COMPLEX SPINORS AND UNIFIED THEORIES

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We were told by Frank Yang in his welcoming speech that supergravity is a phenomenon of _theoretical_ physics. Why, at this time, is it not more than that? Self-coupled extended supergravity, especially for N = 8, seems very close to the overall unified theory for which all of us have yearned since the time of Einstein. There are no quanta of spin >2; there is just one graviton of spin 2; there are N gravitini of spin 3/2, just right for eating the N Goldstone fermions of spin 1/2 that are needed if N-fold supersymmetry is to be violated spontaneously; there are $N(N-1)/2$ spin 1 bosons, perfectly suited to be the gauge bosons for SO_N in the theory with self-coupling. There are $N(N-1)(N-2)/6$ spin 1/2 Majorana particles, and with the simplest assignments of charge and colour they include isotopic doublets of quarks and leptons. The theory is highly non-singular in perturbation theory, and the threatened divergence at the level of three loops has not even been demonstrated. The apparently arbitrary cancellation of huge contributions of opposite sign to the cosmological constant (from self-coupling on the one hand and from spontaneous violation of supersymmetry on the other) has been phrased in such an elegant way that it may be acceptable. (Of course, if we follow Hawking et al., we may not even need to cancel out the cosmological constant!)

What is wrong then? Of course the spontaneous violation of SO_N and of supersymmetry is not known to happen in the supergravity theory. But what seems much worse, the spectrum of elementary particles includes too few spin 1 and spin 1/2 objects to agree with the list that we would like to see on the basis of our experience at energies $\lesssim$ 50 GeV. Of course, looking up at the Planck mass of $\sim 2 \times 10^{19}$ GeV, we are in a position of greater inferiority than an ant staring up at a skyscraper (facing a factor of only 10^6 or so) and it may not be reasonable to expect that what looks elementary to us should be elementary on a grand scale.

Nevertheless, we make the comparison and we find that $SO_8 \supset SU_3^c \times U_1 \times U_1$ but $SO_8 \not\supset SU_3^c \times SU_2 \times U_1$, so that there is no room for the $X^{\pm}$ intermediate bosons of the charged current weak interaction. Among the spin 1/2 particles, we have room for at most two flavours of lepton (say e and ν_e) and four flavours of quark (say u, d, c and s). Even these numbers may be reduced if we try to locate the Goldstone fermions among the elementary spin 1/2 particles or if we use the generator of the second U_1 in a restrictive way.

We would then be forced to regard all or most of the known quarks and leptons as non-elementary, as well as at least two of the intermediate bosons of the weak interaction. The broken Yang-Mills theory of the weak interaction would be only an effective gauge theory, not a fundamental one. All this may prove to be the case, and we will then have to understand the rather complicated relation existing between the elementary particles of the theory and the elementary particles as we perceive them today.

Various investigators have looked into superconformal supergravity, in which one tries to use the full SU_N as a gauge group; such a theory is plagued with particles appearing as multiple poles in propagators, involving difficulties with negative probabilities or lack of causality. Ignoring these serious difficulties, we may ask about the algebraic description of the spin 1/2 fermions in such a theory Apparently they are again connected with third-rank tensor representations, forming part of $(N + \bar{N})_A^3$ of SU_N (where A means totally antisymmetrized) instead of being assigned to $(\tilde{N})_A^3$ of SO_N.

This tendency to assign the spin 1/2 fermions to a tensor representation, probably a third rank tensor, of SO_N or SU_N, exists even in theories having nothing to do with supergravity. We may, for example, consider a composite model of quarks and leptons, in which they are made up of N kinds of fermionic sub-units. We may think of such a scheme in algebraic terms as assigning these sub-units to the representation N of SO_N or SU_N and the quarks and leptons to tensors that are part of $(N)^3$ of SO_N or part of $(N + \bar{N})^3$ of SU_N, provided each known particle is made up of three sub-units. (Of course one might use five or a higher odd number and obtain fifth rank tensors and so forth, but three is much simpler.)

Now what indications come from the attempts to construct a unified Yang-Mills theory? Do they also point to such a third-rank tensor for the spin 1/2 fermions?

