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Implications of a new SU(2) flavour group in early-universe phase transitions

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Mounting evidence suggests that planned and present gravitational-wave detectors may be sensitive to signatures from first-order phase transitions in the early universe. Here, we consider the recently-proposed "flavour transfer" model, where the Standard Model flavour structure is augmented by a new horizontal SU(2) flavour gauge group. For such a model, the new gauge symmetry is broken far above the electroweak scale and constraints are dominated by "flavour-transfer" operators rather than flavour-changing currents. We calculate the finite-temperature corrections to the effective potential and determine the critical temperature at which we expect a phase transition. We examine the parameters for which the phase transition is strongly first order.

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Introduction

Ongoing experimental progress within gravitational-wave (GW) astronomy [1, 2] has stoked interest in the prospect of observing GW signals from the early universe, induced by a possible first-order phase transition. Such a phase transition would correspond to a dramatic shift in the state of the universe as it cools past a "critical temperature", T_c. In so doing, the universe decays from a metastable "false" vacuum to a "true" stable minimum which appears in the effective potential and is separated from the false vacuum by a barrier. The transition to this true vacuum occurs via thermal tunnelling. As the universe cools to the "nucleation temperature" $T = T_n < T_c$, the tunnelling rate becomes fast enough that there is at least one bubble formed (characterised by a critical radius per Hubble volume), which in turn grows to form our observable universe [3].

Since the Standard Model (SM) presents a cross-over phase transition [4], such a GW signal is expected to arise from physics beyond the SM (BSM), providing us with a new phenomenological avenue via which to explore physics at energies as high as 10^5 TeV, exceeding the reach of present and future particle colliders [5]. There are several scenarios by which to promote the cross-over phase transition to a first-order one, predicated on introducing additional bosonic degrees of freedom (dof) in the finite-temperature effective potential. These dof contribute to a cubic term in the effective mass, thereby delaying the phase transition and consequentially strengthening the barrier between the false and true vacuum [6]. Promising studies suggest that the peak of the resultant GW signal for a wide variety of BSM models lies within the sensitivity window of upcoming GW detectors (see Ref. [5] and references therein). However, to predict the frequency of this signal accurately, one must take into account thermal corrections to the effective potential as well as the thermodynamic properties associated with the bubble dynamics following the phase transition.

In this work, we shall focus on these corrections to the effective potential. It is well understood that perturbative expansions fail in the high-temperature limit due to the coupling of long-wavelength bosonic Mastubara modes [7]. This necessitates resummation, where two schemes have been popularised in the literature: the daisy resummation approach [8, 9] and dimensional reduction [10]. A comparison of the theoretical uncertainties of the two methods can be found in Ref. [11]. Here, we focus on applying these to a newly-constructed model in which the SM flavour structure is extended horizontally by a new non-abelian symmetry group, under which light generations of left-handed fermions transform as doublets [12]. As discussed in Ref. [13], the lightness of these generations can be attributed to gauge-invariant dimension-5 operators constructed using a real SM-singlet scalar Φ that transforms as a doublet under $SU(2)_f$. The phase transition we investigate arises from the spontaneous breaking of this new $SU(2)_f$ symmetry by Φ.

The new $SU(2)_f$ flavour-transfer model

When the scalar field Φ acquires a vacuum expectation value (vev) v_Φ, breaking the horizontal $SU(2)_f$ flavour symmetry, the "W-like" flavour gauge bosons become massive. This symmetry breaking then sets the scale at which flavour-violating transitions occur. Furthermore, these gauge bosons facilitate "flavour transfer", where flavour-violating transitions are linked across sectors of the extended SM. That is, a flavour-violating transition (ΔF_f) in one fermionic sector will be pairwise-related to a flavour-violating transition ($\Delta F'_f$) in another, such that four-fermion operators originating from these flavour gauge boson exchanges always satisfy a null sum rule $\Delta F_f + \Delta F'_f = 0$. This ensures that any flavour violation in one sector is compensated by a corresponding violation in another, thereby maintaining overall balance in the flavour structure. This in turn imposes anomaly cancellation restrictions, such that only the $SU(2)_f \otimes SU(2)_f \otimes U(1)_Y$ mixed anomaly is non-zero. Moreover, the large vev of Φ introduces suppression factors for these transitions, such that flavour-violating processes are highly constrained, and suppressed in the low-energy regime.

