hypersurface Stretched Across a Wire Frame Window with One Moveable Side

In physics, future states are inaccessible because nothing can travel faster than the speed of light. Therefore, a boundary in time exists for an observer and, according to the basic postulates of physical chemistry of surfaces, surface tension should and must exist regardless of dimensionality of a hypersurface. What is true for one observer must be true for all observers. Hypersurface tension should result in a uniform negative stress energy holding all matter and energy together within the construct of general relativity, as we shall see, enforcing causality, and avoiding complete temporal dispersion. In four-dimensions, the units of Koppa are J/m^3, which is equivalent to N/m^2[1].

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4. Hypersurface Continuum Mechanics

It would be useful to apply traditional continuum mechanics to describe hypersurface geometry with surface tension. Continuum mechanics offers tools based in Riemannian geometry for relating stress energy to reconfiguration and surface evolution that satisfy conditions of covariance and uniqueness [8]. However, one cannot apply these tools without a change of coordinates. Spacetime geometry is pseudo-Riemannian and, in fact, hyperbolic. According to special relativity,

$$-c^2d\tau^2 = -c^2dt^2 + {dx^1}^2 + {dx^2}^2 + {dx^3}^2$$

where dx^j are spatial coordinates with index, *j*, running from 1 to 3, and *t* is the coordinate of time. This pseudo-Riemannian geometry can be transformed to a Riemannian geometry by rewriting the equation above in complex coordinates, as given by

$$d\tau'^2 = {dx^0}^2 + {dx^1}^2 + {dx^2}^2 + {dx^3}^2$$

where

$$x^0 = ict$$

$$\tau' = ic\tau$$

The parameter, τ, is proper time, a frame independent scalar. The parameter, τ', is referred to as proper distance, but will be called imaginary proper time herein for the following reason. Since all observers agree on the imaginary proper time, τ', one can write a set of equations for the evolution of the system according to τ'. Thus, imaginary proper time may serve as a type of frame independent "clock" in 4-dimensional continuum mechanics.

The substitution of complex coordinates allows traditional continuum mechanics concepts to be applied to spacetime. The remainder of this section is devoted to describing each of the tensor equations of continuum mechanics as they relate to spacetime with hypersurface geometry. Specifically, the Cauchy stress tensor, rate of deformation tensor, equations of motion, and constitutive relationships are presented. In the sections that follow, these geometrical equations are compared with quantum mechanics and general relativity.

Consider again the arbitrary distribution of matter and energy plotted with respect to space and time in Figure 1. One of the projections with time and two coordinates of space from Figure 1 is replotted in Figure 4. A similar plot can be made with time and each pair of spatial coordinates. Recall that dots in the surface represent a particle wave packet or some sort of matter or energy. The distribution of matter and energy in Figure 4 provides an energy density in the hypersurface. The question under consideration is not in regard to specific particles and waves comprising the surface, but how does spacetime evolve due to this bound *surface energy*.

In a rest frame or static spacetime, dx^i is assumed small, $d\tau' \approx dx^o$, and the hypersurface in Figure 4 would be flat. In this work, spacetime is assumed dynamic, and each infinitesimal element in spacetime is moving at some four-vector velocity, u^μ, according to the arbitrary observer. The metric is allowed to evolve beyond flat, deviations in imaginary proper time and coordinate time are permitted at a point, and hypersurfaces are irregular.

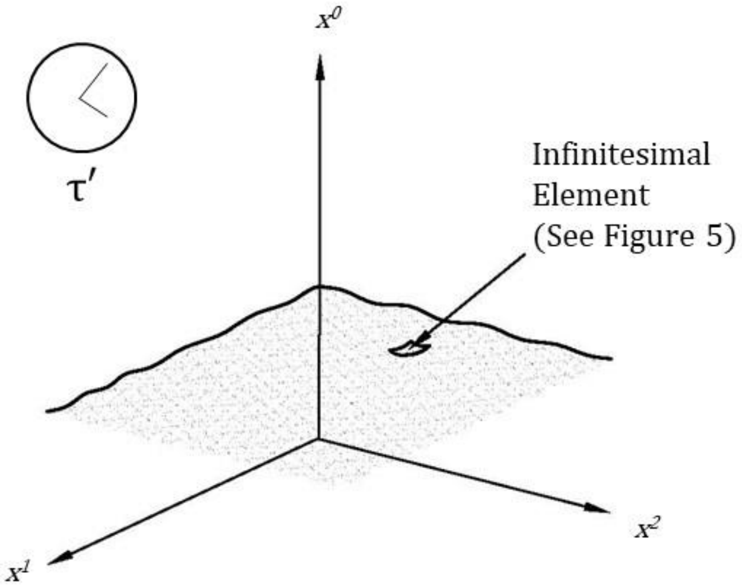

Figure 4. Hypersurface Showing Postulated Curvy Microstructure

An infinitesimal element of hypersurface from Figure 4 is depicted at larger scale in Figure 5. Surface tension is shown along the four spatial sides of the element. A pressure, P, is applied normal to the hypersurface and represents an external volumetric temporal pressure or encapsulated energy density. The infinitesimal element is depicted as curved and surface tension vectors are not co-linear. Shear stresses are omitted along the edges and upper and lower surfaces of the element by choice of principle frame.