We turn, then, to the program of formulating a broken Yang-Mills theory of strong and weak interactions, with an effective energy of unification between 10^{14} GeV and the Planck mass. This program is only slightly less immodest in conception than the overall unification program of self-coupled supergravity. For the sake of expressing all the Yang-Mills coupling constants in terms of a single one, a simple group G is employed. (Actually one could use $G \times G$, $G \times G \times G$, etc. with discrete symmetries connecting the factors, but we shall treat here only the case of a single G.)

The smallest G that has been used is SU_5; the known left-handed spin 1/2 fermions are then assigned to three families, each belonging to the reducible representation $\bar{5} + 10$ of SU_5, where $\bar{5}$ contains $\bar{d}$, e^-, and ν_e for the lowest family, while 10 consists of d, u, $\bar{u}$ and e^+. The combination $\bar{5} + 10$ is anomaly-free. The violation of symmetry takes place in two stages. First the symmetry SU_5 is broken down to $SU_3^c \times SU_2 \times U_1$ by means of a non-zero vacuum expected value of an operator transforming like the adjoint representation 24, with no direct effect on the fermion masses, and then $SU_2 \times U_1$ is broken down to U_1^{e-m} by means of operators transforming like $5 + \bar{5}$, with perhaps an admixture of 45, giving the masses of the quarks and leptons. The detailed work is done using explicit spinless Higgs boson fields, with various constants for mass, for self-coupling and for coupling to fermions, constants that must be delicately adjusted to make the masses of the fermions and of the intermediate boson for weak interactions tiny with respect to the unification mass. A quantity of roughly similar magnitude, the renormalization-group-invariant mass Λ of QCD, is tiny with respect to the unification mass for a totally different reason, namely the smallness of the unified coupling constant, which is $\sim 10^{-2}$ near the unification mass, corresponding to the fine structure constant at low energies, and is proportional to the reciprocal of the logarithm of $(10^{14}$ GeV$)/\Lambda$. Despite some successes, which we mention below, the SU_5 scheme seems to us a temporary expedient rather than a final theory, because of the arbitrariness associated with the Higgs bosons and also because the particles and antiparticles among the left-handed spin 1/2 fermions have no relation to each other (i.e., there is no C or P operator for the theory).

The SU_5 scheme has at least two successes: a roughly correct prediction of the weak angle θ_W and the prediction that after allowing for renormalization $m_b \approx m_\tau$, which works quite well. The violation of $SU_2 \times U_1$ by $5 + \bar{5}$ of SU_5 would also give $m_s \approx m_\mu$ and $m_d \approx m_e$ after renormalization. The second of these relations does not work but might be subject to large corrections because the quantities are so small; the first might work if the usual estimates of m_s are in error – otherwise some admixture of a 45 of SU_5 has been suggested, along with $5 + \bar{5}$, but affecting mainly the two lower families.

We have studied various complex spinor schemes that reduce to the SU_5 system after some symmetry violation. Work on such schemes has also been done by Georgi et al. at Harvard, Susskind and collaborators at Stanford, Wilczek and Zee, Gürsey et al. in the case of E_6, and no doubt by many others. Early investigations of complex spinor assignments were carried out by Fritzsch and Minkowski.

First, let us restrict our attention to a single family, say the third one, assuming that the t quark exists and that $\nu_{\tau L}$ is nearly massless. We note that the reason that $\bar{5} + 10$ of SU_5 is anomaly-free is that the complex spinor representation 16 of SO_{10} breaks up into $1 + \bar{5} + 10$ of SU_5, where the singlet can give rise to no SU_5 anomaly, and all representations of $SO_n (n \neq 6)$ are anomaly-free.

The 16-dimensional spinor possesses a C symmetry to start with, connecting $\tau_L^- \rightleftarrows \tau_L^+$, $b_L \rightleftarrows (\bar{b})_L$, etc., and the 16th particle is just the missing $(\nu_\tau)_L$. Symmetry violations giving fermion masses must correspond to representations contained in the symmetrized square of the fermion representation. We note that in SO_{10} we have $(16)_s^2 = 10 + 126$ and that with respect to SU_5 we have $10 \rightarrow 5 + \bar{5}$ and $126 \rightarrow 1 + 45 + 10 + \overline{15} + \bar{5} + 50$. An operator transforming like the SU_5 singlet piece of 126 would break the SO_{10} symmetry down to SU_5 and would give a Majorana mass term of the form $(\nu_\tau)_L^2 + \nu_{\tau R}^2$ to the unobserved neutrino, one that had better be very large if the scheme is to work.

