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Dark energy and fundamental physics

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Abstract. The acceleration of the expansion of the Universe which has been identified in recent years has deep connections with some of the most central issues in fundamental physics. At present, the most plausible explanation is some form of vacuum energy. The puzzle of vacuum energy is a central question which lies at the interface between quantum theory and general relativity. Solving it will presumably require to construct a quantum theory of gravity and a correspondingly consistent picture of spacetime. To account for the acceleration of the expansion, one may also think of a more dynamical form of energy, what is known as dark energy, or modifications of gravity. In what follows, we review the the basic models for dark energy and the difficulties encountered in each approach, as well as we discuss the vacuum energy problem.

1. Introduction: the dark energy issue

A large set of observational data has led to the conclusion that the expansion of the Universe has recently (compared to the age of the Universe) been accelerating. This may be due to "astrophysical reasons". Two popular proposals are: (i) the back reaction of inhomogeneities would alter the evolution of cosmological parameters, traditionally used to describe a *homogeneous* universe [1]; (ii) a large local inhomogeneity (void) would make us underestimate the average energy density of the Universe that governs its rate of expansion.

Alternatively, this may be due to novel aspects of fundamental physics. I will concentrate in this review on the latter possibility. A starting point for the discussion may be Einstein's equations[2]:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_{_N} T_{\mu\nu} \ . \tag{1}$$

One of Einstein's achievements was to foresee that such an equation could lead to a cosmological scenario, that is, could describe the evolution of the Universe at large. It is true that the first solution proposed was static, through the introduction of the cosmological term (see below), but this rapidly led to time-dependent solutions that paved the way to the discovery of the expansion of the Universe. In parallel, a series of tests indicates that the principles governing General Relativity, as described by Einstein's equations, are tested to an incredible precision: for example, the equivalence principle is presently tested to the 10^{-13} level. This seems to leave us us with very little room to modify the theory of gravity in order to account for the acceleration

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[2] The metric signature we adopt throughout is Einstein's choice: $(+,-,-,-)$.

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of the expansion of the Universe. This is not so because the precise tests of General Relativity are all local, in the sense that they are all performed in the solar system. On the other hand, the rate of expansion concerns the Universe in its largest dimensions. One may thus consider modifications of gravity at large distances that would lead to a different rate of expansion for the Universe.

To be a little more precise, let us assume a homogeneous and isotropic universe and write the metric in its Robertson-Walker form:

$$ds^2 = c^2dt^2 - a^2(t)\ \gamma_{ij}dx^idx^j, \tag{2}$$

$$\gamma_{ij}dx^idx^j = \frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{3}$$

where $a(t)$ is the cosmic scale factor and the constant k takes the values ±1 or 0 depending on whether space is flat (0), closed (+1) or open (−1). Assimilating the content of the universe to a perfect fluid of pressure p and energy density ρ, we write the energy-momentum tensor $T_{\mu\nu}$ present in Einstein's equations (1):

$$T_{\mu\nu} = -pg_{\mu\nu} + (p+\rho)U_\mu U_\nu \quad , \tag{4}$$

where U^μ is the velocity 4-vector ($U^T = 1, U^i = 0$). Then, one can extract from Einstein's equations the Friedmann equation, which gives an expression for the Hubble parameter $H \equiv \dot{a}/a$ measuring the rate of the expansion of the Universe:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}\ . \tag{5}$$

Differentiating the Friedmann equation with respect to time, and using the energy-momentum conservation

$$\dot{\rho} = -3H(p+\rho)\ , \tag{6}$$

one easily obtains

$$\ddot{a} = -\frac{4\pi G_N}{3}a(3p+\rho)\ . \tag{7}$$

which shows that non relativistic matter ($p \sim 0$, $\rho > 0$) can only account for a deceleration of the expansion ($\ddot{a} < 0$).

Modifications of the theory of gravity may induce new terms or a new form for the Friedmann equation, which would lead to a rate of expansion larger than the one foreseen in the context of the standard Big-Bang model. For example, in the case where our Universe is a 4-dimensional brane immersed in a higher-dimensional universe, an extra term appears on the right-hand side of the Friedmann equation (5) which scales like ρ^2 [2]. However, this term is important for high densities, hence in the primordial Universe, and cannot account for a recent acceleration of the expansion.

A popular model for modifying gravity at large distances has been the induced gravity model of Dvali, Gabadadze and Porrati [3]: four-dimensional gravity is induced on the brane (with corresponding Newton's constant G_N), besides the five-dimensional gravity (constant G_5) of the 5-dimensional spacetime in which the brane is immersed. The Friedmann equation is then modified to [4, 5]:

$$H^2 = \left(\sqrt{\frac{8\pi G_N}{3}\rho + \frac{1}{4r_c^2}} + \frac{1}{2r_c}\right)^2 - \frac{k}{a^2}\ , \tag{8}$$

where the critical distance r_c is the ratio $G_5/2G_N$. The extra terms may this time induce an acceleration at late times.

