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On the entropy variation in the scenario of entropic gravity

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A R T I C L E   I N F O

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A B S T R A C T

In the scenario of entropic gravity, entropy varies as a function of the location of the matter, while the tendency to increase entropy appears as gravity. We concentrate on studying the entropy variation of a typical gravitational system with different relative positions between the mass and the gravitational source. The result is that the entropy of the system doesn't increase when the mass is displaced closer to the gravitational source. In this way it disproves the proposal of entropic gravity from thermodynamic entropy. It doesn't exclude the possibility that gravity originates from non-thermodynamic entropy like entanglement entropy.

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1. Introduction

Since the discovery of the thermodynamics of black hole, it has been realized that gravity and thermodynamics have a very close relationship with each other [1–5]. Gravity can have thermodynamic effects, while gravitational equations can be derived from thermodynamic assumptions. These theoretical developments lead Verlinde to propose a totally entropic origin of gravity where gravity is viewed as a kind of entropic force [6,7].

The idea of entropic gravity is simple but powerful. Consider an object of mass m located at a distance from a gravitational source. It is assumed that the entropy of the entire gravitational system changes as a function of the location of the matter. Especially the entropy increases if the matter is located closer to the gravitational source. Then the tendency of the system to increase its entropy appears as the gravity of the gravitational source to the object. Verlinde has postulated a concrete formula for the change of the entropy as

$$\Delta S = 2\pi K_B \frac{mc}{\hbar} \Delta x, \tag{1}$$

where Δx is the displacement of the mass. This formula is the starting point of the derivation in [6] of Newtonian gravity and furthermore Einstein gravity. However, the postulation has never been carefully examined.

In this paper we shall examine this assumption and derive the entropy variation of a gravitational system with different relative positions. (See Fig. 1.) We find that the entropy of the system doesn't increase when the mass is placed closer to the gravitational source, thus it disproves the assumption of entropic gravity. It is worthy to note that in recent years the profound relationship between quantum entanglement and gravity has been gradually disclosed [8–10]. Our result does not exclude the possibility that gravity originates from the entanglement entropy which is not a thermodynamic quantity in essence.

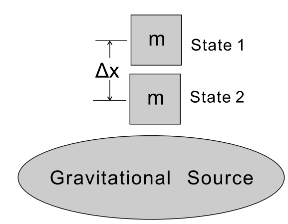

Fig. 1. The system consists of a mass, a gravitational source and the gravitational fields around them. Taking a thermodynamic viewpoint, we can always concern about the energy and entropy for a macroscopic state of the system. The energy difference of the two states is familiar, while whether or not the two states have different entropies as the function of x is to be studied.

2. A spontaneous process

To study the entropy of the gravitational system with different relative positions, we conceive an experiment as shown in Fig. 2 which couples the gravitational system with a standard thermodynamic system filled with some kind of thermodynamic gas. We choose a box of photon gas for simplicity of calculation. The ther-

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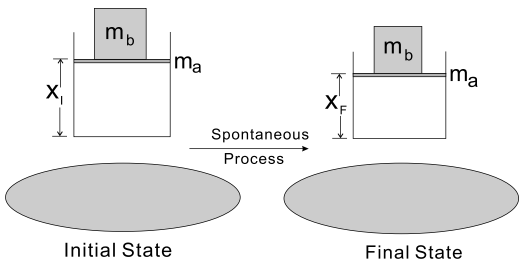

Fig. 2. The gravitational system is coupled with a device which is filled with some kind of thermodynamic gas. Releasing the extra mass m_b, the system will evolve to reach the final state where the total mass $m = m_a + m_b$ is moved closer to the gravitational source. As a spontaneous process, for the entire system there is $\Delta S_{gravity} + \Delta S_{gas} \geq 0$. Since ΔS_{gas} can be easily obtained by thermodynamic analysis, we get an inequality for $\Delta S_{gravity}$.

modynamic quantities of photon gas with certain volume V and temperature T are as follows:

$$P = \frac{\pi^2}{45}T^4,\ \ U = \frac{\pi^2}{15}VT^4,\ \ S = \frac{4\pi^2}{45}VT^3, \tag{2}$$

where we have taken $\hbar = c = K_B = 1$.

