

Shock Acceleration of High-Energy Cosmic Rays: The Importance of the Magnetic-Field Angle

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Abstract. The physics of particle acceleration by collisionless shocks is addressed using analytic theory and numerical simulations. In this paper we focus on the importance of the angle between the shock normal and upstream *mean* magnetic field, $\langle \theta_{Bn} \rangle$, in determining the energy spectrum of the accelerated particles. We show that the acceleration rate is strongly dependent on θ_{Bn} and is a maximum at perpendicular shocks. Moreover, we demonstrate that for a wide range of reasonable parameters, the acceleration efficiency is weakly dependent on the shock normal angle. When applied to acceleration at supernovae blast waves, we find, therefore, for any given time interval, the highest-energy cosmic rays originate from regions in which the shock moves normal to the mean magnetic field. We also find that maximum energy is larger than that obtained using the well-known Bohm-limit.

1. Introduction

The origin of high-energy cosmic rays remains an important unsolved problems in astrophysics. It is currently thought that Galactic cosmic rays up to about 10^{15} eV are accelerated at shocks driven by supernovae explosions via the mechanism of diffusive shock acceleration. In this theory, charged particles are accelerated as they scatter within the converging plasma flow across the shock [1, 2, 3, 4]. The close association of energetic particles with collisionless shocks observed in interplanetary space, as well as those at the Earth's bow shock, provide convincing direct evidence that astrophysical shocks accelerate particles to high energies.

For a supernova blast wave moving into an interstellar plasma containing a magnetic field, \mathbf{B}_{IS} , the angle between the unit normal to the shock and \mathbf{B}_{IS} (θ_{Bn}) can vary along the shock surface, as shown in Figure 1. This can lead to intensity variations of cosmic rays along the shock because, in general, the particle transport is different in the directions perpendicular and parallel to the magnetic field. Because the acceleration depends critically on the particle transport normal to the shock front, θ_{Bn} plays a critical role in determining the resulting energy spectrum of the accelerated particles. Jokipii [5, 6] showed that, in general, the rate of energy gain is highest for perpendicular shocks.

The physics of particle acceleration at nearly perpendicular shocks is not as well developed as that for parallel shocks; nevertheless, it is clearly different as shown in Figure 2. In particular, an important issue has been the well-known injection threshold problem. The problem arises because, until recently, it was assumed that particles move essentially along the lines of force which are convecting through the shock. Therefore, it was thought that there was no means by

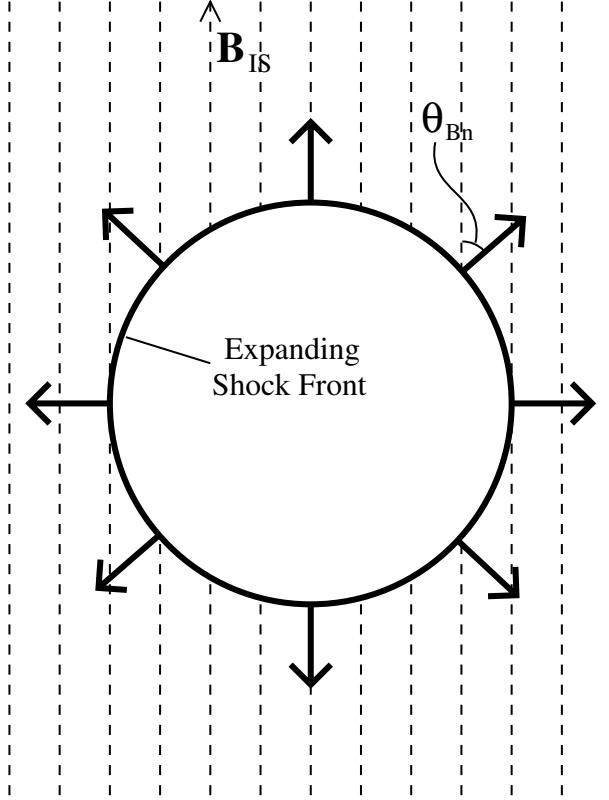


Figure 1. Illustration supernova shock and incident magnetic field

which low-energy, or suprathermal particles could encounter the shock several times, which is required for efficient particle acceleration. Here we show that there is actually no such injection problem and, in fact, the injection does not depend strongly on the shock-normal angle. This can be understood in terms of the increased cross-field transport arising from so-called field-line random walk due to the large-scale (order of a parsec) turbulent interstellar magnetic field.