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It is however quite difficult to modify gravity. Most models suffer from the appearance of ghosts or tachyons, or from the presence of the van Dam-Veltman-Zakharov discontinuity [6, 7], a pathology of theories of massive gravity concerning the light deflection by static sources. This is probably just an illustration of the robustness of the theory of General Relativity as a description of gravity.

We will thus concentrate in this review on a third possibility to account for the acceleration of the expansion of the Universe: we assume that a new unknown component participates to the energy density of the Universe. It is obviously a dark component since it is not observed through its luminous emission. And it is certainly not a new form of matter, since, as we have seen in (7), matter leads to a deceleration of the acceleration. It has thus been named *dark energy*. We will focus, in what follows, on dark energy, discuss its properties, and some of its explicit realizations.

Dark energy does not appear to be clustered. We may thus treat this diffuse background as a perfect fluid with energy density ρ_X, pressure p_X and equation of state:

$$p_X = w_X \rho_X \ . \tag{9}$$

Obviously, acceleration of the expansion implies that this dark energy component dominates the energy density of the Universe and that, according to (7),

$$3p_X + \rho_X < 0 \text{ or } w_X < -1/3 \ . \tag{10}$$

In order to be a little more precise, let us assume that the dominant contributions to the late evolution of the Universe are non relativistic (luminous and dark) matter and dark energy (with an equation of state parameter w_X constant in time) : since then $\rho = \rho_M + \rho_X$ and $p = w_X \rho_X$, the Hubble parameter at redshift z reads from (5), using $a(t) = a_0/(1+z)$ and assuming $k = 0$,

$$H^2(z) = H_0^2 \left[\Omega_M (1+z)^3 + \Omega_X (1+z)^{3(1+w_X)}\right] \ , \tag{11}$$

where the present value $H_0 \sim 70$ km.s^{-1}.Mpc^{-1} of the Hubble parameter sets the critical density scale $\rho_c \equiv 3H_0^2/(8\pi G_N) \sim 10^{-26}$ kg.m^{-3}: $\Omega_M \equiv \rho_{M0}/\rho_c$ and $\Omega_X \equiv \rho_{X0}/\rho_c$ [3]. Obviously, at present time ($z = 0$), we have, from (11),

$$1 = \Omega_M + \Omega_X \ , \tag{12}$$

which is just another way of writing the Friedmann equation (5).

Cosmological observations lead in this simple model to a determination of the two independent parameters: Ω_M and w_X. As is well-known [8, 9], observational data on the flux ϕ received from supernovae of type Ia leads, under the assumption of a constant luminosity L (i.e. if these supernovae behave as standard candles), to a determination of their luminosity distance d_L: $\phi = L/(4\pi d_L^2)$. The dependence of the luminosity distance with respect to the redshift of the supernovae then provides a constraint on the geometry of spacetime. Similarly, data on distant galaxies provide important information on the acoustic oscillations that the coupled baryon-photon fluid underwent before photons decoupled from baryons: the corresponding baryon acoustic peak [10] is similar to the peaks observed in the cosmic microwave background of photons. It provides, just as well, important information on the cosmological parameters. As an illustration, figure 1 gives the constraints obtained by the collaboration SuperNova Legacy Survey [11] on the cosmological parameters (Ω_M, w_X), using as well the baryon oscillation data. The central values correspond to $\Omega_M = 0.3$, $\Omega_X = 0.7$ and $w_X = -1$: the energy density of the Universe is dominated by the dark energy component.

[3] An index 0 indicates a value at present time and we have used (6) to show for example that $\rho_X \sim a(t)^{-3(1+w_X)}$.

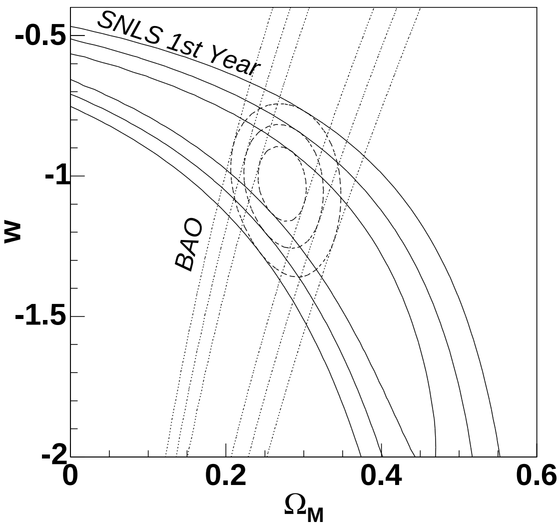

Figure 1. Contours at 68.3%, 95.5% and 99.7% confidence levels for the fit to a flat (Ω_M, w_X) cosmology, from the SNLS Hubble diagram alone, from the SDSS baryon acoustic oscillations alone [10] and the joint confidence contours [11].

