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Quark-meson model in a Tamm-Dancoff inspired approximation ⋆

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Abstract. The nonlinear chiral quark meson U(3)×U(3) model is solved using the Tamm-Dancoff inspired approximation (TDIA) which is described in our earlier paper [1]. The resulting system of 15 coupled nonlinear differential equations self-consistently determines all quark-meson coupling constants. Obtained solutions for quark and meson fields are stable and physically acceptable. These approximate Heisenberg fields resulted from dynamics in which u, d and s quarks were treated on the same footing. They were used to calculate SU(3) baryon octet magnetic moments and axial vector coupling constants. The baryon state vectors containing valence quarks were used. The results strongly indicate that simple state vectors and currents cannot adequately describe physical baryons.

1 Introduction

The Tamm-Dancoff inspired approximation (TDIA) [1] was applied some time ago to the chiral quark meson model based on the SU(2) linear σ-model [2,3]. The results seemed to be comparable to those obtained using the hedgehog Ansätze [4–7]. That is to some extent understandable as both methods lead to similarly looking sets of equations for meson solitons (fields). All details of the TDIA are described in ref. [1]. It is well known that the Tamm-Dancoff method [8] is a better approximation than the perturbation theory. That feature it has in common with the hedgehog based meson field solutions [5–7].

In ref. [1] the linear σ-model was used as a transparent example for the application of TDIA. However since 1996. evidence has been found for the existence of the σ meson [9–11]. It has been stated [10,11] that the linear σ-model with three flavors works much better than what was generally believed. In the linear σ-model one can treat both scalar and pseudoscalar nonets simultaneously. The scalars are the chiral partners of π, η, etc. and the analysis strongly suggests that they, like the pseudoscalars, are $\bar{q}q$ states [10,11]. Such theoretical conclusions made TDIA approach quite attractive as in that approximation mesons (solitons)

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naturally appear, in the lowest order as $\bar{q}q$ states. In TDIA one works in the Heisenberg picture [12], expands field operators in the free field creation and annihilation operators and then truncates the expansion. That leads automatically, after truncations, to meson fields (soliton phases) which depend on bilinear combinations of quark/antiquark operators, i.e. to $\bar{q}q$ structures.

The chiral quark/meson U(3)×U(3) model under consideration has the familiar form which was used previously when the SU(2) model [5,6] was enlarged by cranking involving intrinsic flavor space [7]. The system of nonlinear differential equations obtained here bears some similarity to the systems obtained by using hedgehog ansätze [5–7]. It has been argued that the linear σ-model [10,11] and its close relative the quark-meson model [7] might capture the essential features of QCD in the low energy region, while being easier to handle than the complex exact quark-gluon theory. The TDIA treatment of the U(3)×U(3) quark-gluon model thus might give some physical insights in the baryon structure.

Even with the bag formalism for quarks retained [1,13], thus using the static spherical cavity approximation and with the modest symmetry breaking, the lowest order TDIA leads to the coupled system of 15 nonlinear differential equations and 21 boundary conditions. That problem is completely solvable, as it will be outlined below. The strengths of quark-meson couplings are self consistently determined by the system. In principle the spherical cavity approximation for quarks can be dropped. That would lead to somewhat larger system of equations.

The structure of this model [1] is very transparent and all of its features are always discernible. One can see directly how the approximate baryon states, made of valence quarks only [14,15], perform. In order to do that one calculates the matrix elements of the (approximate) Heisenberg operators. As in TDIA the isospin (and hypercharge) and spin are separably conserved, the solutions can be used to calculate magnetic moments and axial-vector coupling constants for the baryon octet. The results indicate the need for richer structure ($s\bar{s}$ pairs etc.) of baryon state vectors [16] and for the inclusion of exchange current corrections [17]. The inclusion of quark triplet in the dynamical scheme does not seem to be sufficient by itself alone.

2 Model formalism

TDIA has been already described in some detail elsewhere [1]. Here we give some particulars concerning the quark linear σ-model and TDIA approximation. The Lagrangian in which the linear σ-model is embedded in the bag environment has the well known form [1,6,18]

$$\mathcal{L} = \mathcal{L}_\psi \Theta + \mathcal{L}_{int}\delta_S + [\mathcal{L}_\chi + U(\chi)]\overline{\Theta}. \tag{2.1}$$

Here all pieces but the symmetry-breaking one ($\mathcal{L}_{SB}$), are U(3)×U(3) invariant [3,7,11] i.e.