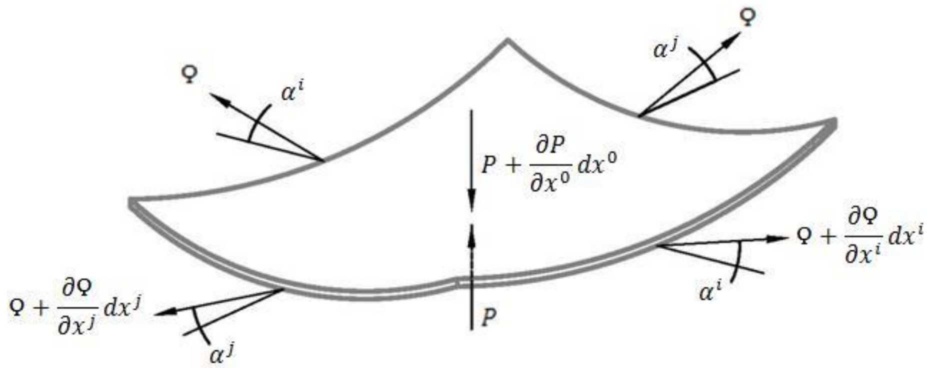

Figure 5. Hypersurface Tension on an Infinitesimal Element of Spacetime

The stresses about the infinitesimal element of spacetime in Figure 5 can be represented by a covariant hypersurface tension stress energy tensor, $T_{\mu\nu}$, given by,

$$T_{\mu\nu} = \begin{bmatrix} P & 0 & 0 & 0 \\ 0 & -Ϙ & 0 & 0 \\ 0 & 0 & -Ϙ & 0 \\ 0 & 0 & 0 & -Ϙ \end{bmatrix} \qquad (1)$$

Note that the sign convention in (1) is opposite that in [1]. Surface tension is generally taken as positive in physical chemistry and membrane mechanics. A sign convention is used herein where tensile stress is negative and compression stress is positive. This convention is more consistent with usual treatments of general relativity.

Reconfiguration of a system in continuum mechanics is represented by a two-point rate of deformation tensor constructed from the gradient of spatial velocity and the conjugate transpose of the gradient. Velocity in continuum mechanics refers to the movement of "material" relative to some arbitrary inertial reference frame. Here, velocity refers to metric dynamics, or movement of spacetime, according to any arbitrary observer. Recall the four-vector velocity of the infinitesimal element of spacetime in Figure 5 is denoted by $u^{\mu} = \frac{dx^{\mu}}{d\tau'}$ and varies according to the observer. The two-point rate of deformation tensor, D^{α}_{β}, for a membrane-like infinitesimal element of spacetime for any observer is written as,

$$D^{\alpha}_{\beta} = \begin{bmatrix} \frac{\partial u^0}{\partial x^0} & \frac{1}{2}\left(\frac{\partial u^0}{\partial x^1} + \frac{\partial u^1}{\partial x^0}\right) & \frac{1}{2}\left(\frac{\partial u^0}{\partial x^2} + \frac{\partial u^2}{\partial x^0}\right) & \frac{1}{2}\left(\frac{\partial u^0}{\partial x^3} + \frac{\partial u^3}{\partial x^0}\right) \\ \frac{1}{2}\left(\frac{\partial u^1}{\partial x^0} + \frac{\partial u^0}{\partial x^1}\right) & \frac{\partial u^1}{\partial x^1} - \frac{1}{2}\left|\frac{\partial u^0}{\partial x^1}\right|^2 d\tau' & \frac{1}{2}\left(\frac{\partial u^1}{\partial x^2} + \frac{\partial u^2}{\partial x^1}\right) - \frac{1}{2}\left|\left(\frac{\partial u^0}{\partial x^1}\right)\left(\frac{\partial u^0}{\partial x^2}\right)\right| d\tau' & \frac{1}{2}\left(\frac{\partial u^1}{\partial x^3} + \frac{\partial u^3}{\partial x^1}\right) - \frac{1}{2}\left|\left(\frac{\partial u^0}{\partial x^1}\right)\left(\frac{\partial u^0}{\partial x^3}\right)\right| d\tau' \\ \frac{1}{2}\left(\frac{\partial u^2}{\partial x^0} + \frac{\partial u^0}{\partial x^2}\right) & \frac{1}{2}\left(\frac{\partial u^2}{\partial x^1} + \frac{\partial u^1}{\partial x^2}\right) - \frac{1}{2}\left|\left(\frac{\partial u^0}{\partial x^2}\right)\left(\frac{\partial u^0}{\partial x^1}\right)\right| d\tau' & \frac{\partial u^2}{\partial x^2} - \frac{1}{2}\left|\frac{\partial u^0}{\partial x^2}\right|^2 d\tau' & \frac{1}{2}\left(\frac{\partial u^2}{\partial x^3} + \frac{\partial u^3}{\partial x^2}\right) - \frac{1}{2}\left|\left(\frac{\partial u^0}{\partial x^2}\right)\left(\frac{\partial u^0}{\partial x^3}\right)\right| d\tau' \\ \frac{1}{2}\left(\frac{\partial u^3}{\partial x^0} + \frac{\partial u^0}{\partial x^3}\right) & \frac{1}{2}\left(\frac{\partial u^3}{\partial x^1} + \frac{\partial u^1}{\partial x^3}\right) - \frac{1}{2}\left|\left(\frac{\partial u^0}{\partial x^3}\right)\left(\frac{\partial u^0}{\partial x^1}\right)\right| d\tau' & \frac{1}{2}\left(\frac{\partial u^3}{\partial x^2} + \frac{\partial u^2}{\partial x^3}\right) - \frac{1}{2}\left|\left(\frac{\partial u^0}{\partial x^3}\right)\left(\frac{\partial u^0}{\partial x^2}\right)\right| d\tau' & \frac{\partial u^3}{\partial x^3} - \frac{1}{2}\left|\frac{\partial u^0}{\partial x^3}\right|^2 d\tau' \end{bmatrix} \quad (2)$$