The cap of the box is movable and has mass m_a. In the initial state, the pressure of the gas is balanced with the weigh of the cap. Thus we have $P_I = m_a g/A$, where A is the surface area of the cap and g represents the absolute value of the gradient of the gravitational field. It follows the temperature as $T_I = (\frac{45 m_a g}{\pi^2 A})^{1/4}$, and the energy and entropy of the gas as

$$U_I = 3P_I V_I = 3m_a g x_I, \tag{3}$$

$$S_I = \frac{4\pi^2}{45} A x_I \left(\frac{45 m_a g}{\pi^2 A}\right)^{\frac{3}{4}}. \tag{4}$$

Then we release an object with mass m_b onto the cap of the box, so the cap of the box will move down to acquire a new balance. The pressure and temperature of the final state are respectively $P_F = \frac{(m_a+m_b)g}{A}$ and $T_F = \left(\frac{45(m_a+m_b))g}{\pi^2 A}\right)^{\frac{1}{4}}$. Due to the law of energy conservation, we have

$$U_I + (m_a + m_b)\, g \Delta x = U_F, \tag{5}$$

where $U_I = 3P_I A x_I$ and $U_F = 3P_F A\,(x_I - \Delta x)$. It is easy to derive $\Delta x = \frac{3}{4}\frac{m_b}{m_a+m_b}x_I$. The gravitational acceleration g is viewed as a constant for small displacement, and the potential energy of the gas has been omitted because of $U_{gas}/c^2 \ll m_a$ with $\sqrt{g x_I} \ll c$. In the final state, the energy and entropy of the photon gas are

$$U_F = 3\,(m_a + m_b)\, g x_I \left(1 - \frac{3}{4}\frac{m_b}{m_a + m_b}\right), \tag{6}$$

$$S_F = \frac{4\pi^2}{45}(1 - \frac{3}{4}\frac{m_b}{m_a + m_b}) A x_I \left(\frac{45\,(m_a + m_b)\, g}{\pi^2 A}\right)^{\frac{3}{4}}. \tag{7}$$

Notice the initial state and the final state of the entire system are attached by a spontaneous process. According to the second law of thermodynamics, the total entropy can never decrease for an isolated system, which means $\Delta S_{gravity} + \Delta S_{gas} \geq 0$. The entropy variation of the gas is

$$\begin{aligned}\Delta S_{gas} &= S_F - S_I \\ &= \frac{4\pi^2}{45} V_I \left(\frac{45 m_a g}{\pi^2 A}\right)^{\frac{3}{4}} \left[\frac{m_a + \frac{m_b}{4}}{(m_a + m_b)^{\frac{1}{4}}\, m_a^{\frac{3}{4}}} - 1\right].\end{aligned} \tag{8}$$

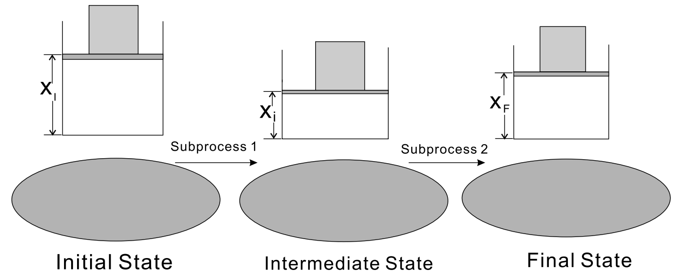

Fig. 3. The initial state and the final state are attached by a quasi-static process, which includes two sub-processes. In the first subprocess, the extra mass m_b is pulled by a string and the wight of the mass is added very slowly to the cap of the box. In the second subprocess, the gas absorbs heat to push the cap back and reach the final state. For a quasi-static process of the system, there is $\Delta S_{gravity} + \Delta S_{gas} = \int \frac{\delta Q_{gas}}{T_{gas}} + \frac{\delta Q_{gra}}{T_{gra}}$. As the difference of a state function, ΔS_{gas} can be directly read from the initial and final states. The right hand side of the equality can be obtained by detailed analysis of the evolvement processes. Then we can get $\Delta S_{gravity}$.

One can check $\Delta S_{gas} \geq 0$ for any ratio of m_b/m_a. Accordingly, it is not enough to know whether the entropy variation of the gravitational system $\Delta S_{gravity}$ is positive or not.

3. A quasi-static process

In fact, one can construct various physical processes attaching the initial and final states. To obtain the exact value of the entropy variation of the gravitational system, we must choose a quasi-static process other than the spontaneous process analyzed above.