A powerful tool is also provided by weak gravitational lensing [12]. The deviation of light rays by an accumulation of matter along the line of sight depends on the distance to the source, and thus on the cosmological parameters. As the Universe accelerates, there is more volume and more lenses between the observer and the object at redshift z.

The result $\Omega_X \sim 0.7$ yields $\rho_X \sim 0.7\rho_c \sim 10^{-26}$ kg.m^{-3}. If fundamental microphysics is to provide the apropriate framework for dark energy, it might be more transparent to work in units where $\hbar = c = 1$. In this case, energy densities have dimensions of mass to the fourth power and we can write:

$$\rho_X \sim \Lambda_X^4 \ , \qquad \Lambda_X \sim 10^{-3} \text{ eV} \ . \tag{13}$$

Is this scale a new fundamental scale associated with the physics of dark energy? In this case, is there any connection with neutrino masses? Or is it only a derived scale with no fundamental meaning? Such questions should receive an answer in the context of specific models.

The deceleration parameter $q \equiv -\ddot{a}a/\dot{a}^2$ reads, using (7),

$$q = \frac{H_0^2}{2H(z)^2}\left[\Omega_M(1+z)^3 + \Omega_X(1+3w_X)(1+z)^{3(1+w_X)}\right] \quad . \tag{14}$$

This shows that, in our simple model, the universe starts accelerating at a redshift value $z_{\rm acc}$ given by

$$1 + z_{\rm acc} = \left[-(1+3w_X)\frac{\Omega_X}{\Omega_M}\right]^{-1/(3w_X)} \quad . \tag{15}$$

Setting $\Omega_X \sim 1 - \Omega_M$ allows to determine $z_{\rm acc}$ in terms of Ω_M: for $\Omega_M \sim 0.3$ and $w_X \sim -1$, we have $z_{\rm acc} \sim 0.6$. This is why a lot of attention has been paid to the region of redshifts around 1. We will see however in what follows that this value is somewhat model dependent.

A central problem that models of dark energy have to address is the following: since matter and dark energy evolve differently, why should they be of the same order at present times? This is known as the cosmic coincidence problem or the "Why now?" problem. As is clear from

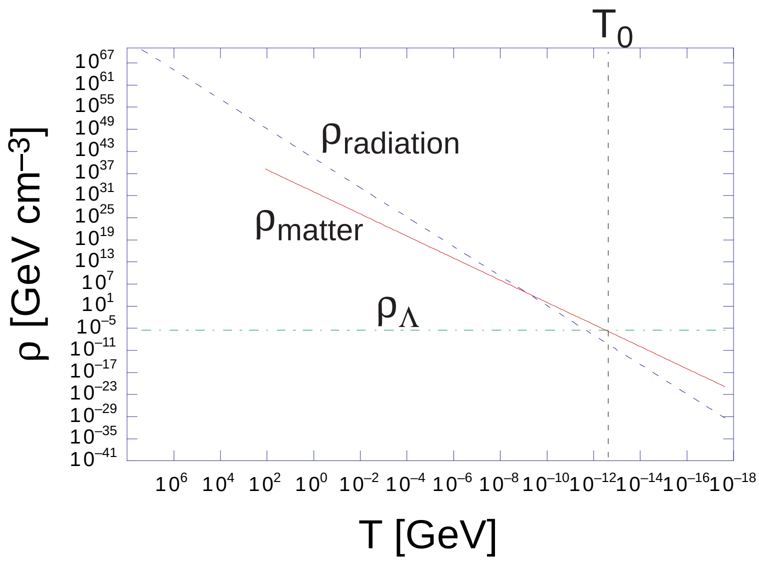

Figure 2. The cosmic coincidence problem illustrated in the case of a cosmological constant i.e. a dark energy component with $w_X = -1$ (see below).

figure 2, this problem is particularly acute in the case (favoured by observation, see figure 1) where $w_X = -1$, that is, where the dark energy energy density remains constant in time: why is it emerging from matter energy density at present time? Or, to avoid phrasing the question in an anthropocentric manner, why is it emerging at a time close to galaxy formation ($z \sim 5$ to 10)? At a time which, at the scale of the age of the Universe, is not so distant from radiation-matter equality ($z \sim 3500$).

Another generic problem associated with dark energy arises if one interprets dark energy as a field. Because the only basic dimensionful parameter in the problem is the Hubble constant H_0 (which has the dimension of an inverse time, i.e. of a mass in units where $\hbar = c = 1$)[4], in generic examples, the mass of the dark energy field is of the order of H_0, that is 10^{-33} eV. This pauses drastic challenges to any dark energy theory of fundamental physics. Not only because this is a small number but also because the exchange of the dark energy field between ordinary quarks or leptons leads to a long range force, which is not observed (or not yet). The consequence has to be that, if it exists, the dark energy field has to be extraordinarily weakly coupled to ordinary matter. Tests of the equivalence principle also induce strong constraints on such couplings. Let us however stress that such restrictions do not apply to couplings to dark matter or neutrinos since these have not been probed to the degree of accuracy reached for ordinary matter. As we will see, this leads to interesting models for dark energy.