Below we discuss the physics of particle acceleration at shocks with arbitrary obliquity using simple arguments based on the theory of diffusive shock acceleration. In the following section, we present results from recent test-particle numerical simulations that reveal the weak dependence of the injection efficiency on the shock-normal angle.

2. Analytic Considerations

2.1. The Parker Equation

For charged particles with a speed w moving in a plasma with a bulk flow speed U , the particle transport, to the first order in U/w , can be well described by the well-known Parker transport equation give by

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_i} \left[\kappa_{ij} \frac{\partial f}{\partial x_j} \right] - U_i \frac{\partial f}{\partial x_i} + \frac{1}{3} \frac{\partial U_i}{\partial x_i} \frac{\partial f}{\partial \ln p} + Q \quad (1)$$

where $f(x_i, p, t)$ is the phase-space distribution of cosmic rays, x_i is the position vector, p is the particle momentum, and t is time. Here, κ_{ij} is the diffusion tensor U_i is the plasma velocity, and Q is any source. Written in this form, the the effects of gradient and curvature drifts associated with the large-scale magnetic field are contained in the off-diagonal elements of the diffusion tensor (the first term on the right-hand side of (1)).

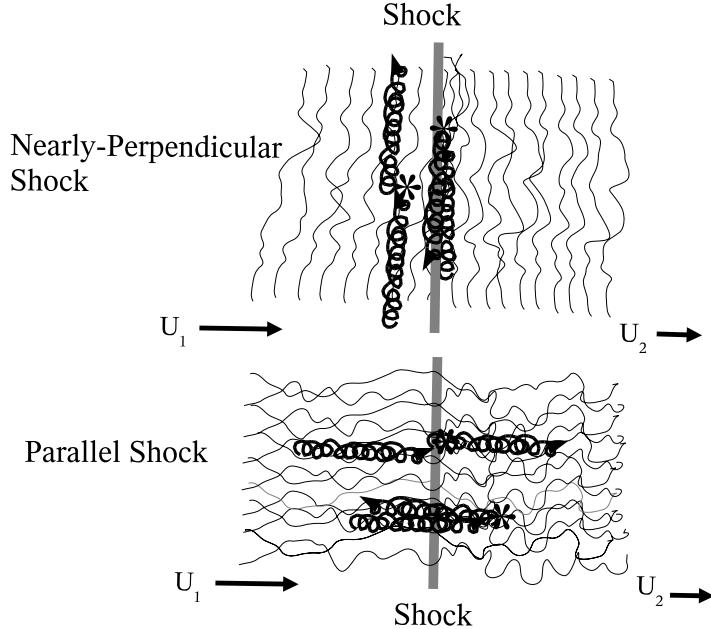


Figure 2. Illustration of particle acceleration at parallel and perpendicular shocks.

In (1) we have, in addition to spatial diffusion, convection and acceleration/deceleration caused by the $\mathbf{U} \times \mathbf{B}$ electric field. Note that the electric field does not appear explicitly – it is nonetheless contained in the terms containing the flow velocity \mathbf{U} . (1) was first written down by Parker [7] and is the basis of most current work on cosmic-ray transport and acceleration. The equation is a good approximation for energetic particles ($U/w \ll 1$) if there is enough scattering by magnetic irregularities that the scattering time is much smaller than the macroscopic time scales of the system and the distribution is nearly isotropic. It applies at shocks (discontinuities in \mathbf{U}) and the entire theory of diffusive shock acceleration [1, 2, 3, 4, 5] may be obtained from the equation by simply putting a step-function flow velocity \mathbf{U} into it. From this we see that it applies equally to parallel and perpendicular shocks [5, 6]. If the divergence of the flow velocity \mathbf{U} is zero, there is no energy change to this order in U/w .