As visualized in Fig. 3, the process can be decomposed as two sub-processes. The first is an adiabatic process. The extra mass m_b is pulled by a string and the wight of the mass is added very slowly to the cap of the box, until all the wight of the mass is released. For an infinitesimal displacement, from the law of energy conservation we can derive the relation $\Delta x = -\frac{3}{4}\frac{\Delta m}{m(x)+\Delta m}x$ where $m(x)$ denotes the total mass that has added to the cap of the box at the height x. Taking the limit of $\Delta m \to 0$, we get the differential equation $\frac{dx}{x} = -\frac{3}{4}\frac{dm}{m}$. Together with the initial value $x|_{m=m_a} = x_I$, we solve that the immediate state has $x_i = \left(\frac{m_a}{m_a+m_b}\right)^{\frac{3}{4}} x_I$. In this subprocess the exterior has done negative work to the entire system which gives

$$\begin{aligned}W &= \int_{x_I}^{x_i} F(x)dx \\ &= -4\left[m_a + \frac{m_b}{4} - (m_a + m_b)^{\frac{1}{4}}\, m_a^{\frac{3}{4}}\right] g x_I.\end{aligned} \tag{9}$$

In the immediate state, the pressure and temperature of the gas are $P_i = \frac{(m_a+m_b)g}{A}$ and $T_i = \left(\frac{45(m_a+m_b))g}{\pi^2 A}\right)^{1/4}$, then the energy and entropy are calculated as

$$U_i = 3\,(m_a + m_b)\, g x_I \left(\frac{m_a}{m_a + m_b}\right)^{\frac{3}{4}}, \tag{10}$$

$$\begin{aligned}S_i &= \frac{4\pi^2}{45}\left(\frac{m_a}{m_a + m_b}\right)^{\frac{3}{4}} A x_I \left(\frac{45\,(m_a + m_b))g}{\pi^2 A}\right)^{3/4} \\ &= \frac{4\pi^2}{45} A x_I \left(\frac{45 m_a g}{\pi^2 A}\right)^{\frac{3}{4}}.\end{aligned} \tag{11}$$

Note that $S_i = S_I$ is reasonable since the gas doesn't absorb heat from exterior in the subprocess.

The next subprocess is an isothermal process. The gas absorbs heat from the exterior heat reservoir to reach the final state, while its temperature is fixed as $T=\left(\frac{45(m_a+m_b))g}{\pi^2 A}\right)^{\frac{1}{4}}$. The thermodynamics of the final state has been listed as Eqs. (6) and (7). To reach it, by energy conservation the heat absorbed by the gas should be

$$\Delta Q_{gas} = (U_F - U_i) + P\,(V_F - V_i) \\ = 4\left[m_a + \frac{m_b}{4} - (m_a+m_b)^{\frac{1}{4}}\, m_a^{\frac{3}{4}}\right] g x_I. \tag{12}$$

Noticing that $W + \Delta Q_{gas} = 0$, the net energy of the entire system obtained form outside is 0 now, which means the total energy of the entire system has reached the finial state. As the result, the gravitational system can not absorb any heat from exterior, so $\Delta Q_{grav} = 0$.

In a quasi-static process, the increase of entropy of a system can be calculated from its absorption of the heat from outside divided by temperature, so we have

$$\Delta S_{gravity} + \Delta S_{gas} = \int \frac{\delta Q_{gas}}{T_{gas}} + \frac{\delta Q_{gra}}{T_{gra}}. \tag{13}$$

Since entropy is a function of state of the system, ΔS_{gas} can be directly determined by the initial and final states, which has been given in eq. (8). On the other hand, ΔQ_{gas} and ΔQ_{grav} depend on the evolvement processes and have been analyzed above. Dividing ΔQ by the temperature, one can easily check that the value of $\int \frac{\delta Q_{gas}}{T_{gas}} + \frac{\delta Q_{gra}}{T_{gra}}$ is equal to ΔS_{gas}. Finally, we get the main result $\Delta S_{gravity} = 0$.

4. Concluding remarks

In conclusion, by coupling the gravitational system with a standard thermodynamics system, we are able to calculate the entropy variation of the gravitational system with different relative positions. The advantage of this approach is that it avoids a direct treatment of the possible quantum-gravitational effects within the two-body gravitational system which is not clear before a complete understanding of quantum gravity. Only the energy conservation principle and the second law of thermodynamics have been used in the analysis. The result shows that the entropy doesn't increase when the mass is located closer to the gravitational source. So the analysis does not support the viewpoint of gravity as an entropic force at least for the conventional thermodynamic entropy.

The fundamental reason that gravity cannot be understood as an entropic force is that gravity is a conservative force [11]. The movement of an object to the gravitational source is commonly a reversible process unless it is accompanied by gravitational radiation. (Nevertheless, we can construct a spherically symmetric process to avoid gravitational radiation.) In contrast, the falling across the black hole horizon is irreversible. So the horizon thermodynamic effects are understandable by intuition, while the generalizations to general gravitational phenomena are questionable.

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