The model that we have used until now to parametrize the data is useful to orient the discussion but oversimplified. Indeed, in most models that we will discuss below, the dynamics associated with dark energy is somewhat more involved. In particular, the equation of state parameter w_X often evolves with time. Various parametrizations have been proposed to account for this effect. Since we are dealing with late time evolution ($a \sim a_0$), it has become popular

[4] See below explicit examples to understand why this is the relevant scale for the mass of the dark energy field, and not the scale Λ_X that we have introduced in (13).

[13] to write

$$w_X = w_0 + (a_0 - a)w_a \ . \tag{16}$$

Observational data favors values for the equation of state parameter of dark energy which are close to -1: $w_0 \sim 1$ and $w_a \ll 1$. This is where the discussion on dark energy reaches its most fascinating aspects: if $w_X = -1$, this would be a clear signal to identify dark energy with the energy of the quantum vacuum. We recall that, in classical quantum physics, the vacuum is the fundamental state of the system considered. Quantum field theory enriches the notion since this vacuum state cannot be isolated from the quantum fluctuations associated with virtual particle production but its energy is not an observable because only differences of energy are measurable (the best example is the Casimir effect where quantum fluctuations of the electromagnetic field induce a force –hence a difference of energy– between two conducting plates separated by the vacuum). But, in a gravitational context, the presence of the energy-momentum tensor in the right-hand side of Einstein's equations (1) shows that absolute energies have an impact on the geometry of spacetime. Indeed, as can be seen from the Friedmann equation (5), it is the absolute energy density that determines the rate of expansion.

As is well-known, the cosmological odyssey of the vacuum energy started with the cosmological term introduced by Einstein in the early days: in order to reproduce what was known at the time i.e. a static universe (our own galaxy), Einstein introduced an extra term in his equation in order to allow for the presence of a static solution [14]:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu} + \lambda g_{\mu\nu} \ . \tag{17}$$

The cosmological term involves the cosmological constant λ which has the dimension of an inverse length squared. Its effect is clearly seen from the Friedmann equation that follows, using the Robertson-Walker metric (2),

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{1}{3}\left(\lambda + 8\pi G_N \rho\right) - \frac{k}{a^2} \ . \tag{18}$$

Thus a positive cosmological constant induces an acceleration of the expansion.

Typically, since we know that the spatial curvature term is presently subdominant, (18) considered at present time implies the following constraint on λ (barring a cancellation between the dynamical ρ and the constant λ):

$$|\lambda| \leq H_0^2 \ . \tag{19}$$

In other words, the length scale $\ell_\Lambda \equiv |\lambda|^{-1/2}$ associated with the cosmological constant must be larger than the Hubble length $\ell_{H_0} \equiv cH_0^{-1} \sim 10^{26}$ m, and thus be a cosmological distance.

This may not be a problem as long as one remains classical: ℓ_{H_0} provides a natural cosmological scale for our present Universe. The problem arises when one tries to combine gravity with quantum theory. Indeed, from Newton's constant *and* the Planck constant $\hbar$, we can construct the Planck mass scale $m_P = \sqrt{\hbar c/(8\pi G_N)} = 2.4 \times 10^{18}$ GeV/c^2, the corresponding length scale being the Planck length $\ell_P = \hbar/(m_P c) = 8.1 \times 10^{-35}$ m.

The above constraint now reads:

$$\ell_\Lambda \equiv |\lambda|^{-1/2} \geq \ell_{H_0} = \frac{c}{H_0} \sim 10^{60} \ \ell_P. \tag{20}$$

In other words, there are more than sixty orders of magnitude between the scale associated with the cosmological constant and the scale of quantum gravity.

It might be just as well to consider that the cosmological constant is altogether vanishing. Indeed, Einstein soon withdrew the cosmological term, writing his equation in the form (1), and claiming that this was his "biggest blunder". But the devil was now out of the box: assuming that there is a non-vanishing vacuum energy i.e. $\langle T_{\mu\nu}\rangle = \rho_{\rm vac} g_{\mu\nu}$ [5], Einstein's equation (1) reads

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu} + 8\pi G_N \rho_{\rm vac} g_{\mu\nu} \ . \tag{21}$$

The last term is thus interpreted as an effective cosmological constant [15]:

$$\lambda_{\rm eff} = 8\pi G_N \rho_{\rm vac} \equiv \frac{\Lambda^4}{m_P^2} \ . \tag{22}$$