The rate of deformation tensor shown here is similar to the ordinary three-dimensional dynamical strain tensor in continuum mechanics except it is expanded to four-dimensions and contains extra terms to account for temporal strain, $\frac{\partial x^0}{\partial x^{\mu}}$, and out-of-plane deformations, $\frac{1}{2}\left|\left(\frac{\partial u^0}{\partial x^i}\right)\left(\frac{\partial u^0}{\partial x^j}\right)\right| d\tau'$, in time. Out-of-plane deformations are movements in the time direction in Figure 5 and resulting curvature. Equations for out-of-plane deformations can be found in common texts on mechanics of thin shells without bending (membrane theory) [9,10,11] and are described in terms of Riemannian geometry in [8]. Spatial components of tensor (2) were derived in [1].

Rate of deformation of an element of spacetime over an infinitesimally small increment in imaginary proper time can be assumed to be very small compared to unity. The Riemannian metric is related to the rate of deformation tensor by the following approximation derived in [1] with opposite sign convention,

$$g^{\alpha}_{\beta} = \boldsymbol{I} - 2D^{\alpha}_{\beta} d\tau' \qquad (3)$$

Written in this way, the reconfiguration of the system is a linearization of Lagrangian and Eulerian strains by a process known as infinitesimal strain theory. It has many similarities to linearized gravity. By the underlying assumption of small changes in the metric, one might conclude that infinitesimal strain theory cannot be used to describe dynamics of spacetime with a high degree of curvature. This conclusion is false as infinitesimal strain theory is used frequently in numerical modeling to describe highly curved systems provided numerical elements are small compared to the degree of curvature.

Equations of motion describing evolution of an infinitesimal element of spacetime with imaginary proper time were derived in [1]. The derivation followed from a simple application of Newton's laws in four-dimensions taking advantage of local Euclidian geometry in the tangent space at a point. These laws of motion can be written as,

$$\begin{bmatrix} Ϙ\left(\frac{\partial^2 x^0}{\partial {x^1}^2}+\frac{\partial^2 x^0}{\partial {x^2}^2}+\frac{\partial^2 x^0}{\partial {x^3}^2}\right)-dP & \frac{1}{2}\left(\frac{\partial Ϙ}{\partial x^1}-dP\right) & \frac{1}{2}\left(\frac{\partial Ϙ}{\partial x^2}-dP\right) & \frac{1}{2}\left(\frac{\partial Ϙ}{\partial x^3}-dP\right) \\ \frac{1}{2}\left(\frac{\partial Ϙ}{\partial x^1}-dP\right) & \frac{\partial Ϙ}{\partial x^1} & \frac{1}{2}\left(\frac{\partial Ϙ}{\partial x^1}+\frac{\partial Ϙ}{\partial x^2}\right) & \frac{1}{2}\left(\frac{\partial Ϙ}{\partial x^1}+\frac{\partial Ϙ}{\partial x^3}\right) \\ \frac{1}{2}\left(\frac{\partial Ϙ}{\partial x^2}-dP\right) & \frac{1}{2}\left(\frac{\partial Ϙ}{\partial x^2}+\frac{\partial Ϙ}{\partial x^1}\right) & \frac{\partial Ϙ}{\partial x^2} & \frac{1}{2}\left(\frac{\partial Ϙ}{\partial x^2}+\frac{\partial Ϙ}{\partial x^3}\right) \\ \frac{1}{2}\left(\frac{\partial Ϙ}{\partial x^3}-dP\right) & \frac{1}{2}\left(\frac{\partial Ϙ}{\partial x^3}+\frac{\partial Ϙ}{\partial x^1}\right) & \frac{1}{2}\left(\frac{\partial Ϙ}{\partial x^3}+\frac{\partial Ϙ}{\partial x^2}\right) & \frac{\partial Ϙ}{\partial x^3} \end{bmatrix}$$

