Gravitational Properties of Light

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As Einstein's equations tell us that all energy is a source of gravity, light must gravitate. However, because changes of the gravitational field propagate with the speed of light, the gravitational effect of light differs significantly from that of massive objects. In particular, the gravitational force induced by a laser pulse is due only to its creation and annihilation and decays with the inverse of the distance to the pulse. We can expect the gravitational field of light to be extremely weak. However, the properties of light are premises in the foundations of modern physics: they were used to derive special and general relativity and are the basis of the concept of time and causality in many alternative models. Studying the back-reaction of light on the gravitational field could give new fundamental insights to our understanding of space and time as well as classical and quantum gravity. In this article, recent work by the authors on the gravitational properties of light is reviewed. The gravitational field of a laser pulse of finite lifetime is investigated in the framework of linearized gravity.

The question of the gravitational properties of light in general relativity was raised already quite early, in 1931, in an article by Tolman, Ehrenfest and Podolsky [1]. In the article, the gravitational effect of an infinitesimally thin, cylindrical pulse of unpolarized light of finite lifetime was investigated. The central result of the article was that two light pulses that are propagating parallel do not affect each other gravitationally if they are co-propagating. In contrast, it was found that the two pulses attract each other if they are counter-propagating.

Subsequently, several other authors investigated similar situations using Einstein's equations. In an article by Bonnor [2], light was described as a null-fluid of massless particles and the corresponding full solution of Einstein's equations was constructed. In an article by Aichelburg and Sexl [3], the Lorentz-boosted Schwarzschild-metric of a point mass in the limit $v \to c$, $m \to 0$ was derived. Recently, Van Holten derived exact plane wave solutions of the coupled Maxwell-Einstein theory [4].

Here, we will shortly review the model of the laser pulse propagating in vacuum that we use and the properties of its gravitational field. A full derivation is given in our previous article about the gravitational properties of a laser pulse [6]. We start by assuming that the laser pulse is well described by the paraxial approximation. Furthermore, we assume that we can neglect the transversal spread of the pulse and that the length of the pulse is much larger than its width. Then, we model the pulse as an effectively 1-dimensional "needle of null stuff" propagating with the speed of light. The energy momentum of the pulse is given by "boxing" the energy momentum of a plane wave of a given polarization. With this model, we catch the essential ingredients of a laser pulse, which are (1) its localizability, (2) its masslessness, and (3) its polarization.

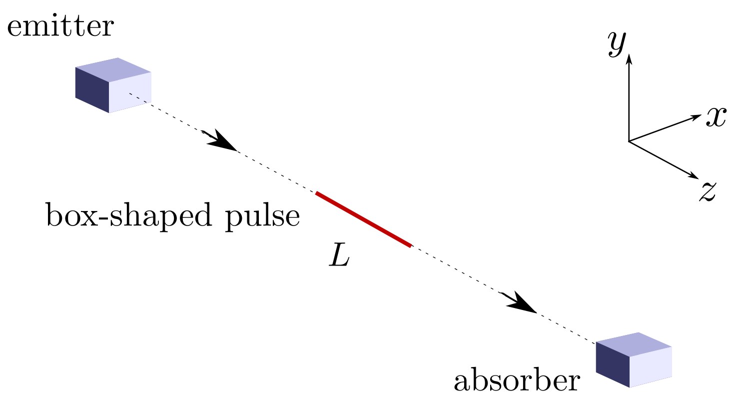

Figure 1 – The laser pulse is modeled with finite extension L in the direction of propagation, but negligible extension $\Delta(z)$ in the transverse x/y-directions, $\Delta(z) \ll L$. It travels from the emitter to the absorber over a distance D along the z-axis. The figure was originally published in Rätzel et al. NJP 18 023009 (2016) under the CC Attribution 3.0 license https://creativecommons.org/licenses/by/3.0/.

Additionally, we consider the emission and the absorption process (see Figure 1) by continuously letting the energy momentum appear and vanish. The continuity equation of general relativity would then require the inclusion of the energy momentum of the emitter and the absorber and their evolution during the pulse emission. However, we have shown in our previous article[6] that these effects are negligible for small distances to the pulse trajectory. To give a full description of the situation, we have derived the full gravitational field of the whole emission process for a specific situation - the emission of two light pulses from a single atom in another article[7].