Meanwhile, the 10 of SO_{10} would give rise to equal Dirac masses for b and τ (apart from renormalization) and also to equal Dirac masses for t and ν_τ. The Dirac mass for the neutrino leads directly to a small effective mass $m(\nu_{\tau L})$ $\sim m_{Dirac}^2/m_{Majorana}$. If $m(\nu_{\tau L}) \approx 1$ eV, then neutrinos account for a modest fraction of the missing matter in the universe and give a moderate contribution to the

gravitational closure of galaxies and clusters of galaxies. Putting $m_{Dirac} \approx m_t$ ≈ 30 GeV at a guess, the corresponding value of $m_{Majorana}$ would be $\approx 10^{12}$ Gev. If $m_{Majorana}$ is very much smaller than that, the cosmological effects become too large; if $m_{Majorana}$ is much larger, that is harmless, but the cosmological effects become negligible.

We can examine SO_{10} in a different way by using the decomposition $SO_{10} \supset SO_6 \times SO_4$ where algebraically SO_6 is equivalent to SU_4 and SO_4 to $SU_2 \times SU_2$. We have, then, effectively $SO_{10} \supset SU_2 \times SU_2 \times SU_4$, where the first SU_2 is that of the weak interactions, the second one the corresponding SU_2 for left-handed antiparticles or right-handed particles and SU_4 is the generalization of SU_3 introduced in a different connection by Pati and Salam, in which leptons appear as having a fourth colour. The representations of SO_{10} then decompose as follows:
$$\underset{\sim}{16} \to (2,1,4) + (1,2,\bar{4}), \quad \underset{\sim}{\overline{16}} \to (1,2,4) + (2,1,\bar{4}), \quad \underset{\sim}{10} \to (2,2,1) + (1,1,6),$$
$$\underset{\sim}{126} \to (2,2,15) + (1,1,6) + (3,1,10) + (1,3,\overline{10}).$$
The representations $\underset{\sim}{1}$, $\underset{\sim}{4}$, $\underset{\sim}{\bar{4}}$, $\underset{\sim}{15}$, $\underset{\sim}{10}$ and $\underset{\sim}{\overline{10}}$ of SU_4 each contain one colour singlet. We see that the Dirac mass term coming from $\underset{\sim}{10}$ is just of the form $(2,2,1)$, while the Majorana mass term for $(\bar{\nu}_e)_L$ or ν_R coming from $\underset{\sim}{126}$ is a component of $(1,3,\overline{10})$. We must not use $(3,1,10)$, which would introduce an unwanted triplet violation of SU_2^{weak} and would give a mass directly to the left-handed neutrino. A possible danger is that radiative corrections might give rise to a large or uncontrollable term of that kind anyway, in addition to the term $m_{Dirac}^2/m_{Majorana}$, since the left-handed neutrino Majorana mass is not prohibited by a selection rule.

Such a selection rule exists in the SU_5 scheme, where an ungauged quantity that distinguishes $\underset{\sim}{5}$ from $\underset{\sim}{10}$ and a gauged generator of SU_5 are simultaneously violated, preserving a linear combination, which is the baryon number minus the lepton number. The conservation of this quantity prohibits neutrino mass altogether. Here an unwanted massless spinless Goldstone boson is fed to an unwanted massless spin 1 gauge boson to give a massive spin 1 boson. This trick, which we have studied in connection with conserving baryon number (perhaps an obsolete idea now) can be applied whenever there is a reducible representation of G for the fermions (or even for spinless elementary particles if there are some).

A further generalization of SU_5 for one family might make use of the lowest complex spinor representation $\underset{\sim}{27}$ of E_6, which breaks down to $\underset{\sim}{16} + \underset{\sim}{10} + \underset{\sim}{1}$ of SO_{10}. Here one would have to marry the new SO_{10}-singlet neutrino to the unwanted $(\bar{\nu})_L$ of $\underset{\sim}{16}$, allowing them to share a huge Dirac mass, and one would have to do it in such a way as to leave the ν_L of $\underset{\sim}{16}$ with a small mass or none at all. At the same time one would have to assign high masses to the members of the $\underset{\sim}{10}$ of SO_{10} in the $\underset{\sim}{27}$ of E_6, in order to get them out of the way, leaving just the fifteen fermions of the SU_5 scheme.