$$= -Ϙ\begin{bmatrix} \frac{\partial u^0}{\partial \tau'} & \frac{1}{2}\left(\frac{\partial u^0}{\partial \tau'}+\frac{\partial u^1}{\partial \tau'}\right) & \frac{1}{2}\left(\frac{\partial u^0}{\partial \tau'}+\frac{\partial u^2}{\partial \tau'}\right) & \frac{1}{2}\left(\frac{\partial u^0}{\partial \tau'}+\frac{\partial u^3}{\partial \tau'}\right) \\ \frac{1}{2}\left(\frac{\partial u^1}{\partial \tau'}+\frac{\partial u^0}{\partial \tau'}\right) & \frac{\partial u^1}{\partial \tau'} & \frac{1}{2}\left(\frac{\partial u^1}{\partial \tau'}+\frac{\partial u^2}{\partial \tau'}\right) & \frac{1}{2}\left(\frac{\partial u^1}{\partial \tau'}+\frac{\partial u^3}{\partial \tau'}\right) \\ \frac{1}{2}\left(\frac{\partial u^2}{\partial \tau'}+\frac{\partial u^0}{\partial \tau'}\right) & \frac{1}{2}\left(\frac{\partial u^2}{\partial \tau'}+\frac{\partial u^1}{\partial \tau'}\right) & \frac{\partial u^2}{\partial \tau'} & \frac{1}{2}\left(\frac{\partial u^2}{\partial \tau'}+\frac{\partial u^3}{\partial \tau'}\right) \\ \frac{1}{2}\left(\frac{\partial u^3}{\partial \tau'}+\frac{\partial u^0}{\partial \tau'}\right) & \frac{1}{2}\left(\frac{\partial u^3}{\partial \tau'}+\frac{\partial u^1}{\partial \tau'}\right) & \frac{1}{2}\left(\frac{\partial u^3}{\partial \tau'}+\frac{\partial u^2}{\partial \tau'}\right) & \frac{\partial u^3}{\partial \tau'} \end{bmatrix}$$

$$-\begin{bmatrix} u^0\frac{\partial Ϙ}{\partial \tau'} & 0 & 0 & 0 \\ 0 & u^1\frac{\partial Ϙ}{\partial \tau'} & 0 & 0 \\ 0 & 0 & u^2\frac{\partial Ϙ}{\partial \tau'} & 0 \\ 0 & 0 & 0 & u^3\frac{\partial Ϙ}{\partial \tau'} \end{bmatrix} \qquad (4)$$

Diagonal terms in (4) are those derived in [1] except that one-sided temporal pressure has been replaced by differential pressure across the hypersurface to be more rigorous. Off-diagonal terms describing rotation and shear have been added for completeness. The right side of (4) differs in sign from [1] due to convention (e.g. compression stress is positive).

In order to maintain a stable hypersurface for the arbitrary observer, it is proposed that spacetime behave in accordance with an elastic continuum. An elastic continuum exhibits a constitutive relationship between stress energy and rate of deformation tensors. The simplest example of a stable constitutive relationship is the isotropic and linear elastic tensor given by $\boldsymbol{EI}$ where $\boldsymbol{I}$ is the identity matrix and E is a constant known in continuum mechanics as Young's Modulus. This was the approach taken in [1] where, with some math errors corrected, the constitutive relationship was set as $T_{ii} = (c\hbar/2)D_i^i$ where $\hbar$ is the reduced Plank's constant. This isotropic relationship was chosen to enforce spatial symmetries and cause spacetime geometry to resemble the appearance of some equations in quantum mechanics.

Constitutive relationships between temporal components of stress energy and rate of deformation were omitted in the original model [1]. It is necessary to include temporal components for comparison with general relativity. However, if the scalar elastic moduli in [1] is applied to temporal terms, it would result in gravity that is about 57 orders of magnitude higher than reality. A scalar constitutive relation does not provide for a unified model for quantum mechanics and gravity.

In continuum mechanics, there are many stable constitutive relationships between stress energy and rate of deformation. To say a constitutive relationship is stable, is to say it is nondegenerate and invertible. A scalar elastic tensor is just one of many stable possibilities. Another possibility is a stable *anisotropic* linear elastic tensor. Such a tensor would allow for different elastic moduli for spatial and temporal energy terms.