Since the gravitational field of light can be expected to be small, we make use of the framework of linearized gravity. The metric is split as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is metric of the background, which we assume to be Minkowski space, and $h_{\mu\nu}$ is a small perturbation. We choose the set of Cartesian coordinates (ct, x, y, z) in which the experiment is at rest and the background metric takes the form $\eta = \mathrm{diag}(-1, 1, 1, 1)$. To be a small perturbation means then $|h_{\mu\nu}| \ll 1$ for all μ, ν. By imposing the gauge condition

$$\partial^\mu \left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h_\alpha{}^\alpha \right) = 0 \,, \tag{1}$$

where $h_\alpha{}^\alpha = h_{\alpha\beta}\eta^{\alpha\beta}$, the Einstein field equations assume the well known form of linearized gravity

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right] h_{\mu\nu} = \frac{16\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T_\alpha{}^\alpha \right) \,, \tag{2}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of the lase pulse, G is Newton's gravitational constant, c is the speed of light and $T_\alpha{}^\alpha = T_{\alpha\beta}\eta^{\alpha\beta}$. It is well known from classical electrodynamics that an equation like (2) can be solved by a retarded potential

$$h_{\mu\nu}(x, y, z, t) = \frac{4G}{c^4} \int \frac{\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T_\alpha{}^\alpha \right)(x', y', z', t_{\mathrm{ret}})}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}} dx' dy' dz' \,, \tag{3}$$

with t_{ret} the retarded time, $t_{\mathrm{ret}} = t - \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}/c$.

The energy momentum tensor of the laser pulse in our model is given as $T_{00} = T_{zz} = -T_{0z} = -T_{z0} = u$, with $u = \frac{\varepsilon_0}{2}\vec{E}^2 + \frac{1}{2\mu_0}\vec{B}^2$ the energy density of the electromagnetic field, where the index 0 corresponds to ct. We obtain the non-vanishing components of the metric

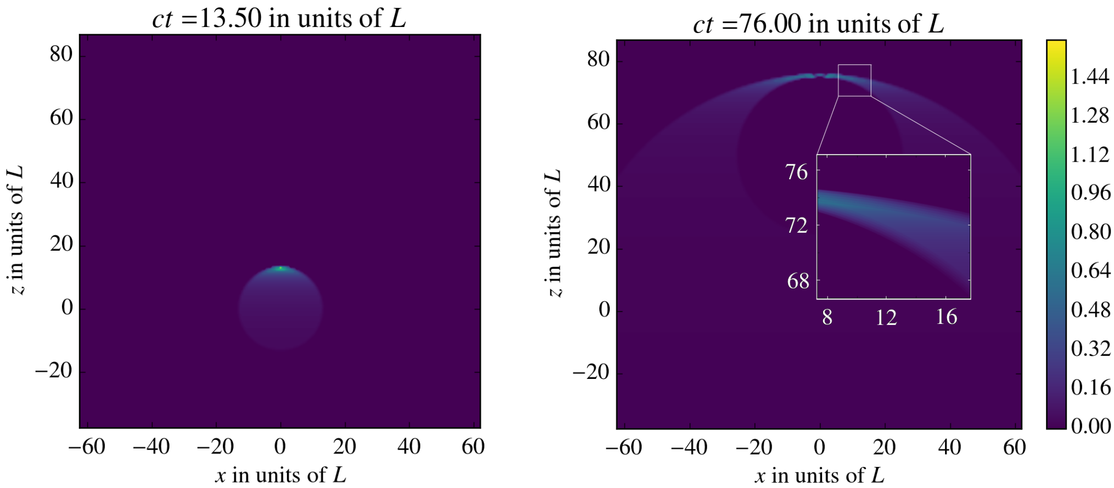

Figure 2 – The plots show the metric perturbation $h^{\mathrm{p}} = h_{00} = h_{11} = -h_{10} = -h_{01}$ for a pulse of length L in the coordinates (ct, x, y, z) in the (x, y)-plane for different times t. h^{p} is normalized to units of κ and then the logarithm of the logarithm is taken. The figures were originally published in in Rätzel et al. NJP 18 023009 (2016) under the CC Attribution 3.0 license https://creativecommons.org/licenses/by/3.0/.

as $h^{\mathrm{p}}_{00} = h^{\mathrm{p}}_{zz} = -h^{\mathrm{p}}_{0z} = -h^{\mathrm{p}}_{z0} = h^{\mathrm{p}}$, where h^{p} can be read off from (3). Plots for the function h^{p} in the x-z-plane for different times after the events of the emission of the pulse can be seen in Figure 2. It is interesting to note that the metric perturbation approaches the form of a plane fronted parallel propagating wave (pp-wave) for distances to the trajectory of the pulse much smaller than the distance from the observer to the emitter (see Figure 3). This situation coincides with the full solution of Einstein's equations constructed by Bonner [2].