What we have seen from the example of one family is that a complex spinor representation, while it involves us in delicate questions of neutrino mass, does permit the description of left-handed fermions by a single irreducible representation of G and in such a way that the asymmetry between the SU_2^{weak} assignments of particle and antiparticle is rather natural, while the whole system possesses an initial symmetry C between left-handed particles and left-handed antiparticles, a symmetry that interchanges SU_2^{weak} and another SU_2.

It is also clear that in such a scheme the dimensions of the representations that violate the symmetry, for example in the generation of fermion masses, tend to be large and that the arbitrary character of the violation scheme employing elementary Higgs bosons is strongly emphasized. It seems to us that one must hope for a situation in which, somehow, spontaneous symmetry violation is achieved dynamically.

Although we do not, of course, exclude the existence of some spinless elementary fields, provided they are not the arbitrary ones of the elementary Higgs boson method, we may look as an example at a theory with just gauge bosons and elementary spin 1/2 fields and imagine what hypothetical dynamical spontaneous symmetry breaking would be like.

We would like to point out first that if the (say) left-handed fermion representation in such a theory is reducible, then ungauged quantum numbers arise that commute with the gauge group. When these are violated spontaneously, that necessarily leads to unwanted massless Goldstone bosons unless the trick described above is used and global conservation laws result. If all the irreducible representations are inequivalent, then such globally conserved quantities are Abelian and tolerable, but if there are equivalences among representations, as in the case of several families transforming alike, then an ungauged non-Abelian family group arises and that would have to be matched with an isomorphic subgroup of G with resulting global conservation of a third isomorphic non-Abelian group relating the families. That would not agree at all with observation, and we conclude therefore that having united each family in an irreducible representation of SO_{10} or E_6 we had better consider all the fermions as belonging to a single irreducible representation of the gauge group G.

This can be done in two different ways. Either we go to a higher-dimensional representation of the same group that we used for one family or else we enlarge the group and assign the fermions to a relatively low-lying representation of the bigger group. In the case of complex spinor representations, we could try, as an example of the first approach, the 1728 of E_6, contained in 27×78. As examples of the second approach, we can take the lowest-dimensional complex spinors of larger groups, and the only larger groups possessing such spinors are SO_{14} (lowest dimensional spinor 64), SO_{18} (lowest-dimensional spinor 256), SO_{22} (lowest-dimensional spinor 1024), etc. We have studied both possibilities, but we shall describe here the case of the lowest spinors of SO_{4n+2}.

A great deal of thought has been devoted to the question of what dynamical spontaneous symmetry breaking would be like for a theory containing elementary fields for gauge bosons and fermions only. Weinberg, Dimopoulos and Susskind, and various other theorists have drawn some important conclusions, including the following, which we specialize to the case of an irreducible fermion representation.

Symmetry reduction occurs through "condensations", that is non-zero vacuum expected values of operators that break symmetries. If the symmetry group of the kinetic energy is H and if $G_1 \subset G$ and $H_1 \subset H$ are the subgroups left invariant by these condensations, then the generators of G_1 correspond to exact conservation laws and massless gauge bosons, those of G/G_1 to massive gauge bosons, those of H_1/G_1 to modified Goldstone bosons that acquire mass a a result of the gauge coupling and those of $(H/G)/(H_1/G_1)$ to approximate conservation laws, broken by the gauge coupling . The flavour-non-singlet pseudoscalar mesons would be modified Goldstone bosons, and the PCAC condensation $<\bar{q}q>_{vac} \sim \Lambda^3$ presumably occurring in QCD would contribute only $\sim e\Lambda$ to the masses of the weak intermediate bosons. If one or more additional factors of the exactly conserved strong colour group exist (we prefer to call them primed colour, etc.), then these could have higher renormalization-group-invariant masses Λ', etc., and a primed colour group with $\Lambda' \sim 10^3$ GeV could give a condensation of fermions possessing primed colour that would account for the weak intermediate boson masses. Some of the corresponding pseudoscalar primed mesons would serve as effective Higgs bosons to be eaten by these intermediate gauge bosons. The mixing between these primed mesons and ordinary pseudoscalar mesons would be rather small. There would be no real ultra-violet fermion masses, but only medium-frequency or infra-red masses of order Λ for quarks, Λ' for fermions possessing primed colour, and so forth, and then masses obtained by sharing these medium-frequency masses through radiative corrections - these last would simulate ultra-violet masses up to fairly high energies.