Generically, $\rho_{\rm vac}$ receives a non-zero contribution from symmetry breaking: for example, the scale Λ would be typically of the order of 100 GeV in the case of the electroweak gauge symmetry breaking. But the constraint (20) now reads:

$$\Lambda \leq 10^{-30}\, m_P \sim 10^{-3} \text{ eV}. \tag{23}$$

It is this very unnatural fine-tuning of parameters (in explicit cases $\rho_{\rm vac}$ and thus Λ are functions of the parameters of the theory) that is referred to as the cosmological constant problem, or more accurately the *vacuum energy problem.*

There is one spacetime symmetry that ensures a vanishing vacuum energy: it is supersymmetry. Indeed, from the supersymmetry algebra (the commutator of two supersymmetry transformations is a spacetime translation $\{Q_r, \bar{Q}_s\} = 2\gamma^\mu_{rs}\, P_\mu$), one may derive the following expression for the Hamiltonian $H = P_0$

$$H = \frac{1}{4} \sum_r Q_r^2 \quad . \tag{24}$$

Thus the vacuum energy $\langle 0|H|0\rangle$ vanishes if and only if the vacuum is supersymmetric i.e. $Q_r|0\rangle = 0$ for all r. Supersymmetry thus seems the apropriate framework to discuss vanishing or small vacuum energies. However, we know that in nature, supersymmetry is broken by a large amount, say larger than 1 TeV, which gives a contribution of the same order to the vacuum energy, hence much larger than the 10^{-3} eV allowed above.

We have seen above that observational data is consistent with the possibility that the acceleration of the expansion is due to the cosmological constant, its value is as large as the upper bounds just obtained:

$$\lambda \sim H_0^2 \ , \quad \ell_\Lambda \sim \ell_{H_0} \ , \quad \Lambda \sim 10^{-3} \text{ eV} \ . \tag{25}$$

Considering the latter scale Λ, which characterizes the vacuum energy ($\rho_{\rm vac} \equiv \Lambda^4$), one may note the interesting numerical coincidence:

$$\frac{\hbar c}{\Lambda} \sim \sqrt{\ell_{H_0} \ell_P} \sim 10^{-4} \text{ m} \ . \tag{26}$$

This relation underlines the fact that the vacuum energy problem involves some deep connection between the infrared regime (the infrared cut-off being ℓ_{H_0}) and the ultraviolet regime (the ultraviolet cut-off being ℓ_P), between the infinitely large and the infinitely small. We will return to this interesting question in the last question.

For the time being, we discuss generic models of dark energy using the dynamics of scalar fields.

[5] We note here that this corresponds to a perfect fluid with equation of state $p = -\rho$ ($T_{00} = \rho$ and $T_{ij} = p\delta_{ij}$ from (4)).

2. Scalar field models of dark energy

2.1. Why scalar fields?

Scalar fields easily provide a diffuse cosmological background and are thus a favourite candidate for dark energy. More precisely, the relevant quantity when discussing perturbations associated with a field is the speed of sound

$$c_s^2 \equiv \frac{\delta p}{\delta \rho} \ . \tag{27}$$

It is a measure of how the pressure of the field resists gravitation clustering. In most models with scalar fields, we have $c_s^2 \sim 1$, which explains why scalar dark energy does not cluster: its own pressure resists gravitational collapse.

A typical candidate for dark energy is a scalar field ϕ slowly evolving in its potential $V(\phi)$. More explicitly, let us consider the following action

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[-\frac{m_P^2}{2} R + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) \right] , \tag{28}$$

which describes a real scalar field ϕ minimally coupled with gravity. Computing the corresponding energy-momentum tensor, we obtain the energy density and pressure

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad , \tag{29}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad . \tag{30}$$

The corresponding equation of motion is, if one neglects the spatial curvature ($k \sim 0$),

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi} \ , \tag{31}$$

from which we deduce as expected

$$\dot{\rho}_\phi = -3H(p_\phi + \rho_\phi) \ . \tag{32}$$

We have for the equation of state parameter

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \geq -1 \quad . \tag{33}$$

If the kinetic energy is subdominant ($\dot{\phi}^2/2 \ll V(\phi)$), we clearly obtain $-1 \leq w_\phi \leq 0$, which thus provides a potential dark energy candidate (see (10)).

Particle physics models, in particular in the context of supersymmetry or string theory, provide numerous models of dark energy scalar fields. We present some of them in the remainder of this Section.

2.2. The example of quintessence

We start with the simplest class of models. Historically, they correspond to the first models studied, already back in 1988 [16, 18, 17].

For illustration, let us consider the case of the Ratra-Peebles [17, 18] potential:

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^\alpha} \ , \quad \alpha > 0 \ , \tag{34}$$

in a universe dominated by a background component with equation of state parameter w_B (the cosmic scale factor $a(t)$ evolves as t^{2/n_B} with $n_B = 1 + 3w_B$; $n_B = 4$ for radiation, 3 for nonrelativistic matter).