Vector-like fermions (VLFs) are introduced in the model to lift the mass rank of the lighter fermions from one to two, which helps generate the mass hierarchy between generations. They

Table 1: *Particle content of the model [13], where we highlight the two transverse and one longitudinal dof for the $SU(2)_f$ gauge boson. Φ breaks the $SU(2)_f$ symmetry to produce Yukawa matrices of rank 1. VLFs then lift the rank of the Yukawa matrices to 2, generating the hierarchy between generations. LQs contribute to radiative mass generation in the first generation, lifting the Yukawa matrices to rank 3.*

$Rank$	$Field$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$SU(2)_f$	dof
1	q_L^α	3	2	1/6	2	$3\times2\times2=12$
1	q_L^3	3	2	1/6	1	$3\times2\times2=12$
1	u_R^p	3	1	2/3	1	$3\times2\times3=18$
1	d_R^p	3	1	−1/3	1	$3\times2\times3=18$
2	$Q_{L,R}$	3	2	1/6	1	$3\times2\times2\times2=24$
1	ℓ_L^α	1	2	−1/2	2	$2\times2\times2=8$
1	ℓ_L^3	1	2	−1/2	1	$2\times2=4$
1	e_R^p	1	1	−1	1	$2\times2\times3=12$
2	$L_{L,R}$	1	2	−1/2	1	$2\times2\times2=8$
1	H	1	2	1/2	1	1
1	Φ	1	1	0	2	1
3	R_u	3	2	7/6	1	$3\times2\times2=12$
3	R_d	3	2	1/6	1	$3\times2\times2=12$
3	S	3	1	2/3	2	$3\times2\times2=12$
1	V	1	1	0	3	$3\times(2+1)=9$

interact with the scalar field Φ, and their large masses ensure that they can contribute to flavour mixing while preserving an anomaly-free gauge symmetry. VLFs participate in higher-dimensional operators and play a key role in generating suppressed masses for the first two generations through loop effects, complementing the contributions of the new flavour gauge bosons in the flavour transfer mechanism. Note that the third generation of fermions remains unaffected by the new $SU(2)_f$ symmetry, distinguishing it from the lighter generations. As discussed in Ref. [13], leptoquarks (LQs) can also be introduced in the model to generate masses for the lighter fermions. Specifically, these scalar LQs introduce additional contributions to the mass hierarchy, particularly for the first generation, by enabling radiative mass generation through loop interactions with dimension-5 operators. The LQs are involved in the flavour transfer mechanism by mediating interactions between quarks and leptons at loop-level. Here, R_u and R_d are doublets under $SU(2)_L$ and singlets under $SU(2)_f$, while S is a singlet under $SU(2)_L$ and a doublet under $SU(2)_f$. The new field content and corresponding dof are summarised in Table 1. Note that we work in the Landau gauge, such that Goldstone and longitudinal dof are counted separately [14].

In this work, we consider the symmetry-breaking scale to be $v_\Phi \sim 10^3$ TeV, with masses of the VLFs scaling as $M_{VLF} \gg v_\Phi \gg v_{EW}$, where v_Φ is the vev for Φ and v_{EW} is the electroweak vev. We choose to set the masses of the LQs as $M_{R_u}, M_{R_d} \gg M_S > v_\Phi$. As such, all LQs decouple from the thermal bath and do not play a significant role in the phase transition. The mass of the flavour gauge bosons M_f^2 are tied to the symmetry breaking scale, but depend on the strength of the $SU(2)_f$ gauge coupling g_f. A small g_f therefore leads to lighter gauge bosons with a lighter M_f.

Constructing the effective potential

Here, we choose to follow the "Parwani" [8] or "Truncated Full Dressing" [15] formalism in which leading-order thermal mass corrections to the zero-temperature mass are incorporated directly into the loop-level effective potential: that is, $m_i^2 \to m_i^2 + \sum_j \pi_i^j T^2$ for each species i from interactions with species j in the thermal bath. These corrections arise from the "daisy diagrams", the loop corrections to the propagator of particle i from its interactions with j that become significant at high temperature. While there are known theoretical uncertainties associated with this method (see Ref. [11] for details), it serves as a fairly economical means of determining whether the breaking of the $SU(2)_f$ gauge symmetry corresponds to a first-order phase transition. Since our present objective is to determine if the $SU(2)_f$ can produce a first-order phase transition, we reserve the more detailed calculation for a later work.