A great deal of the algebraic behaviour of such symmetry-breaking schemes should be simulated by generalized non-linear σ-models. If those are embedded in linearized σ-models, then one has some connection with the algebraic properties of explicit Higgs boson theories.

An important question is whether the many condensations required for symmetry breaking in a unified theory can be explained by the strong long-range interactions that appear in the same theory. This is a problem, for example, in connection with any condensation leading to Majorana masses for the unwanted neutrinos.

Now let us return to the notion that G might be SO_{4n+2} with the left-handed fermions placed in the 2^{2n}-dimensional complex spinor representation. Let us consider the example of SO_{18}, which evidently contains $SO_8 \times SO_{10}$. We can decompose the 256 of SO_{18} as $(8_{sp},\underline{16}) + (8'_{sp},\underline{16})$ of $SO_8 \times SO_{10}$, where 8_{sp} and $8'_{sp}$ are the two real inequivalent spinors of SO_8. We can now write $SO_8 \supset Sp_4 \times SU_2$, where the vectorial octet 8_v of SO_8 can be made to give $(4,2)$ of $Sp_4 \times SU_2$ and $8'_{sp}$ of SO_8 likewise, while 8_{sp} of SO_8 gives $(1,3) + (5,1)$ of $Sp_4 \times SU_2$. The 256 of SO_{18} then becomes $(1,3,\underline{16}) + (5,1,\underline{16}) + (4,2,\overline{\underline{16}})$ of $Sp_4 \times SU_2 \times SO_{10}$. If we interpret Sp_4 as a supplementary factor of the exactly conserved colour group, which becomes $SU_3^C \times Sp_4^{C'}$, and SU_2 as a gauged family subgroup of SO_{18}, we see that the only fundamental left-handed fermions without primed colour are three families of 16-dimensional spinors of SO_{10}, and we glimpse a possible agreement with experiment.

We note that $Sp_4^{C'}$, if there were no fermions would have the same renormalization-group behaviour as SU_3^C in lowest order, and we would need a special explanation for its reaching the strong-coupling regime at a much higher mass than SU_3^C. The differing fermion corrections might make a difference; so might the possibility that as we come down in mass from the unification region SO_8 remains undivided over a considerable interval before splitting into $Sp_4 \times SU_2$.

In the same way, $SO_{14} \supset SO_4 \times SO_{10}$ and SO_4 is actually algebraically equivalent to $Sp_2 \times SU_2$; the 64 of SO_{14} decomposes into $(1,2,\underline{16}) + (2,1,\overline{\underline{16}})$ of $Sp_2 \times SU_2 \times SO_{10}$ and we would have two families lacking primed colour. Similarly, $SO_{22} \supset SO_{12} \times SO_{10}$ and $SO_{12} \supset Sp_6 \times SU_2$; the 1024 of SO_{22} decomposes into $(1,4,\underline{16}) + (14,2,\underline{16}) + (6,3,\overline{\underline{16}}) + (14',1,\underline{16})$ and we would have four families lacking primed colour.

As far as representations giving fermion mass are concerned, we have the following situation:

SO_{10}: $(\underline{16})_S^2 = (\underline{10})_A^5 (\text{self-dual}) + (\underline{10})^1 = 126 + 10$;

SO_{14}: $(\underline{64})_S^2 = (\underline{14})_A^7 (\text{self-dual}) + (\underline{14})^3 = 1716 + 364$;

SO_{18}: $(\underline{256})_S^2 = (\underline{18})_A^9 (\text{self-dual}) + (\underline{18})_A^5 + (\underline{18})^1 = 24310 + 8568 + 18$;

SO_{22}: $(\underline{1024})_S^2 = (\underline{22})_A^{11} (\text{self-dual}) + (\underline{22})_A^7 + (\underline{22})_A^3 = 352,716 + 170,544 + 1540$;

and so forth. It looks in each case as if the Majorana mass term comes from the highest-dimensional representation and the Dirac masses of the familiar fermions from the next-highest-dimensional one, if such a scheme is to work. The Dirac masses then obey an important constraint, which equates a function of the charged lepton mass matrix with the same function of the $Q = -1/3$ quark mass matrix. Since in each case the matrix is dominated, according to experience, by the highest mass, these two masses must be roughly equal, and for three families that explains the relation $m_b \sim m_\tau$ after renormalization.

