Equation of State of Isentropic Hot Neutron Star Matter

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Introduction

The equation of state (EoS) of neutron star matter (NSM) at finite temperature T is the crucial input in the simulation studies of astrophysical phenomena, such as; binary neutron star merger (BNSM) and core-collapsing supernovae and formation of proto-neutron star (PNS) [1,2]. Although there are several EoSs constructed under different formulations for the purpose, but still the area is understood to a less extent and the subject is of crucial contemporary research interest. In this work we shall formulate the EoS of hot isentropic neutrino-trapped NSM of neutron (n), proton (p), electron (e) and muon (μ) composition in charge neutral beta-stable condition. The EoS has keen relevance in the core-collapsing supernovae leading to formation of PNS.

Formalism

The neutrino-trapped NSM of n + p + e + μ composition in equilibrium under the weak interaction is subject to the condition

$$\mu_n - \mu_{\nu_l} = \mu_p + \mu_l \quad , \qquad (1)$$

where the lepton l = e or μ, μ_i , i= n, p, e , μ, is the respective chemical potential at equilibrium and ν_l is the corresponding neutrino for the leptons e and μ. The charge neutrality condition is

$$Y_p = Y_e + Y_\mu \quad , \qquad (2)$$

where $Y_i = (\rho_i/\rho)$, i= n, p, e , μ, ν_l, is the respective particle fractions having density ρ_i and $\rho = \rho_n + \rho_p$ is the total density of NSM. The chemical potentials $\mu_i(\rho, Y_p, T)$ are functions of density ρ, equilibrium proton fraction Y_p and temperature T. For a given ρ and T, the beta-equilibrium charge neutrality conditions in Eqs. (1) and (2) are to be solved simultaneously for conserved lepton number at constant entropy S in order to obtain the equilibrium particle fractions. We have used the finite range simple effective interaction model (SEI) to compute the nucleonic chemical potentials μ_n and μ_p by exactly solving the asymmetric nuclear matter at temperature T self-consistently. The details of the self-consistent evaluation procedure is discussed in our recent work in Ref. [3]. The leptonic chemical potentials are obtained in the frame work of relativistic Fermi gas model at temperature T. So our evaluation is exact and thermodynamically consistent.

Results and Discussion

The EoS of SEI used in the present calculation is the same as has been used in the work of Ref. [3]. The Eqs. (1) and (2) are evaluated for total lepton fraction $Y_L = Y_l + Y_{\nu_l}$=0.35 and for the values of constant entropy per particle S=1,2,3 (in the unit of k_B, the Boltzmann constant) which are typical for supernovae matter [4,5]. The particle fractions are shown in panel (a) of Fig.1 for the three isentropic EoSs corresponding to S=1, 2, 3. The particle fractions for these different EoSs differ little and the difference is indistinguishable in the scale shown in the figure. It has been verified that our predictions in Fig.1 compare well with the predictions of the G-matrix calculation using realistic interaction [6]. In panel (b) we have shown the chemical potentials of the particles as a function of density which also differ little from each other for the three EoSs. The EoSs of the isentropic supernovae matter for S =1,2 and 3 are shown in panel (a) of Fig.2 where we have shown the pressure as a function of energy density. In panel (b), the isentropes, which is the temperature profile as a function of density in the supernovae matter, are shown for the three values of constant entropy, S=1,2 and 3. The EoSs for the three cases of S=1,2 and 3 differ little while the isentropes show remarkable differences. For higher S the temperature profile inside the supernovae matter is considerably stiff.