To maintain the dimensionality of spacetime, an anisotropic elastic tensor must have linearly independent eigenfunctions and, thus, be eigen-decomposable. A simple example of an eigen-decomposable tensor is diagonal. A diagonal elastic tensor has zero off-diagonal terms and is manifestly symmetric which ensures conservation of angular momentum (torsion free equations). The components of the diagonal should be spatially isotropic to ensure rotational symmetry in time and conservation of momentum and mass energy. To be invertible, the matrix must have a non-zero determinant. Any diagonalizable matrix with non-zero determinate can be written as the product of a scalar and a matrix with determinate equal to one. A possible elastic tensor meeting these criteria is shown in Voigt notation below

$$T_{\mu\nu} = C^{\beta}_{\mu\nu\alpha} \cdot D^{\alpha}_{\beta} \qquad (5)$$

$$\begin{bmatrix} T_{00} \\ T_{11} \\ T_{22} \\ T_{33} \\ \vdots \end{bmatrix} = A \begin{bmatrix} b^3 & 0 & 0 & 0 & \cdots \\ 0 & \frac{1}{b} & 0 & 0 & \cdots \\ 0 & 0 & \frac{1}{b} & 0 & \cdots \\ 0 & 0 & 0 & \frac{1}{b} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix} \begin{bmatrix} D^0_0 \\ D^1_1 \\ D^2_2 \\ D^3_3 \\ \vdots \end{bmatrix} \qquad (6)$$

where A and b are non-zero constants. Voigt notation is commonly used in continuum mechanics to represent a higher order elastic tensor in lower order. As can be seen, the determinant of the elastic tensor in (6) is equal to one for all values of b.

As in earlier work [1], a geometry suggestive of quantum mechanics can be derived if constants A and b are assigned values such that the spatial components of the constitutive relation equal $c\hbar/2$. In a later section, tangent space in continuum mechanics is pulled back to four-dimensional gravitational geometry. In order to match gravity at slow speed approximations, it is important that the time-time constitutive term in (6) have a value equal to the inverse of Einstein's constant since moving the elastic tensor to the stress energy side of (7) will invert the eigenvalues. Thus, an elastic tensor satisfying these conditions can be found by solving the following simultaneous equations,

$$Ab^3 = \frac{c^4}{8\pi G}$$

$$\frac{A}{b} = \frac{c\hbar}{2}$$

which yields, with some algebraic simplification,

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$$C_{\mu\nu\alpha}^{\beta} = \frac{c\hbar}{2}\begin{bmatrix} \frac{1}{4\pi {l_p}^2} & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix} \quad (7)$$

where l_p is the Plank length. Note that the determinant of (7) is the Einstein constant. This is an important result that enables the elastic tensor to replace the Einstein constant in general relativity and maintain scale between energy and curvature for observers.

The elastic tensor for a four-dimensional continuum is actually a 4x4x4x4 matrix containing 256 terms. The problem was greatly simplified by imposing a maximum of internal symmetries. Also, shear moduli (diagonal terms to bottom right) are not shown in (7). Shear moduli may prove to be important in transformations and may need to be added at a later time. It is possible that nature may be more complex, so (7) should be viewed as a starting point for exploration of other possible spacetime anisotropies.

5. Quantum Mechanical Similarities

In this section, the mechanical model of spacetime with hypersurface tension is dissected and compared with mathematical expressions found in quantum mechanics. Similar comparisons were made in earlier work [1], but are shown here with more rigor. Concerns have arisen regarding the haphazard placement of imaginary number, *i*, in earlier work. The appearance of *i* is not haphazard, but arises from moving between the complex coordinates used here for Reimannian geometry of continuum mechanics and the normal "laboratory" coordinates of pseudo-Riemannian spacetime geometries. More clarity regarding the appearance of imaginary numbers is given below.

The first similarity between the mechanics of spacetime with hypersurface tension and quantum mechanics is found in the time-time component of (4). For problems in physical chemistry, the change in surface tension with time, $\frac{\partial Ϙ}{\partial \tau}$, is generally ignored for short-duration, micro-scale phenomena as this term has to do with overall changes in temperature of the system. The temperature independent time-time component in (4) is given by,

$$\frac{\partial^2 x^0}{\partial {x^1}^2} + \frac{\partial^2 x^0}{\partial {x^2}^2} + \frac{\partial^2 x^0}{\partial {x^3}^2} - \frac{dP}{Ϙ} = -\frac{\partial^2 x^0}{\partial \tau'^2}$$

When this equation is converted from complex coordinates back to natural units, we find,

$$ic\left(\frac{\partial^2 t}{\partial {x^1}^2} + \frac{\partial^2 t}{\partial {x^2}^2} + \frac{\partial^2 t}{\partial {x^3}^2}\right) - \frac{dP}{Ϙ} = -ic\,\frac{\partial^2 t}{-c^2 \partial \tau^2}$$

which simplifies to,

$$\frac{\partial^2 t}{\partial {x^1}^2} + \frac{\partial^2 t}{\partial {x^2}^2} + \frac{\partial^2 t}{\partial {x^3}^2} - \frac{dP}{icϘ} = \frac{1}{c^2}\frac{\partial^2 t}{\partial \tau^2} \quad (8)$$

The wave function in (8) is the observer's natural coordinate of time, t, at different points in space. These waves are known as capillary waves in physical chemistry of surfaces. As can be seen in (8), the waves move at the speed of light independent of frequency and wavelength. Since capillary waves move along hypersurfaces, they exhibit polarization and follow the overall curvature of spacetime. Capillary waves do not carry mass. Electromagnetic radiation behaves in accord with this analog. Equation (8) is similar in mathematical structure to the Klein-Gordon equation in quantum mechanics.