We are looking for a scaling solution where the ϕ energy density scales as a power of the cosmic scale factor:

$$\rho_\phi \propto a^{-n_\phi} \ , \quad n_\phi \ \mathrm{cst} \ . \tag{35}$$

Then $\dot\rho_\phi/\rho_\phi = -n_\phi H$. In this case, using (32), one obtains

$$w_\phi = \frac{n_\phi}{3} - 1 \ . \tag{36}$$

Hence the equation of state parameter needs to be constant.

Using (29,30), we have $\dot\phi^2 \sim V(\phi) \sim \rho_\phi \sim t^{-2n_\phi/n_B}$. Thus ϕ behaves as $t^{1-(n_\phi/n_B)}$ and, since $\phi^{-\alpha} \sim t^{-2n_\phi/n_B}$, one obtains

$$n_\phi = \frac{\alpha n_B}{\alpha + 2} \quad \mathrm{or} \quad w_\phi = -1 + \frac{\alpha(1+w_B)}{\alpha+2} \ . \tag{37}$$

The complete solution of the equation of motion (31) is

$$\phi = \left(\frac{\alpha(\alpha+2)^2 n_B \ M^{4+\alpha} t^2}{2\left[6(\alpha+2) - n_B \alpha\right]} \right)^{\frac{1}{\alpha+2}} \ . \tag{38}$$

It turns out that such scaling solutions correspond to attractors in the cosmological evolution of the scalar field.

We have thus found an attractor scaling solution $\phi \propto a^{n_B/(2+\alpha)}$, $\rho_\phi \propto a^{-\alpha n_B/(2+\alpha)}$ in the case where the background density initially dominates. The scalar field energy density ρ_ϕ decreases at a slower rate than the background density ($\rho_B \propto a^{-n_B}$) and tracks it until it becomes of the same order, at a given value a_Q. We thus have:

$$\frac{\phi}{m_P} \sim \left(\frac{a}{a_Q}\right)^{n_B/(2+\alpha)} \quad , \quad \frac{\rho_\phi}{\rho_B} \sim \left(\frac{a}{a_Q}\right)^{2n_B/(2+\alpha)} \ . \tag{39}$$

Since $\rho_B \sim m_P^2/t^2$ and $\rho_\phi \sim M^{\frac{2(\alpha+4)}{\alpha+2}} t^{-\frac{2\alpha}{\alpha+2}}$, the energy density ρ_ϕ overcomes the background value ρ_B at a time $t_Q \sim m_P^{\frac{\alpha+2}{2}} M^{-\frac{\alpha+4}{2}}$.

Shortly after ϕ has reached for $a = a_Q$ a value of order m_P, it satisfies the standard slow roll conditions

$$\epsilon \equiv \frac{1}{2}\left(\frac{m_P V'}{V}\right)^2 = (\alpha/2)(m_P/\phi)^2 \ll 1 \ , \quad \eta \equiv \frac{m_P^2 V''}{V} = \alpha(\alpha+1)(m_P/\phi)^2 \ll 1 \ , \tag{40}$$

and (37) provides a good approximation to the present value of w_ϕ. Thus, at the end of the matter-dominated era, this field may provide the quintessence component that we are looking for.

Two features are interesting in this respect. One is that this scaling solution is reached for rather general initial conditions, i.e. whether ρ_ϕ starts of the same order or much smaller than the background energy density [19].

The second is the present value of ρ. Typically, since in this scenario ϕ is of order m_P when the quintessence component emerges, we must choose the scale M in such a way that $V(m_P) \sim \rho_c$. The constraint reads:

$$M \sim \left(H_0^2 m_P^{2+\alpha}\right)^{1/(4+\alpha)} \ . \tag{41}$$

We may note that this gives for $\alpha = 2$, $M \sim 10$ MeV, not such an atypical scale for high energy physics.

Models of dynamical supersymmetry breaking easily provide a potential of the Ratra-Peebles type discussed above [20]. Let us consider supersymmetric QCD with gauge group $SU(N_c)$ and $N_f < N_c$ flavors, *i.e.* N_f quarks Q_g (resp. antiquarks $\bar{Q}^g$), $g = 1 \cdots N_f$, in the fundamental $\mathbf{N_c}$ (resp. anti-fundamental $\bar{\mathbf{N}}_\mathbf{c}$) of $SU(N_c)$. At the scale of dynamical symmetry breaking Λ where the gauge coupling becomes strong, boundstates of the meson type form: ${M_f}^g = Q_f \bar{Q}^g$. The dynamics is described by a superpotential which can be computed non-perturbatively using standard methods:

$$W = (N_c - N_f) \frac{\Lambda^{(3N_c - N_f)/(N_c - N_f)}}{(\det M)^{1/(N_c - N_f)}} \ . \tag{42}$$

Such a superpotential has been used in the past but with the addition of a mass or interaction term (*i.e.* a positive power of M) in order to stabilize the condensate. One does not wish to do that here if M is to be interpreted as a runaway quintessence component. For illustration purpose, let us consider a condensate diagonal in flavor space: ${M_f}^g \equiv \phi^2 \delta_f^g$. Then the potential for ϕ has the form (32), with $\alpha = 2(N_c + N_f)/(N_c - N_f)$. Thus,

$$w_\phi = -1 + \frac{N_c + N_f}{2N_c}(1 + w_B), \tag{43}$$

which clearly indicates that the meson condensate is a potential candidate for a quintessence component.