$$\begin{aligned}
\mathcal{L}_\psi &= \frac{i}{2}(\overline{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\overline{\psi}\gamma^\mu\psi) \\
\mathcal{L}_{\text{int}} &= \frac{g}{2}\overline{\psi}(\sigma_a + i\pi_a\gamma_5)\lambda^a\psi \\
\mathcal{L}_\chi &= \frac{1}{2}\left(\partial^\mu\sigma_a\partial_\mu\sigma_a + \partial^\mu\pi_a\partial_\mu\pi_a\right) \\
U(\chi) &= -\frac{1}{2}\mu^2(\sigma_a^2 + \pi_a^2) - \frac{1}{4}\lambda^2(\sigma_a^2 + \pi_a^2)^2 + \mathcal{L}_{SB}.
\end{aligned} \tag{2.2}$$

Here (σ^a, π^a, a=0,1,...,8) are (scalar, pseudoscalar) U(3) nonets. The symmetry is broken in a minimal way by the vacuum expectation values of U(3) scalars σ and ζ

$$\begin{aligned}
\mathcal{L}_{SB} &= m_\pi^2 f_\pi \sigma + \frac{(2m_K^2 f_K - m_\pi^2 f_\pi)}{\sqrt{2}}\zeta \\
\sigma_{\text{vac}} &= f_\pi \Rightarrow \sigma \to \sigma - f_\pi \\
\zeta_{\text{vac}} &= \frac{(2f_K - f_\pi)}{\sqrt{2}} \Rightarrow \zeta \to \zeta - \zeta_{\text{vac}}.
\end{aligned} \tag{2.3}$$

That leaves pseudoscalar (scalar) masses in the corresponding U(3) nonets degenerate.

The standard variational procedure leads to the coupled system which contains equations of motion, linear boundary and derivative boundary conditions involving quantum fields. However as system retains lot of symmetry in TDIA this gets reduced to a smaller set of c-equations. Here we sketch TDIA procedure and list nonlinear system of c-equations which will be solved numerically.

The "driving" Ansätze are the ones for the quark fields. For the massless u and d fields one uses:

$$\begin{aligned}
\psi_f^c &= \frac{N_0}{\sqrt{4\pi}}\left[\begin{pmatrix} f_0 \\ i\,(\boldsymbol{\sigma}\hat{r})\,g_0 \end{pmatrix}\chi_\mu^f b_{\mu,f}^c + \begin{pmatrix} (\boldsymbol{\sigma}\hat{r})\,g_0 \\ if_0 \end{pmatrix}\chi_\mu^{f\dagger} d_{\mu,\bar{f}}^{c\dagger}\right] \\
f_0 &= j_0\left(\frac{\omega_0 r}{R}\right), \qquad g_0 = j_1\left(\frac{\omega_0 r}{R}\right) \\
N_0^2(\omega_0) &= \frac{1}{R^3}\left[j_0^2(\omega_0) + j_1^2(\omega_0) - \frac{2j_0(\omega_0)j_1(\omega_0)}{\omega_0}\right]^{-1}.
\end{aligned} \tag{2.4}$$

The SU(3)-flavor symmetry is explicitly broken by assuming that s-quark has a mass $m_s \neq 0$, with corresponding Ansatz

$$\begin{aligned}
\psi_f^c &= \frac{N_m}{\sqrt{4\pi}}\left[\begin{pmatrix} f_m \\ i\,(\boldsymbol{\sigma}\hat{r})\,g_m \end{pmatrix}\chi_\mu^f b_{\mu,f}^c + \begin{pmatrix} (\boldsymbol{\sigma}\hat{r})\,g_m \\ if_m \end{pmatrix}\chi_\mu^{f\dagger} d_{\mu,\bar{f}}^{c\dagger}\right] \\
f_m &= \sqrt{\frac{E+m_s}{E}}\,j_0\left(\frac{\omega_m r}{R}\right), \quad g_m = \sqrt{\frac{E-m_s}{E}}\,j_1\left(\frac{\omega_m r}{R}\right) \\
E(m,R) &= \frac{1}{R}\sqrt{\omega^2 + (m_s R)^2}
\end{aligned}$$

$$N_m^2(\omega_m) = \frac{N_0^2(\omega_m)}{1 + N_0^2(\omega_m) N_R}, \; N_R = \frac{m_s j_0(\omega_m) j_1(\omega_m) R^3}{E\omega_m}. \tag{2.5}$$

Here the indices c, f and μ denote color, flavor and spin respectively.