A second similarity with quantum mechanics is found in the diagonal spatial terms of (1), (2), (4), (5) and (7). Begin by relating the diagonal spatial terms of (1) and (2) in accord with (5) and (7) to find,

$$-\text{Ϙ} = \frac{c\hbar}{2}\left(\frac{\partial u^j}{\partial x^j} - \frac{1}{2}\left|\frac{\partial u^0}{\partial x^j}\right|^2 d\tau'\right)$$

where j is an index running from 1 to 3 to indicate spatial components only. Next, the partial derivative of the above expression for Ϙ with respect to x^j is given by,

$$\frac{\partial \text{Ϙ}}{\partial x^j} = -\frac{c\hbar}{2}\left(\frac{\partial^2 u^j}{\partial {x^j}^2} - \left|\frac{\partial u^0}{\partial x^j}\right|\frac{\partial^2 u^0}{\partial {x^j}^2} d\tau'\right)$$

The component $\frac{\partial^2 u^j}{\partial {x^j}^2}$ represents the change of in-plane[2] strain with spatial distance, which is a stretching of the hypersurface. The component $\left|\frac{\partial u^0}{\partial x^j}\right|\frac{\partial^2 u^0}{\partial {x^j}^2} d\tau'$ is the change in curvature with spatial distance. Substituting the simplified lead-order result for $\frac{\partial \text{Ϙ}}{\partial x^j}$ into (4) and again ignoring $\frac{\partial \text{Ϙ}}{\partial \tau}$ (temperature dependence) yields the energy dispersion equation,

$$\frac{c\hbar}{2}\left(\frac{\partial^2 u^j}{\partial {x^j}^2}\right) = \text{Ϙ}\,\frac{\partial u^j}{\partial \tau'}$$

Converting this equation from complex coordinates to natural units, noting that $u^j = \frac{\partial x^j}{\partial \tau'} = \frac{1}{ic}\frac{\partial x^j}{\partial \tau}$, we find,

$$\frac{c\hbar}{2}\left(\frac{\partial^2 u^j}{\partial {x^j}^2}\right)\frac{1}{ic} = \text{Ϙ}\,\frac{\partial u^j}{\partial \tau}\frac{1}{-c^2}$$

which simplifies to,

$$\frac{\hbar}{2}\frac{\partial^2 u^j}{\partial {x^j}^2} = -i\frac{\text{Ϙ}}{c^2}\frac{\partial u^j}{\partial \tau} \qquad (9)$$

The wave function in (9) is the spatial velocity of a point in space relative to the arbitrary observer. Velocity is related to energy states in transformed reference frames. This equation governs how stress and, therefore, mass/energy moves in space. Equation (9) is similar in appearance to the one-dimensional time-dependent Schrödinger equation. Surface energy density divided by the speed of light

[2] The terms *in-plane* and *out-of-plane* refer to the plane of the hypersurface in spacetime located at the arbitrary observer's time coordinate as described in Figures 1, 4, and 5.

squared, $\text{Ϙ}/c^2$, plays the role of mass density. In the geometry of continuum mechanics, mass/energy moves through space according to Schrödinger-like dispersion equations.

A third similarity with quantum mechanics is found in the constitutive relation for diagonal spatial terms in (5) and (7), which can be written as

$$\frac{T_{jj}}{D_j^j} = \frac{c\hbar}{2}$$

Recalling that hypersurface tension, T_{jj}, is the energy contained in a volume of spacetime and dynamical strain, D_j^j, has units of the inverse of imaginary proper time, this constitutive relation is simply a linear proportionality of energy and time. The expression holds for a change in time and a change in energy such that,

$$\Delta\text{Ϙ}\ \Delta\tau' = \frac{c\hbar}{2}$$

which in natural units is given by,

$$\Delta\text{Ϙ}\ \Delta\tau = -\frac{i\hbar}{2}$$

If both sides are squared, then we arrive at,

$$(\Delta\text{Ϙ})^2(\Delta\tau)^2 = \left(\frac{\hbar}{2}\right)^2 \qquad (10)$$

In continuum mechanics, equation (10) is essentially a version of Hooke's law. It governs how perturbations in spacetime oscillate and are quantized in accord with the elastic modulus. In this model, the elastic modulus for objects moving in space is the reduced Planck's constant. The form of (10) is similar to the generalized Heisenberg uncertainty principle with the exception of the equal sign.