2.2. Diffusive Shock Acceleration

The basic physics of shock acceleration is that particles gain energy as they move back and forth across the shock. The energy gain comes from moving against the motional electric field $\mathbf{E} = -(1/c)\mathbf{U} \times \mathbf{B}$. For the case of a parallel shock, the particles move back and forth across the shock and either gain energy as they are scattered by an approaching magnetic irregularity in the upstream flow, or lose energy as they scatter off of a retreating fluctuation in the downstream flow. There is a net gain in energy because the gains are larger than the losses due to the difference in flow speeds across the shock. For a perpendicular shock, the particle drifts along the shock front due to the gradient in the magnetic field at the thin shock and crosses it many times in its gyromotion. This drift is in the same direction as the electric field, thus the particles gain energy. Another way of seeing this is to note that at the perpendicular shock, due to its gyromotion, the particle crosses the shock many times, gaining an energy increment each time, in one parallel mean free path. These differences are illustrated in Figure 2.

A quantitative solution can be obtained by solving the (1) for a shock-like discontinuity. The usual approach is to solve (1) in the upstream and downstream plasma separately, and then

match the solutions at the shock. Assuming that the shock is located at $x = 0$, it is readily found that the steady solution to (1) for a one-dimensional planar shock is given by

$$f(p) = Ap^{-3r/(r-1)}H(p - p_0)F(x, p) \quad (2)$$

where r is the ratio of the downstream to upstream plasma density, p_0 is the injection momentum, $H(p)$ is the Heaviside step function, and

$$F(x, p) = \begin{cases} \exp(U_1 x / \kappa_{xx}) & x \leq 0 \\ 1 & x > 0 \end{cases}$$

Here κ_{xx} is the component of the diffusion tensor normal to the shock. In terms of θ_{Bn} , the acute angle between the shock normal and mean magnetic field, $\kappa_{xx} = \kappa_{\perp} \sin^2 \theta_{Bn} + \kappa_{\parallel} \cos^2 \theta_{Bn}$.

The key result is that the downstream spectrum depends only on the shock compression ratio! In the limit of a strong shock, where $r \rightarrow 4$ the momentum dependence becomes $f(p) \propto p^{-4}$, which corresponds to an energy spectrum $j = p^2 f \propto p^{-2}$ which is not far from the observed Galactic cosmic-ray spectrum at relativistic energies.

2.3. Acceleration Rate

Of course, the spectra observed in space are only power laws up to a particular energy. At higher energies they roll over and their dependence on energy is more complicated. There are a number of causes of the spectral rollover. These include free-escape losses, finite size of the shock (see below), and time dependence. Of course, the power law does not arise instantaneously at the shock. The acceleration to high energies takes time. Thus, one needs to solve the time-dependent transport equation (1), for particles injected at a momentum p_0 at $t = 0$. The result reveals that the spectrum above p_0 is the same power law given in (2), but with a rollover at a cutoff momentum, p_c . This cutoff increases with time with the following rate:

$$\nu_{acc} = \frac{1}{\tau_{acc}} \sim \frac{U_1^2}{\kappa_{\parallel} \cos^2 \theta_{Bn} + \kappa_{\perp} \sin^2 \theta_{Bn}} \quad (3)$$

Thus, taking $\nu_{acc,\parallel}$ to be the acceleration rate at a parallel shock ($\theta_{Bn} = 0$), and $\epsilon = \kappa_{\perp}/\kappa_{\parallel}$, we obtain

$$\frac{\nu_{acc}}{\nu_{acc,\parallel}} = \frac{1}{\cos^2 \theta_{Bn} + \epsilon \sin^2 \theta_{Bn}} \quad (4)$$

Equation (4) is plotted as a function of θ_{Bn} for the case of $\epsilon = 0.02$ in the left panel of Figure 3 (A). Clearly the acceleration rate is a maximum at a perpendicular shock.