Let us begin with the tree-level potential after symmetry breaking,

$$V_{\text{tree}}(\phi) = -\frac{1}{2}\mu_\phi^2\phi^2 + \frac{1}{4}\lambda_\phi\phi^4 + \frac{1}{2}\mu_s^2|s|^2 + \frac{1}{4}\lambda_s|s|^4 + \frac{1}{2}\lambda_{\phi s}\phi^2|s|^2 \,, \tag{1}$$

where we assume ϕ weakly couples to the complex scalar LQ, $\lambda_{\phi s} \sim 0$. Following the Minimal Subtraction $\overline{MS}$ renormalisation scheme, the loop-level zero-temperature correction is given by the Coleman-Weinberg potential,

$$V_{1-\text{loop}}(\phi) = \sum_{i=\phi,\chi,f,s} \pm\frac{n_i}{64\pi^2}m_i^4\left[\log\left\{\frac{m_i^2}{\mu^2}\right\} - C_i\right] \,, \tag{2}$$

where μ is the renormalisation scale and $C_i = 5/6\ (3/2)$ for gauge bosons (scalars/fermions). Here, $+\ (-)$ is used for bosons (fermions), and $m_i = m_i(v_\Phi)$, which is the tree-level mass calculated at the vev $s = 0$, $\phi = v_\Phi$. The number of dof n for each species is summarised in Table 1, where the hierarchy of scales $M_{VLF} > M_{R_u}, M_{R_d} \gg M_S > v_\phi \gg v_{EW}$ suggests that the heavy species decouple from the thermal bath and therefore play a negligible role in the dynamics of the phase transition. As such, the dof and the masses of the contributing species are, respectively, $n_{\{\phi,\chi,s,f\}} = \{1, 3, 12, 9\}$ and $m^2_{\{\phi,\chi,s,f\}} = \{3\lambda_\phi\phi^2 - \mu_\phi^2, \lambda_\phi\phi^2 - \mu_\phi^2, \lambda_s\phi^2/2 + \mu_s^2, g_f^2\phi^2/4\}$, with the Goldstone dof being χ.

For bosons (B) and fermions (F), the loop-level finite temperature corrections are given by

$$V_T(\phi, T) = \sum_i \frac{n_i T^4}{2\pi^2} J_B\left(\frac{m_i^2}{T^2}\right) + \sum_i \frac{n_i T^4}{2\pi^2} J_F\left(\frac{m_i^2}{T^2}\right) \,, \tag{3}$$

$$J_{B/F}\,(a) = \pm\int_o^\infty dy y^2 \log\left[1 \mp e^{-\sqrt{y^2+a}}\right] \tag{4}$$

for $a = m_i^2/T^2$ [15, 16], where only bosonic dof contribute in this case study. For high temperature expansions ($a = m_i^2/T^2 \ll 1$), we can expand the thermal integral J_B as

$$J_B^{high}\,(a) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12}a - \frac{\pi}{6}a^{3/2} - \frac{a^2}{32}\left(\log(a) - c_B\right) \,, \tag{5}$$

for $c_B = 3/2 - 2\gamma_E + 2\log(4\pi)$ and $\gamma_E \approx 0.5772$. At low temperature ($a = m_i^2/T^2 \gg 1$),

$$J_{B,F}^{low}\,(a) \approx -\sqrt{\frac{\pi}{2}}a^{3/4}e^{-\sqrt{a}}\left(1 + \frac{15}{8}a^{-1/2} + \frac{105}{128}a^{-1}\right) \,, \tag{6}$$

We connect the high and low temperature contributions using [16]

$$J_B\,(a) \approx e^{-\left(\frac{a}{6.3}\right)^4} J_B^{high}\,(a) + \left(1 - e^{-\left(\frac{a}{6.3}\right)^4}\right) J_B^{low}\,(a) \,, \tag{7}$$

We substitute these into Eq. (3) for each species $i = \phi, \chi, f, s$.