The forth similarity with quantum mechanics is found by equating the off-diagonal spatial terms in (1) and (2) in accord with (5) and (7). One finds the following relation arising from the fluidity of spacetime,

$$\frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) - \frac{1}{2}\left|\left(\frac{\partial u_0}{\partial x_1}\right)\left(\frac{\partial u_0}{\partial x_2}\right)\right| = 0$$

In natural units, this expression reduces to,

$$\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} - ic\left|\left(\frac{\partial u_0}{\partial x_1}\right)\left(\frac{\partial u_0}{\partial x_2}\right)\right| = 0 \qquad (11)$$

Membrane-like behavior provides solutions in spacetime geometry where in-plane strains are counterbalanced by out-of-plane displacements leading to rotation and local curvature. In membrane mechanics, these rotating geometries are known as vortices. Vortices can have short duration rotational geometry or perpetual irrotational geometry. Vortices carry specific quantized amounts of angular momentum and energy that depend on the elastic properties of the hypersurface. The expression (11) is one of several possible perturbations in the rate of deformation tensor that resemble Weyl-type equations in quantum mechanics. The other possibilities are located in the off-diagonal temporal components and perturbations between P and $\wp$ along the diagonal.

One of the interesting mathematical solutions of (11) is a stable pair of geometrically entangled vortices. By geometrical entanglement, what is meant is a solution consisting of two oppositely rotating vortices whose flow lines are joined above or below the hypersurface. In this model, the flow lines would form a micro-wormhole that would appear as a string fixed on both ends to the hypersurface by Dirichlet boundary conditions.

Although continuum mechanics is founded on geometry with elasticity, and quantum mechanics is interpreted based on probability with uncertainty, the mathematical similarities between these two arenas are interesting and worth investigation. Many physicists are working to relate quantum mechanics and gravity through a probabilistic interpretation (i.e. quantum gravity); others study the stochastic mechanics of random fluctuations [12]. The direction taken here is to lay roots for a geometrical interpretation of quantum mechanics with gravitation. This work is in its infancy; a focus of future research will be to further investigate if continuum mechanics of spacetime in complex coordinates has additional similarities to quantum mechanics beyond just appearance.

6. Gravitational Similarities

The previous section showed that spacetime geometry with surface tension described in Equations (1) to (7) shares some similarities with equations in quantum mechanics. If the warping of spacetime geometry in the model resembles quantum mechanics, how does the model also describe warping of spacetime with gravitation? In this section, the model will be re-written in index notation and compared with general relativity.

The rate of deformation tensor (2) can be re-written as,

$$D_\beta^\alpha = \frac{1}{2}\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right) - \frac{1}{2}\left|\left(\frac{\partial u_0}{\partial x_\alpha}\right)\left(\frac{\partial u_0}{\partial x_\beta}\right)\right| d\tau' \qquad (12)$$

In continuum mechanics, the right term on the right side of (12) represents membrane or thin shell curvature. In fact, it is the Gaussian Curvature in 3+1 space, which is half the Scalar Curvature, R. In continuum mechanics, the left term on the right side of (12) represents the change in volume or strain. The mathematical significance of this strain term was misquoted in [1]. According to [8], this traditional strain component is the push forward of the Lie derivative of the metric tensor. Thus, Equation (12) can be written in this form,

$$D_\beta^\alpha = \frac{1}{2} g^{\mu\nu} \mathcal{L}_\nu \boldsymbol{g} - \frac{1}{2} R$$

In order to obtain the rate of deformation tensor with two covariant indices, it is necessary to lower one index by multiplication of both sides by the metric tensor with two covariant components,

$$D_\beta^\alpha g_{\mu\nu} = \frac{1}{2} g_{\mu\nu} g^{\mu\nu} \mathcal{L}_\nu \boldsymbol{g} - \frac{1}{2} R g_{\mu\nu}$$

Note that the metric with two covariant components and the metric with two contravariant components cancel in the first term on the right side. According to [13], the Lie derivative of the metric, $\frac{1}{2}\mathcal{L}_v \boldsymbol{g}$, is the extrinsic curvature in 3+1 space, which in its natural form is the Ricci Curvature Tensor, $R_{\mu\nu}$, in four-dimensions [14]. Since α and β are dummy indices, one can change indices such that $D^{\alpha}_{\beta} g_{\mu\nu} = D^{\alpha}_{\mu} g_{\alpha\nu} = D_{\mu\nu}$. Hence,

$$D_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \qquad (13)$$

The rate of deformation tensor with two covariant components is the Einstein Tensor given in [14].