The question is, of course, left open as to <u>why</u> the mass matrix for three families is so close to

$$\begin{pmatrix} C & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

for each kind of particle. With the families described as a triplet of SU_2 rather than a triplet of SU_3, that is rather mysterious, since it corresponds to a miraculous compensation of a scalar and a quadrupole term under SU_2. Under SU_3, of course, they would combine to form a 6 of SU_3 and the approximate matrix above would correspond to the intervention of the component of 6 invariant under the maximal little group SU_2. Unfortunately we are not dealing here with a family SU_3.

The Dirac masses of neutrinos and of $Q = +2/3$ quarks would obey the same relation as that for the charged leptons and $Q = -1/3$ quarks. The Majorana masses of the neutrinos are also subject to a constraint if they come from the

highest-dimensional representation for the fermion mass. Of course, the mass matrix for neutrinos is not easy to detect and at best requires delicate experiments that we shall describe elsewhere.

In summary, the idea of assigning left-handed spin 1/2 fermions to a complex spinor representation of a gauge group SO_{4n+2} (or conceivably E_6) has a number of attractive features, although some difficulties as well. As alternatives for an irreducible representation, we have, of course, the possibility of a real or pseudoreal representation, giving a vector-like theory in which all the known fermions must be accompanied by heavy partners that have weak interactions of opposite handedness; or a complex representation of a unitary group, which when irreducible generally leads to anomalies and thus to divergences, and is also rather hard to reconcile with observation.

If we suppose that the familiar quarks and leptons are really to be assigned to a complex spinor representation of a group SO_{4n+2} or E_6, can we reconcile that idea with the notion that there is some truth in extended supergravity where the spin 1/2 fermions are placed in a third rank antisymmetric tensor representation of SO_N?

We have looked, in collaboration with Jon Rosner, for an analogue of supersymmetry that might lead to a theory with assignments like $\underset{\sim}{1}$ of E_6 for $J_z = 2$, $\underset{\sim}{27}$ of E_6 for $J_z = 3/2$, adjoint $\underset{\sim}{78}$ of E_6 for $J_z = 1$, and for $J_z = 1/2$ some representation contained in 27×78, like $\underset{\sim\sim\sim}{1728}$ of E_6. We have searched for the same kind of scheme using $S\widetilde{O}_{10}$, and we have even tried non-associative systems in an effort to find something that would work. So far we have had no success.

It seems likely anyway that if supergravity or some similar future theory is correct, then there must be only an indirect relation between the elementary fields of the theory and the particles that appear to us today to be elementary. If the known fermions behave, for a given handedness, like a complex spinor representation of SO_{4n+2} or E_6, then the relation is not even that of a composite model. All or most of the familiar particles would have to correspond to particle-like solutions of the fundamental equations, with a different algebraic behavior from that of the fundamental fields.

In this talk we have only sketched the subject of complex spinor representations and related topics. Elsewhere we present a proper account of our own work and adequate references to the work of others.

We have also taken a rather schizophrenic approach, shuttling back and forth between extended supergravity on the one hand and a particular kind of unified Yang-Mills theory on the other. The ideas underlying the two approaches have to be compared more carefully.

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In the second expression of eqs.(14) the third term on the r.h.s. should read:

$$+ \tfrac{1}{2}\kappa\bar{\psi}_{[\,\mu}{}^{[\,i}\phi_{\nu]}{}^{j]}$$

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The third line of reference [2] should read:

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In lines 6 and 7 of the 4th paragraph, the expressions 'H_1/G_1' (line 6) and '$(H/G)/(H_1/G_1)$' (line 7) should be interchanged.