It is interesting to note that the acceleration rate at a perpendicular shock is larger than the result from Bohm diffusion, which was previously thought to be the smallest possible diffusion coefficient [8]. This can be shown by determining the ratio of the acceleration time to that for Bohm diffusion given by:

$$\frac{\nu_{acc,\perp}}{\nu_{acc,Bohm}} = \frac{(1/3)wr_g}{\kappa_{\perp}} = \frac{\kappa_A}{\kappa_{\perp}} = \frac{1}{\epsilon\eta} \quad (5)$$

For reasonable parameters, this quantity is larger than unity for the highest-energy particles (note that η is expected to decrease with energy according to the quasi-linear theory, while ϵ is independent of energy [9]). Thus, for a given time interval, higher energies can be achieved by acceleration at a perpendicular shock compared to the predictions using Bohm diffusion.

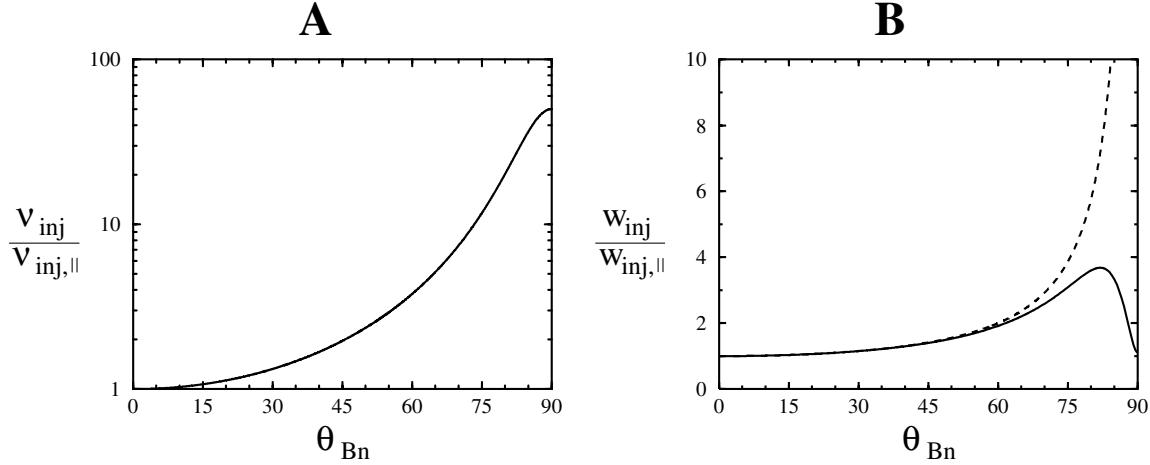


Figure 3. (A) Acceleration rate, normalized to that at a parallel shock, as a function of θ_{Bn} . (B) Injection velocity derived from the diffusive streaming anisotropy for the case of field-line random walk (solid line) normalized to that at a parallel shock. The dashed curve assumes the scatter-free approximation. See text for details.

2.4. Effects of Spherical Geometry

The previous discussion has considered a planar shock. This is a good approximation if the scale of variation of the energetic particles upstream of the shock, $L_{cr} \approx \kappa_{nn}/U_1$ (where κ_{nn} is the component of the diffusion tensor in the direction of the unit normal to the shock) is significantly less than the shock radius, r_{sh} . If L_{cr} is of the order of r_{sh} or greater, the scale becomes approximately r_{sh} .