Returning attention to the constitutive relationship (5), it can be seen that,

$$T_{\mu\nu} = C^{\beta}_{\mu\nu\alpha} \cdot D^{\alpha}_{\beta} = C^{\beta}_{\mu\nu\alpha} \cdot \frac{D_{\mu\nu}}{g_{\mu\nu}}$$

Contracting the elasticity tensor, moving it to the stress energy side, and inserting (13) yields an equation closely analogous to general relativity,

$$T_{\mu\nu} \cdot C^{\alpha}_{\beta} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \qquad (14)$$

except the Einstein constant from [15] is replaced by a symmetric nondegenerate anisotropic elasticity tensor,

$$C^{\alpha}_{\beta} = g_{\mu\nu} C^{\mu\nu\alpha}_{\beta} = \frac{2}{c\hbar} \begin{bmatrix} 4\pi {l_p}^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (15)$$

The proposed anisotropic constitutive relationship provides an energetic spatial geometry resembling quantum mechanics and a lower energy temporal geometry consistent with general relativity. The fact that (15) has independent eigenvalues implies that whatever energy is inserted as rest mass, *P*, in the time-time term of (1) will continue to be multiplied by Einstein's constant through Lorentz transformations. Whereas the negative stress energy inserted for surface tension, *Ϙ*, will continue to be multiplied by Planck's constant through Lorentz transformations.

7. Discussion

The concept that spacetime itself might behave as a multi-dimensional membrane long has been suggested [16]. Much of the interest in relativistic membranes in recent decades stems from the realization that action in membrane dynamics is equivalent to Nambu-Goto action in bosonic string theory. Earlier works require membranes to be embedded in higher dimensional spaces. The work [16] and the present work suggest spacetime itself may exhibit membrane-like properties.

In the Stueckelberg covariant quantum theory [17, 18], time and space are dynamical observables that evolve according to an invariant parameter, τ, related to proper time on the mass shell. The mass shell is described as the tangent space of freely falling clocks of general relativity. Proper time and the parameter, τ, are allowed to deviate on the mass shell and mass becomes dynamic. Of interest to the present work, the Stueckelberg theory alludes to a tension-like mechanism that holds a particle's mass to its original mass-shell value and drives parameter, τ, back to the proper time outside the influence of an electric field. The earlier works of [17, 18] provide more sophisticated treatments of electromagnetic interactions and could serve as a roadmap for further development of the current model.

Others have compared continuum mechanics with general relativity as recently as [19, 20, 21]. There is a general consensus on the definition of the linearized spacetime metric with infinitesimal strain. The main difference between past endeavors [19, 20] and the present is the work here is based on membrane behavior without plate bending stresses. Plate bending requires simultaneity of more than one hypersurface. In [19], a Poisson's ratio of 1 was required to explain gravitational waves. The present work used a Poisson's ratio of 0 and has yet to consider gravitational waves.

Few examples of spacetime anisotropic elastic behavior are found in literature. A form of spacetime elastic anisotropy was considered in a recent work [22]. To find a material explanation for dark energy, [22] decomposed the stress energy tensor into potential energy and viscosity components and suggested separate elastic moduli for each. The two elastic tensors in this prior work were isotropic, but acted on different components of the stress energy tensor. The twin elastic moduli suggested in [22] are mathematically similar to the anisotropic constitutive relation proposed here in combined tensor form.

Despite differences in approach, several authors of past works [19, 22] make excellent arguments about the importance in the study of continuum mechanics in general relativity. The authors [19] state that, "Over the past century, Solid Mechanics and General Relativity have advanced independently". The authors [19] also state that models of spacetime based on solid mechanics, "allows General Relativity problems to be formulated as Solid Mechanics problems" and vice-versa, "Thus, ideas, methodologies and tools from each field become available to the other field." The author [22] describes their work as a "gate-way" for importing the physics of solid mechanics into cosmology. The mathematical similarities between continuum mechanics and general relativity are intriguing and should inspire others to join in the study of this approach.

8. Conclusions

Spacetime is a three surface in the observer's coordinate of time. Simple statistical thermodynamics requires that surfaces of any dimension must exhibit surface tension. Ordinary continuum mechanics is applied in four-dimensions to write tensors governing the evolution of spacetime with surface tension. Tensors for stress energy, rate of deformation, and equations of motion for an infinitesimal element of spacetime are presented. In the model, the stress energy tensor, $T_{\mu\nu}$, with surface tension contains positive mass and negative stress components and has signature +,-,-,-.

A stable anisotropic elastic tensor is proposed for relating stress energy, $T_{\mu\nu}$, to curvature. Such a constitutive relation provides gravitation in the manifold of the observer while also providing wave equations of motion resembling quantum mechanics in the tangent space. The fact that equations of motion of spacetime resemble quantum mechanics does not alone give access to the quantum theory.

More research is needed to evaluate if quantum mechanics can be fully embodied in complex geometry. The proposed negative stress terms in the stress energy tensor and stable anisotropic elastic tensor represent a significant modification to equations of general relativity that require additional careful evaluation.

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