Because the value of the quintessence field ϕ is of order m_P in the period of acceleration of the expansion, one must take into account all non-renormalisable interactions of order $(\phi/m_P)^n$. For example, in a supersymmetric context, the full supergravity corrections must be included. One may then argue [21] that this could put in jeopardy the positive definiteness of the scalar potential, a key property of the quintessence potential and one reason to consider supersymmetry.

The couplings of scalar fields are defined in supergavity by two functions: a real function $K(\phi_\imath \bar{\phi}_{\bar{\jmath}})$, known as the Kähler potential (the second derivatives $g_{\imath\bar{\jmath}} \equiv \partial^2 K / \partial \phi_\imath \partial \bar{\phi}_{\bar{\jmath}}$ also determine the field-dependent normalisation of the kinetic terms: $g_{\imath\bar{\jmath}} \partial^\mu \phi_\imath \partial_\mu \bar{\phi}_{\bar{\jmath}}$) and a complex holomorphic function $W(\phi)$, the superpotential. More precisely,

$$V = e^{K/m_P^2} \left[\left(\frac{\partial W}{\partial \phi_\imath} + \frac{\partial K}{\partial \phi_\imath} \frac{W}{m_P^2} \right) g^{\imath\bar{\jmath}} \left(\frac{\partial \bar{W}}{\partial \bar{\phi}_{\bar{\jmath}}} + \frac{\partial K}{\partial \bar{\phi}_{\bar{\jmath}}} \frac{\bar{W}}{m_P^2} \right) - \frac{3}{m_P^2} |W|^2 \right] \ , g^{\imath\bar{\jmath}} g_{\bar{\jmath}k} = \delta^i_k \ . \tag{44}$$

The problem is the term $-3|W|^2/m_P^2$ (which vanishes in the limit $m_P \to \infty$ where one recovers rigid supersymmetry). One possibility is to consider no-scale models: the presence of 3 moduli fields T^i with Kähler potential $K = -\sum_i \ln(T^i + \bar{T}^i)$ cancels the negative contribution $-3|W|^2/m_P^2$ in the supergravity potential.

Alternatively, one may consider models where $\langle W \rangle = 0$ (but not its derivatives).Let us take the Brax and Martin model [21, 22] as an example. It involves three fields X, Y and the quintessence field ϕ:

$$K(X, Y, \phi) = \bar{\phi}\phi + \bar{X}X + \bar{Y}Y(\bar{\phi}\phi)^p / m_P^{2p} \ , \quad W(X, Y, \phi) = \lambda X^2 Y \ . \tag{45}$$

Assuming that, in the minimum, $\langle X \rangle = \xi$ and $\langle Y \rangle = 0$, the potential reads

$$V(\phi) = e^{(\bar{\phi}\phi + |\xi|^2)/m_P^2} \frac{\lambda^2 |\xi|^4 m_P^{2p}}{(\bar{\phi}\phi)^p} \ . \tag{46}$$

This has a form similar to the Ratra-Peebles potential (34), but with an extra exponential factor which stabilizes the quintessence field at large values.

We have just seen that Kähler potentials with $g_{\imath\bar{\jmath}} \neq \delta_{\imath\jmath}$ lead to nontrivial kinetic terms. Dynamics may be hidden in such nontrivial terms, which proves to be well suited to describe dark energy, as we now see.

2.3. Non-trivial kinetic terms

In order to introduce this class of models, we will take our inspiration from the description of a relativistic particle in special relativity. From the Lagrangian $L = -m\sqrt{1-\dot{q}^2}$, where m is the mass of the particle and $q(t)$ its one-dimensional position, one derives its energy $E = m/\sqrt{1-\dot{q}^2}$ and momentum $k = m\dot{q}/\sqrt{1-\dot{q}^2}$. Moving to field theory, one may replace $q(t)$ by $\phi(\mathbf{x},t)$, $\dot{q}^2$ by $\partial^\mu\phi\partial_\mu\phi$ (which would simply read $\dot{\phi}^2$ in the case of a homogeneous field) and even make the mass m a field-dependent function $\mu(\phi)$. The corresponding Lagrangian density is thus

$$\mathcal{L} = -\mu(\phi)\sqrt{1-\partial^\mu\phi\partial_\mu\phi} \ . \tag{47}$$