Also, the accelerated particles may escape from the shock along the magnetic field with diffusion coefficient $\kappa_{||}$. The relevant scale length is r_{sh} . This give a loss time

$$\tau_{loss} \lesssim \frac{r_{sh}^2}{\kappa_{||}} \quad (6)$$

But, in the quasi-perpendicular part of the shock, they are accelerated with a time scale $1/\nu_{acc,\perp}$ given by

$$\tau_{acc} \approx 4 \frac{\kappa_{\perp}}{U_1^2} \quad (7)$$

For acceleration to proceed, clearly we must satisfy

$$\tau_{acc} < \tau_{loss} \quad (8)$$

or, putting in numbers and setting $\kappa_{\perp} = \epsilon \kappa_{||}$ and writing $U_{sh} = 10^9$ cm/sec and $r_{sh} = r_{pc}^*$ parsecs we obtain

$$\sqrt{\epsilon} \kappa_{||} < 6.2 \times 10^{27} r_{pc}^* \text{ cm}^2/\text{s} \quad (9)$$

For $\epsilon = 0.02$ (as before), and $r_{pc}^* = 1$, we find that this condition gives $\kappa_{||} < 4.4 \times 10^{28}$ cm²/s, which can be satisfied even with the ambient value of κ estimated from secondary cosmic rays of the order of 10²⁸ cm²/sec [10].

2.5. Injection Problem

We have shown that perpendicular shocks are more rapid accelerators of particles and can probably account for acceleration of cosmic rays to 10^{15} eV, and perhaps a bit beyond this, at a supernova blast wave. However, it is widely thought that perpendicular shocks are inefficient accelerators because of the well-known injection threshold problem. We now show that this is not the case.

The main assumption in diffusive shock acceleration is that the pitch-angle distribution is nearly isotropic. By requiring the diffusive streaming anisotropy to be small, one can readily derive an expression for the “injection velocity,” w_{inj} (c.f. [9]). The most general expression is given by:

$$w_{inj} = 3U_1 \left[1 + \frac{\kappa_A^2 \sin^2 \theta_{Bn} + (\kappa_{\parallel} - \kappa_{\perp})^2 \sin^2 \theta_{Bn} \cos^2 \theta_{Bn}}{(\kappa_{\perp} \sin^2 \theta_{Bn} + \kappa_{\parallel} \cos^2 \theta_{Bn})^2} \right]^{1/2} \quad (10)$$

where κ_{\perp} , and κ_{\parallel} , are the components of the diffusion tensor perpendicular and parallel to the mean magnetic field, respectively, and the antisymmetric component of the diffusion tensor is $\kappa_A = vr_g/3$.

For the case in which the correlation scale of the turbulent magnetic field is much larger than the gyroradius of the particles of interest, it has been shown from numerical simulations that $\kappa_{\perp}/\kappa_{\parallel}$ is independent of energy [9]. Thus, taking $\epsilon = \kappa_{\perp}/\kappa_{\parallel} \ll 1$ and $\eta = \lambda_{\parallel}/r_g$, where $\lambda_{\parallel} = 3\kappa_{\parallel}/w$ is the parallel mean-free path, (10) can be rewritten as:

$$w_{inj} = w_{inj,\parallel} \left[1 + \frac{(1/\eta)^2 \sin^2 \theta_{Bn} + \sin^2 \theta_{Bn} \cos^2 \theta_{Bn}}{(\epsilon \sin^2 \theta_{Bn} + \cos^2 \theta_{Bn})^2} \right] \quad (11)$$

where $w_{inj,\parallel} = 3U_1$ is the injection velocity for a parallel shock.

Shown in right panel of Figure 3 (B) is the solution to (11) for $\eta = 100$ and $\epsilon = 0.02$. The dashed curve is $\sec \theta_{Bn}$, which is the scatter-free approximation which is clearly invalid for the case of a turbulent magnetic field. Note that at low-energies, the injection velocity at a perpendicular shock approaches $3U_1$, which is the same as that obtained for a parallel shock [11]!

Thus, we can conclude that enhanced motion normal to mean field by field-line random walk significantly decreases the injection velocity threshold for acceleration. Thus, the theory predicts that there should not be an injection problem at nearly perpendicular shocks. Therefore, we conclude that perpendicular shocks are both efficient and rapid accelerators of charged particles are most important in producing high-energy cosmic rays in a wide variety of astrophysical plasmas.