We shall now resort to the neutron star (NS) phenomenology in order to understand this prediction of stiffer isentrope for higher entropic condition. The mass-radius and mass-central density relations obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equation for the

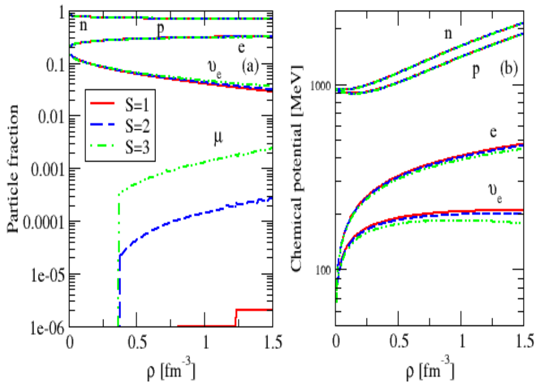

Fig. 1: Particle fractions as a function of ρ (Panel a), Chemical potentials as a function of ρ (Panel b).

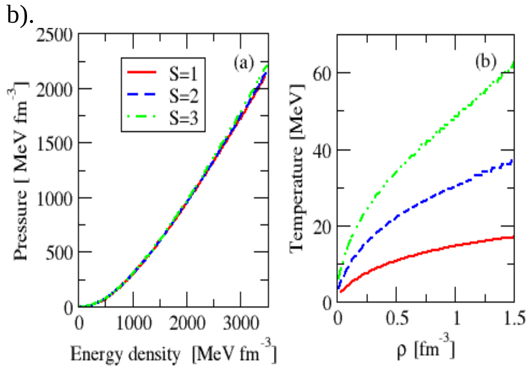

Fig. 2: Pressure of NSM as a function of Energy density (Panel a), Temperature as a function of ρ (Panel b).

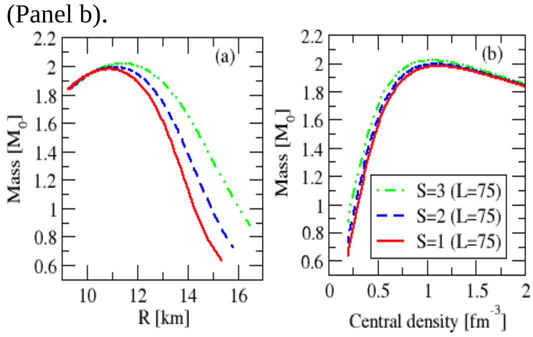

Fig. 3: Mass as a function of Radius (Panel a), Mass as a function of Central density (Panel b).

three EoSs of S=1,2 and 3 are shown in panel (a) and (b) of Fig.3. For a given mass M, let us consider M =1.4 M_0, the radius $R_{1.4}$ for the three EoSs of S=1,2 and 3 are 13.506 km,13.994 km and 14.824 km, respectively. The compactness C = (GM/Rc^2), where G is the gravitational constant and c is the velocity of light, of 1.4 M_0 NS for these three EoSs of S=1, 2 and 3 are 0.1529, 0.1476 and 0.1393 respectively, which shows that the number of particles in a given volume decreases with an increase in S. Since entropy is the measure of disorderness in the system, a large temperature is required for a system having less number of particles to attain the same order of disorderness. This makes a probable explanation of stiffer isentrope for higher S although the change in the EoSs of different S is negligible. It has also been verified that the isentropes have crucial dependence on the slope parameter L, whereas, these isentropes have almost no dependence on the incompressibility parameter.

Conclusion

The composition and EoS of neutrino-trapped n + p + e + μ beta stable charge neutral hot isentropic NSM is computed in exact and thermodynamically consistent manner using the finite range SEI model and relativistic Fermi-gas model at temperature T. Matter in the core collapsing supernovae leading to the formation of PNS is supposed to remain in such low isentropic condition where the constant entropy per particle has value in the range S= 1-3 (in the unit of k_B). The leptons in the matter are dominantly electronic ones, which is also the results of our calculation shown in Fig.1, where the muon fraction is found to be negligibly small. The results of the particle fractions in Fig.1 compares well with the predictions of the G-matrix calculation using realistic interaction. In our computation of EoSs in the low entropic isentropic NSM, there is negligible change in the EoS for the variation of S from 1 to 3. However, the temperature profile inside supernovae matter shows large variation becoming high as S increases. This happens because of decrease in compactness as S increases.

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