Boundary conditions involving quark fields determine (by use of Ansätze (2.4) and (2.5)), the Ansätze for the meson fields. This matching then automatically produces mesons "made out of quark pairs", as suggested in the σ-model analysis [9–11]. One needs for pseudoscalar fields, for example:

$$\begin{aligned}\pi^+ &= \pi_s^+(r)(b_{m,d}^{c\,\dagger}\, d_{m',\bar{u}}^{c\,\dagger} + d_{m,\bar{d}}^{c}\, b_{m',u}^{c})\chi_m^\dagger \mathbf{1} \chi_{m'} + \\ &+ \pi_p^+(r)(b_{m,d}^{c\,\dagger}\, b_{m',u}^{c} - d_{m',\bar{u}}^{c\,\dagger}\, d_{m,\bar{d}}^{c})\chi_m^\dagger (\boldsymbol{\sigma}\hat{r})\chi_{m'}\end{aligned} \tag{2.6}$$

Both scalar (π_s, K_s, η_s) and pseudoscalar ($\pi_p \boldsymbol{\sigma}\hat{r}, \eta_p \boldsymbol{\sigma}\hat{r}$ etc.) components of the pseudoscalar mesons are induced by the boundary conditions. The scalar parts formally correspond to physical "mesons" while the pseudoscalar ones are connected with the solitons. The solitons contribute to the baryonic current matrix elements. All that are just U(3)×U(3) generalizations of our earlier U(2) based results [1]. For scalar fields, scalar and pseudoscalar contributions are reversed. Everything is again driven by boundary conditions. which require the following

The system of q-equations is in TDIA transformed in a system of differential c-equations. The operator equalities are expressed through Ansätze (2.4)-(2.6). They are then sandwiched between suitable states. An example for that can be found in ref. [1], equation (2.16).

One ends with the profile function and with some Pauli matrices and spinors. In that way all the creation (annihilation) operators from Ansätze can be contracted and one ends with the system of 20 equations of motion, 8 linear boundary conditions and 18 derivative boundary conditions.

3 The numerical procedure

The numerical procedure is analogous to the one used by ref. [1]. It relies on the code COLSYS, the collocation system solver developed by Ascher, Christiansen and Russel [19]. However, one should keep in mind that here one deals with much larger system, which contains many novel features, and which streches COLSYS to its upper bounds.

The parameters assume the following values

$$\begin{aligned} m_\pi &= 140\,\text{MeV}, & f_\pi &= 92.6\,\text{MeV} \\ m_K &= 494\,\text{MeV}, & f_K &= 113\,\text{MeV} \\ m_s &= 125\,\text{MeV}, & R &= 5\,\text{GeV}^{-1}. \end{aligned} \tag{3.1}$$

The parameters μ and λ from $U(\chi)$ (2.2) were selected by the requirement that all the profile functions appearing in (3.1), vanish at the infinity.

Using that requirement we have:

$$\mu^2 = -1.29525 \cdot 10^{-2}\ \text{GeV}^2, \qquad \lambda = 9.95484.$$

The coupling constants g_M (M=$\eta, \pi, \dots$) in (2.2) are connected with the linear boundary conditions. This cannot be satisfied by an universal coupling constant g which figures in (2.2) and one encounters, as it was found before [1], some dynamical symmetry breaking. The $U(3) \times U(3)$ model determines all coupling constants g_M leading to the values, shown in Table 3.1.

Table 3.1. The quark-meson dimensionless coupling constants.

g_M	g_σ	g_π	g_K	g_η	$g_{\eta'}$	g_{a_0}	g_κ	g_ζ
	10.7	4.0	7.8	4.0	3.1	1.5	3.9	10.5

The model g_π value is, interestingly, close to the estimated value in ref. [17]. The corresponding ω values are

$$\omega_0 = 2.0; \qquad \omega_m = 2.28 \tag{3.2}$$

In Fig. 3.1 the radial dependencies of $r^2\phi^2(r)$ (ϕ=π_p, K_p, σ_s, $a_{0,s}$) are plotted. The function corresponding to scalar fields ($r^2\sigma_s^2$, $r^2 a_{0,s}^2$) are much smaller than the contributions associated with pseudoscalars (π_p and K_p).

As one has solved the complex coupled system, which contains both nonstrange and strange profile functions, one can say that u, d, π etc. profile functions "feel" the presence of the s-quark dynamics.

4 Results and Conclusions

Our model formalism in TDIA is used for the evaluation of the magnetic moments and the axial vector coupling constants of the nonstrange and strange baryons.