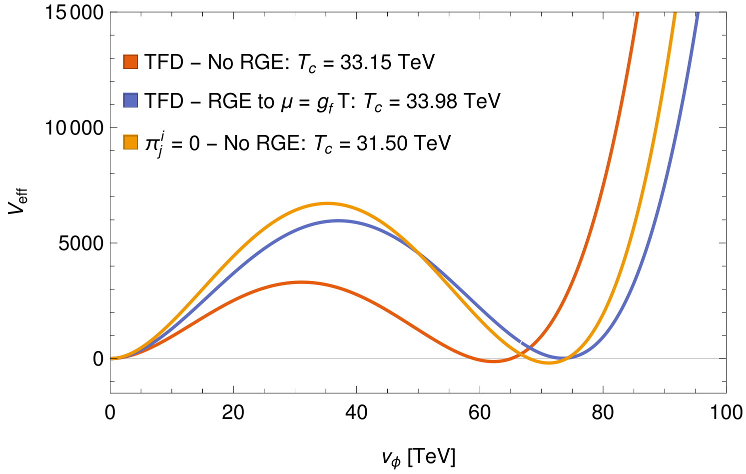

Figure 1: *Effective potential, with and without RG-running, and in the absence of thermal corrections to the mass (i.e. excluding Debye mass terms). For* $\mu = 50$ *TeV,* $\mu_\phi = \mu_s = 100$ *and* $\{\lambda_\phi, g_f^2, \lambda_s\} \sim \{0.0075, 0.7500, 0\}$*; at* $\mu = 30$ *TeV,* $\{\lambda_\phi, g_f^2, \lambda_s\} \sim \{0.0036, 0.7618, 0\}$.

Finally, we must contend with the higher-order thermal corrections from scalars and from the longitudinal polarisations of the gauge bosons, with species i in the centre of the "daisy diagram" and the relevant bosonic dof j in the outside rings. For the ϕ field, π_ϕ must include the self-interaction term π_ϕ^ϕ and the contribution from the gauge bosons $\pi_\phi^{\rm f}$; the same applies to the Goldstone [17, 18]. Similarly, the thermal corrections to the longitudinal polarisations of the $SU(2)_f$ gauge bosons, $\pi_{\rm f}^L$, include the self-interaction term $\pi_{\rm f}^{\rm f}$, as well as contributions from the ϕ field $\pi_{\rm f}^\phi$ and the SM fermionic dof [16, 18]. Altogether, these are, respectively,

$$\pi_\phi = \pi_\chi = \frac{\lambda_\phi}{2} + \frac{9}{48} g_f^2 \,, \quad \pi_{\rm f}^L = \frac{3}{2} g_f^2 \,. \tag{8}$$

As such, the finite-temperature contributions to the potential are produced by evaluating Eqs (2) and (3) at $m_i^2 \to m_i^2 + \sum_j \pi_i^j T^2$, in conjunction with the approximations of Eq. (7). For the flavour gauge bosons, terms representing transverse and longitudinal polarisations in the potential are evaluated separately such that thermal corrections are applied only to the latter. The full finite temperature effective potential $V_{eff}(\phi, T)$ is the sum of Eqs (1), (2), and (3), which we plot in Fig. 1. A strong first-order phase transition satisfies the criterion $\phi_c/T_c \geq 1$, where $V_{eff}(\phi, T)$ develops degenerate minima between the broken and unbroken phase at $T = T_c$, and ϕ_c is the vev of the broken phase.

Discussion

To observe a first-order phase transition, we find that the quartic of the potential must be small, likely due to the sensitivity of the parameters to renormalisation group (RG) running. In other words, the potential is very sensitive to the gauge-induced term, which is large and thus varies quite fast with the running. For this reason, we set $\mu_\phi = \mu_s = 100$ TeV, and for the couplings $\{\lambda_\phi, g_f^2, \lambda_s\} = \{0.0075, 0.75, 0\}$. We set the RG-running from $\mu = 50$ TeV to $\mu \sim g_f T_f$ (approximately 30 TeV). For these, we evaluate the criterion $\phi_c/T_c \geq 1$ using parameters with RG-running, without RG-running, and with neither RG-running nor thermal resummation. Respectively:

$$\text{no RG} : \frac{62.21}{33.15} \approx 1.88 \,, \quad \text{RG} : \frac{73.48}{33.98} \approx 2.16 \,, \quad \text{no RG } \pi_{\rm j}^{\rm i} = 0 : \frac{71.19}{31.50} \approx 2.26 \,. \tag{9}$$