3. Numerical Calculations

3.1. Test-Particle Simulations

We now consider non-diffusive test-particle numerical simulations to better address the physics of acceleration at low energies. This work has recently appeared in the *Astrophysical Journal* ([12]). In these calculations, the trajectories of an ensemble of test particles are integrated by numerically solving the Lorentz force on each particle using pre-specified electric and magnetic fields. The mean magnetic field makes an angle θ_{Bn} with respect to the shock-normal direction. Superimposed on this is a fluctuating component that is determined from a pre-specified power spectrum that resembles the usual Kolmogorov spectrum. The correlation scale of the turbulent magnetic field is taken to be $2000 U_1/\Omega_i$, where U_1 is the upstream flow speed and Ω_i is the ion cyclotron frequency. Both components satisfy Maxwell’s equations. Test particles (protons) are released with an energy of 3 times the plasma-ram energy in the local fluid frame just behind the shock front. Each particle’s trajectory is integrated until it escapes downstream by convection (based on a probability of return criterion), or reaches an arbitrary high-energy cutoff (taken to be 2×10^5 times the plasma-ram energy).

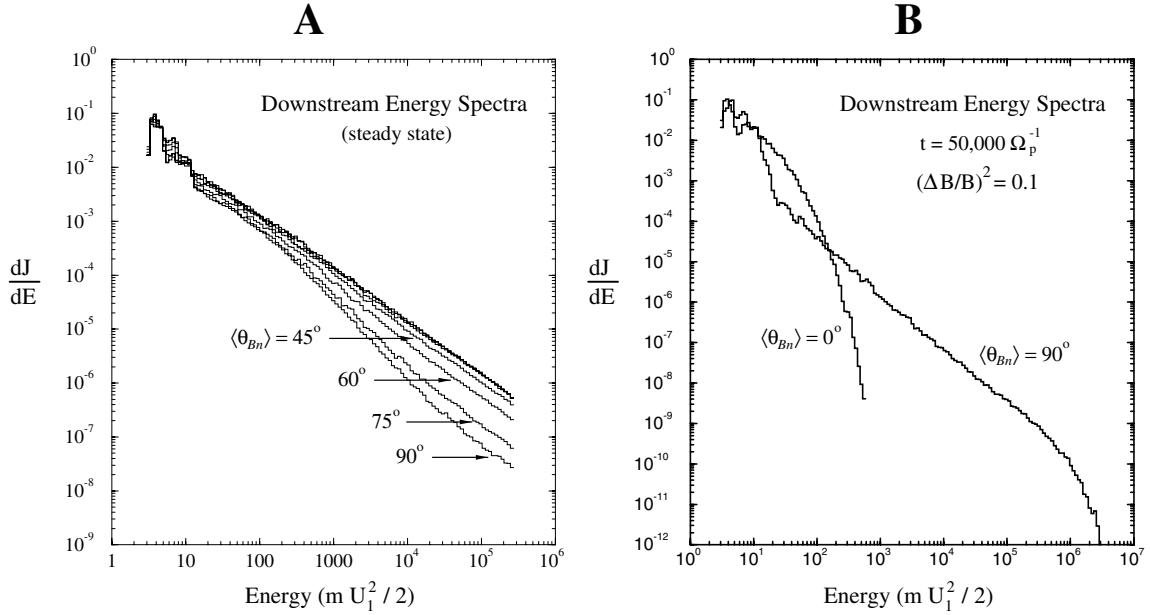


Figure 4. Downstream energy spectra for test-particle numerical simulations. (A) Steady-state spectra obtained from simulations using different values of the shock-normal angle. (B) Time-dependent spectra for two different shock-normal angles and weaker turbulence. These figures are from Giacalone [12].