The baryon magnetic moments are determined by quark $\mu^{(Q)}$ and meson $\mu^{(M)}$ pieces. As the flavor SU(3) is broken only by $m_s \neq 0$, the quark piece has the contribution coming from the u, d quarks $\mu_0^{(Q)}$ and the contribution coming from the s quark $\mu_s^{(Q)}$. The meson pieces depend on the pion soliton $\mu_\pi^{(M)}$ and the kaon soliton $\mu_K^{(M)}$. Their values are:

$$\mu_0^{(Q)} = 1.886, \qquad \mu_s^{(Q)} = 1.695 \tag{4.1}$$

$$\mu_\pi^{(M)} = \frac{8\pi}{3}\int_{R_{bag}}^{\infty} r^2 dr\ \pi_p^2(r) = 0.027, \tag{4.2}$$

$$\mu_K^{(M)} = \frac{8\pi}{3}\int_{R_{bag}}^{\infty} r^2 dr\ K_p^2(r) = 0.020. \tag{4.3}$$

In Table 4.1 the model values are compared with experimental results.

Table 4.1. Baryon magnetic moments.

Baryon	μ_Q	μ_M	μ	μ_{exp}	$\Delta\mu$ %
p	1.886	0.027	1.913	2.793	46
n	−1.257	−0.026	−1.284	−1.913	49
Λ	−0.564	−0.020	−0.584	−0.613	8
Σ^0	0.607	0.010	0.617	-	-
$\Sigma^0 \to \Lambda$	1.089	0.021	1.110	1.610	45
Σ^-	−0.650	0.000	−0.650	−1.160	78
Σ^+	1.864	0.020	1.884	2.458	31
Ξ^0	−1.172	−0.020	−1.191	−1.250	5
Ξ^-	−0.543	0.000	−0.543	−0.651	20

Both quark Q and meson M phases were calculated in a model which includes s quarks. However the simplest "valence" proton state vectors were used. The same "valence" approximation [14,15] was used for the other baryon state vectors.

The s-quark admixture in the nonstrange baryon state vectors would pick up additional contributions from quark and meson fields calculated in TDIA. That would change both the theoretical expressions for the magnetic moments and for the axial vector coupling constants. However, from the point of view of the present work, that would require a substantial addition to the model.

A very similar conclusion follows from the investigation of the axial vector coupling constants.

Table 4.2. Diagonal axial vector constants.

Constant	$g_A^{(Q)}$	$g_A^{(M)}$	g_A	Experiment	Δg in %
g_A^3	1.110	0.184	1.294	1.267	2
g_A^0	0.666	0.111	0.777	0.280	178
g_A^8	0.666	0.111	0.777	0.579	34

It seems reasonable to assume that the discrepancies are again caused by the too poor structure of the proton state vectors. It is usually stated [16] that $s-$ quark admixture in the proton state vector must be important. However the prediction for the isovector axial vector coupling constant g_A^3 is very good. This seems to be some general characteristic od the chiral models which are constructed to satisfactory reproduce $g_A^{I=1}$. Moreover the present nonlinear. nonperturbative approach seems to work somewhat better than some simple expansions which might lead to too large $g_A^{I=1}$.

As shown in Table 4.3 the calculated g_A's, for the semileptonic decays, seem reasonable in two cases. All signs are correctly predicted, absolute magnitude of the Λ-decay constant is 14% too large, Σ-decay constant is 53% too small and the Ξ^--decay constant is 13% too large.

Table 4.3. g_A in semileptonic decays.

Decay	$(g_A)_Q$	$(g_A)_M$	g_A	exp.	Δg in %
$\Lambda \to p + e^- + \bar{\nu}_e$	-0.758	-0.059	-0.817	-0.718	14
$\Sigma^- \to n + e^- + \bar{\nu}_e$	0.206	0.016	0.222	0.340	53
$\Xi^- \to \Lambda + e^- + \bar{\nu}_e$	-0.253	-0.029	-0.282	-0.250	13

Here, as in Tables 4.1-4.2 the meson phase contribution is noticeably smaller than the quark phase contributions. This might look as a support for the simple quark models [14,15]. However our model which contains the spherical cavity as an essential ingredient, might be biassed in that direction. Thus in the future one should attempt to solve a model in which a quark bound state does not need a bag.

In its present form this nonlinear self consistent model shows interesting features. For example π and K contributions are considerably larger than the σ and a_0 contribution. One is tempted to conclude that this reflects the fact that in baryonic processes the presence of scalars was hard to detect. Generally speaking the model offers the stable and physically acceptable [9–11] solutions.

In this model the complete problem with u, d and s quarks and two meson nonets has been solved in TDIA. Quite complicated nonlinear operator dynamics has been reduced to the highly nontrivial, but solvable, nonlinear system.

All model dependent quantities, Tables 4.1-4.3 have acceptable orders of magnitude. All relative signs for μ and g_A are correctly predicted. The discrepancies with the experimental magnitudes reflect the exploratory character of the present TDIA solution. They might be connectable to the too simple description of the baryon state vectors [16] and to the absence of the exchange current corrections [17]. A future development of TDIA based solution might lead to better predictions.

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