If we take into account RG-running and thermal mass corrections, both ϕ_c and T_c are increased; it is when RG-running is not taken into account that we see the most pronounced effect on the vev. However, in all three scenarios, the criterion for a first-order phase transition is satisfied; we can expect a GW signal from this model that falls within the observational window. With this promising result in place, we shall follow up this work with a subsequent investigation into the thermal corrections using the demonstrably more accurate "dimensional reduction" scheme [11], as well as the computation of the subsequent thermodynamic properties required to predict the spectrum of the GW signal.

References

[1] KAGRA, VIRGO, LIGO Scientific collaboration, *GWTC-3: compact binary coalescences observed by LIGO and Virgo during the second part of the third observing run*, *Phys. Rev. X* **13** (2023) 041039 [2111.03606]

[2] NANOGrav collaboration, *The NANOGrav 15 yr data set: evidence for a gravitational-wave background*, *Astrophys. J. Lett.* **951** (2023) L8 [2306.16213]

[3] G.A. White, *A pedagogical introduction to electroweak baryogenesis*,

[4] K. Kajantie, M. Laine, K. Rummukainen and M.E. Shaposhnikov, *Is there a hot electroweak phase transition at $m_H \gtrsim m_W$?*, *Phys. Rev. Lett.* **77** (1996) 2887 [hep-ph/9605288]

[5] R. Caldwell et al., *Detection of early-universe gravitational-wave signatures and fundamental physics*, *Gen. Rel. Grav.* **54** (2022) 156 [2203.07972]

[6] F. Csikor, Z. Fodor and J. Heitger, *Endpoint of the hot electroweak phase transition*, *Phys. Rev. Lett.* **82** (1999) 21 [hep-ph/9809291]

[7] A.D. Linde, *Infrared Problem in Thermodynamics of the Yang-Mills Gas*, *Phys. Lett. B* **96** (1980) 289

[8] R.R. Parwani, *Resummation in a hot scalar field theory*, *Phys. Rev. D* **45** (1992) 4695 [hep-ph/9204216]

[9] P.B. Arnold and O. Espinosa, *The effective potential and first order phase transitions: beyond leading-order*, *Phys. Rev. D* **47** (1993) 3546 [hep-ph/9212235]

[10] K. Kajantie, M. Laine, K. Rummukainen and M.E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the Standard Model*, *Nucl. Phys. B* **458** (1996) 90 [hep-ph/9508379]

[11] D. Croon, O. Gould, P. Schicho, T.V.I. Tenkanen and G. White, *Theoretical uncertainties for cosmological first-order phase transitions*, *JHEP* **04** (2021) 055 [2009.10080]

[12] L. Darmé, A. Deandrea and F. Mahmoudi, *Gauge $SU(2)_f$ flavour transfers*, *JHEP* **05** (2024) 313 [2307.09595]

[13] A. Greljo and A.E. Thomsen, *Rising through the ranks: flavor hierarchies from a gauged SU(2) symmetry*, *Eur. Phys. J. C* **84** (2024) 213 [2309.11547]

[14] H.H. Patel and M.J. Ramsey-Musolf, *Baryon washout, electroweak phase transition, and perturbation theory*, *JHEP* **07** (2011) 029 [1101.4665]

[15] D. Curtin, P. Meade and H. Ramani, *Thermal resummation and phase transitions*, *Eur. Phys. J. C* **78** (2018) 787 [1612.00466]

[16] W.-C. Huang, F. Sannino and Z.-W. Wang, *Gravitational waves from pati-salam dynamics*, *Phys. Rev. D* **102** (2020) 095025 [2004.02332]

[17] M.E. Carrington, *The effective potential at finite temperature in the Standard Model*, *Phys. Rev. D* **45** (1992) 2933

[18] J. Ellis, M. Lewicki and J.M. No, *On the maximal strength of a first-order electroweak phase transition and its gravitational wave signal*, *JCAP* **04** (2019) 003 [1809.08242]