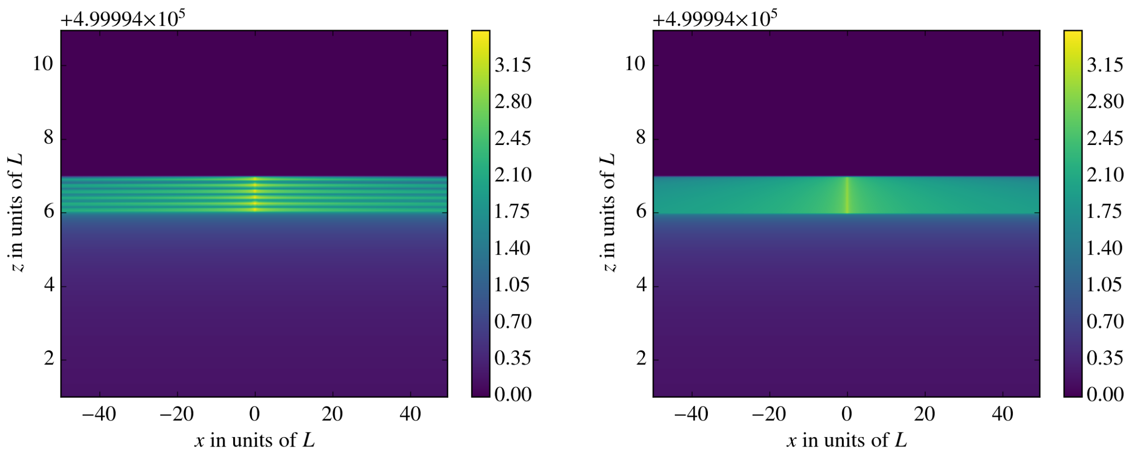

Figure 3 – These plots show the double logarithm of the metric perturbation h^{p} for a linearly polarized pulse of length L and central wavelength $\lambda = \frac{2\pi c}{\omega} = \frac{2}{3}L$ in the x-y-plane at $t = 50000L/c$, after its emission at $z = 0$. h^{p} is normalized to units of $\kappa = 4GAu_0/c^4$ and then the logarithm of the logarithm is taken. The figures were originally published in in Rätzel et al. NJP 18 023009 (2016) under the CC Attribution 3.0 license https://creativecommons.org/licenses/by/3.0/.

To understand the implications of the metric perturbation $h_{\mu\nu}$, it is best to study the Riemann curvature tensor $R^{\mu}{}_{\rho\sigma\alpha}$, which is a simple combination of second derivatives of the metric perturbation in first order in the metric perturbation. The Riemann tensor has a direct physical and geometrical interpretation as it appears naturally in the geodesic deviation equation for the relative acceleration between two infinitesimally close geodesics. This relative acceleration can be interpreted as the effect of tidal forces on neighboring test particles. One component of the curvature tensor, R^{p}_{0x0x}, is plotted for different times in Figure (4).