Figure 4 is from Giacalone [12]. The left panel (A) shows the steady-state energy spectra downstream of the shock for 7 numerical simulations in which the only varying parameter is θ_{Bn} . Note that the spectra for the cases of $\theta_{Bn} = 0^\circ, 15^\circ, 30^\circ$ all lie on top of one another indicating that there is no dependence on this parameter at all for quasi-parallel shocks. The right panel (B) is for the case of a time-dependent acceleration process. Here, weaker turbulence was used and two different shock-normal angles are considered (as indicated).

The results shown in Figure 4 indicate the injection energy, and therefore, the acceleration efficiency does not have a strong dependence on the shock-normal angle. However, as shown in the right panel of Figure 4, for any given time interval to accelerate the particles, perpendicular shocks produce the highest-energy particles. This is because, as we discussed above, the acceleration rate is strongly dependent on the shock normal angle, provided $\kappa_\perp \ll \kappa_\parallel$. This is discussed further in Giacalone [12].

3.2. Self-Consistent Hybrid Simulations

Recently, Giacalone [13] performed massive-scale two-dimensional hybrid simulations of perpendicular shocks propagating into a turbulent upstream magnetic field. It was shown that a fraction of thermal particles encountering the shock are accelerated to high energies. The physics of this process is similar to that which we have already described above. However, the source of the high-energy particles comes directly from the thermal population, which had not been seen in previous self-consistent plasma simulations. It has been long known that a fraction of thermal ions are specularly reflected by the shock and begin to gyrate within the shock ramp before becoming thermalized downstream. For the case in which the shock moves into an upstream region containing large-scale magnetic fluctuations, some of these ions can move upstream along these lines of force before returning to the shock. These ions can gain considerable energy because they can achieve multiple interactions with the shock.

The efficiency for the acceleration in these large-scale hybrid simulations is difficult to estimate because the spatial domain is still rather limited by computation resources. However, it was estimated that the efficiency is probably comparable to that obtained for a parallel shock, or about 10-20% [14].

4. Summary

We have addressed the physics of charged-particle acceleration by shocks. We have shown that the perpendicular shocks are as efficient as parallel shocks in accelerating particles to high energies using reasonable parameters. For these same parameters, perpendicular shocks are much more rapid accelerators. Thus, we conclude that perpendicular shocks are important sites of acceleration and can produce high-energy cosmic rays in a wide variety of astrophysical plasmas. Application of these ideas to a supernova shock suggests that the standard energy limits obtained by considering acceleration at a parallel shock must be reconsidered. It seems likely that energies up to the galactic cosmic-ray knee at $\sim 10^{15}$ eV are attainable.

5. Acknowledgements

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6. References

- [1] Krymsky, G. F., 1977, *Dokl. Akad. Nauk, SSSR*, **234**, 1306
- [2] Bell, A. R., 1978, *Mon. Not. R. Astron. Soc.*, **182**, 147
- [3] Axford, W. I., E. Leer, and G. Skadron, 1978, *Proc. 15th ICRC*, **11**, 132
- [4] Blandford, R. D., and J. P. Ostriker, 1978, *Astrophys. J.*, **221**, L29
- [5] J.R. Jokipii 1982 *Astrophys. J.* **255** 716
- [6] J.R. Jokipii 1987 *Astrophys. J.* **313** 842
- [7] Parker, E. N., 1965, *Planet. Space Sci.*, **13**, 9.
- [8] Lagage, P. O., and C. J. Cesarsky, 1983, *A & A*, **125**, 249.
- [9] Giacalone, J. and J.R. Jokipii 1999 *Astrophys. J.* **520** 204.
- [10] Ginzburg, V. L., and S. I. Syrovatsky, 1964, *Origin of Cosmic Rays*, Permagon Press, London and New York.
- [11] Giacalone, J., 2003, *Planet. & Space Sci.* **51** 659.
- [12] Giacalone, J., 2005, *Astrophys. J.* **624** 765.
- [13] Giacalone, J., 2005, *Astrophys. J. Lett.* in press.
- [14] Giacalone, J., D. Burgess, S. J. Schwartz, and D. C. Ellison, 1997 *J. Geophys. Res.* **102** 19,789.