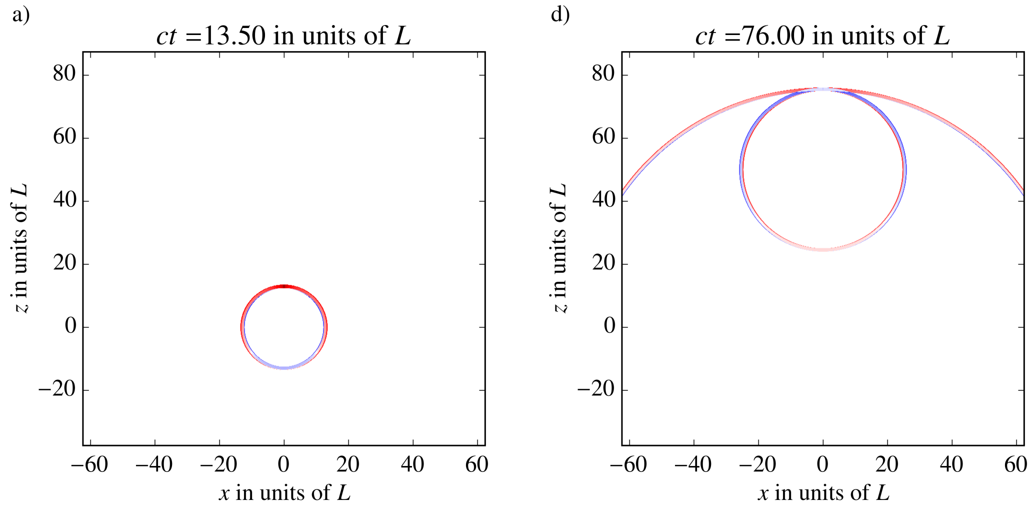

Figure 4 – The plots show the curvature component R^{p}_{0x0x} for the metric perturbation $h^{\mathrm{p}}_{\mu\nu}$ induced by a laser pulse in the coordinates (ct, x, y, z) in the (x, z)-plane for different times t. The logarithm of value of R^{p}_{0x0x} is encoded in the opacity of the color. Red is a negative value of R^{p}_{0x0x} and blue a positive value. White stands for zero. The figures were originally published in in Rätzel et al. NJP 18 023009 (2016) under the CC Attribution 3.0 license https://creativecommons.org/licenses/by/3.0/.

The most interesting feature is that the laser pulse induces curvature only on spherical shells of thickness L (pulse length) that expand from the point of creation and annihilation. If the curvature tensor vanishes in one set of coordinates, it vanishes in all sets of coordinates. Therefore, we can say that any gravitational effect due to the laser pulse must be confined to these spherical shells of non-zero curvature. It is even more interesting to note that the shells are causally disconnected from the pulse during its free propagation. Hence, we conclude that the physical effect of the laser pulse is due to its creation and annihilation alone as it was argued by Bonnor recently from a different perspective [8]. In contrast to our case, in the case of laser pulses running in wave guides [5], test particles would witness gravitational effects in the whole region causally connected with the timeline of the laser pulse propagating with $v < c$. Hence, the localization of the gravitational effect is due to the luminal motion of the laser pulse. A similar situation arises in electrodynamics with massless charges [9].

After we know about the localization of the physical effect, we can learn about the actual effect on test particles located in the spherical shells of non-zero curvature. To this end, the geodesic equation governing the trajectory of free test particles $\gamma^\mu(\lambda)$ is employed $\ddot{\gamma}^\mu = -\Gamma^\mu_{\rho\sigma}\dot{\gamma}^\rho\dot{\gamma}^\sigma$, where we have to assume $g_{\mu\nu}\dot{\gamma}^\mu\dot{\gamma}^\nu = -1$ for massive test particles and $g_{\mu\nu}\dot{\gamma}^\mu\dot{\gamma}^\nu = 0$ for massless test particles as in additional condition. We find that the emission induces an attraction towards the position of the pulse in the coordinates (ct, x, y, z). The absorption induces a repulsion away from the position the pulse would be at if it would have continued to propagate freely. In agreement with the authors of [1], we find from the geodesic deviation equation that massless test particles moving in the same direction as the pulse are not deflected while counter-propagating pulses are deflected four times more strongly than massive particles at rest containing the same energy.

Today, the strongest laser pulses available have a pulse power in the range of 10^{15}W. This means that there would be an acceleration of about $\ddot{\gamma}^x \approx -\frac{4GP}{c^3 x} \approx -10^{-18}\frac{\mathrm{m}}{\mathrm{s}^2}$ for a massive test particle at a distance of 2.5mm from the pulse trajectory. This acceleration can be compared with the acceleration experienced by a test particle in the Newtonian potential induced by a small spherical, massive object. We find that a mass of only $M = 10^{-13}$kg would be necessary to provide the same acceleration as the laser we considered above at a distance of $r = 2.5$mm. We find a slightly different situation, if we assume a periodically pulsed laser. The induced gravitational field is periodically varying and close to the beamline, the corresponding tidal forces can be compared to those induced by a gravitational wave. If we assume a laser power of 10^{15}W again, the induced tidal forces are in the same order as those due to a gravitational wave of angular frequency $\omega = 10^3$Hz and amplitude $h_+ = 10^{-22}$. The order is similar to that of the strain induced by gravitational waves that were detected by LIGO. Of course, this comparison only holds inside a very small region close to the beamline and there is still no chance to detect the gravitational field of light with state of the art technology. It would be necessary to develop completely new technologies for narrow band detectors of oscillating tidal forces on millimeter scales. Promising approaches can be found in [10] and [11].

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