Such non-trivial structure in the kinetic terms of scalar fields often appears in the context of string and brane theory. For example, non-BPS Dp-branes suffer from an instability under which all open string states disappear: a tachyonic mode is present and the system should relax to the minimum of the tachyonic potential [23, 24]. This is described at the level of the effective field theory by a Dirac-Born-Infeld (DBI) action

$$\mathcal{S} = \int d^{p+1}x \ V(\phi)\sqrt{-\det\left[g_{mn} + 2\pi\alpha' F_{mn} + \partial_m\phi\partial_n\phi\right]} \ , \tag{48}$$

where ϕ is the tachyon field, $V(\phi)$ the tachyon potential and $\alpha' \equiv M_S^{-2}$ the string constant. Disregarding the gauge field, we obtain

$$\mathcal{S} = \int d^{p+1}x \ V(\phi)\sqrt{-g}\sqrt{\det\left[\delta^m_n + g^{mr}\partial_r\phi\partial_n\phi\right]} = \int d^{p+1}x \ V(\phi)\sqrt{-g}\sqrt{1+g^{mn}\partial_m\phi\partial_n\phi} \ , \tag{49}$$

where $g = \det(g_{mn})$. The same action provides the effective description of a Dp-brane anti-Dp-brane system: ϕ then describes the distance between the two branes.

More recently, a similar system has been considered: a probe $D3$-brane travelling down a five-dimensional *warped* throat geometry. The warping means that the $d+1 = 4$-dimensional metric on the brane is ϕ-dependent, namely $f(\phi)^{-1})g_{\mu\nu}$. Thus, assuming a constant potential, one obtains [25, 26]

$$\mathcal{S} = \int d^4x \ \sqrt{-g}f^{-2}(\phi)\sqrt{1+f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} \ . \tag{50}$$

In the case of throats coming from IIB flux compactifications [26],

$$f(\phi) \sim \frac{\lambda}{\phi^4} \ , \tag{51}$$

and an additive potential term arises from the couplings of the D-brane to background RR fluxes.

Models with non-trivial kinetic terms have been proposed to account for dark energy [27]: *k-essence* models are based on the following generic action

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{m_P^2}{2}R + \mathcal{L}(X,\phi)\right] \ , \quad \text{where} \quad X \equiv \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \ . \tag{52}$$

Variation of the action with respect to ϕ yields the energy-momentum for the scalar field:

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta\mathcal{S}}{\delta g^{\mu\nu}} = \mathcal{L}_{,X}\partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\mathcal{L} \ . \tag{53}$$

This has a hydrodynamic description. Indeed, introducing (cf the example that started this section)

$$U_\mu \equiv \frac{\partial_\mu \phi}{\sqrt{2X}} \ , \tag{54}$$

$T_{\mu\nu}$ has the perfect fluid form (4) with

$$p(X,\phi) \equiv \mathcal{L}(X,\phi) \ , \quad \rho(X,\phi) \equiv 2X\mathcal{L}_{,X} - \mathcal{L} \ . \tag{55}$$

The equation of state parameter thus reads

$$w = \frac{p}{2Xp_{,X} - p} \ . \tag{56}$$

The speed of sound can be expressed as [28]

$$c_s^2 = \frac{p_{,X}}{\rho_{,X}} = \frac{p_{,X}}{p_{,X} + 2Xp_{,XX}} \ . \tag{57}$$

In the case where $\mathcal{L}$ depends only on X, then one has $p = p(\rho)$ and one recovers the usual $c_s^2 = \partial p/\partial\rho$.

The equation of motion for the scalar field reads

$$\tilde{G}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2X\mathcal{L}_{,X\phi} - \mathcal{L}_{,\phi} = 0 \ , \tag{58}$$

where [29]

$$\tilde{G}^{\mu\nu} \equiv \mathcal{L}_{,X} g^{\mu\nu} + \mathcal{L}_{,XX} \ \partial^\mu\phi\partial^\nu\phi \ . \tag{59}$$

Following the examples that we started with, we will make the simplifying assumption that the dependences in X and ϕ of $\mathcal{L}(X,\phi)$ factorize:

$$p(X,\phi) \ \equiv \ K(\phi)\tilde{p}(X) \ , \tag{60}$$

$$\rho(X,\phi) \ \equiv \ K(\phi)\tilde{\rho}(X) \ , \quad \tilde{\rho}(X) = 2X\tilde{p}_{,X} - \tilde{p} \ . \tag{61}$$

It proves to be useful to parametrize differently the function $\tilde{p}(X)$:

$$\tilde{p}(X) \equiv \frac{g(y)}{y} \ , \quad y \equiv X^{-1/2} \ . \tag{62}$$

Then

$$\tilde{\rho}(X) = -g'(y) \ , \tag{63}$$

and

$$w = -\frac{g}{yg'} \ , \quad c_s^2 = \frac{g - g'y}{g''y^2} \ . \tag{64}$$

The positivity of energy (63) implies that $g' > 0$. Assuming that $\tilde{p}(X)$ is increasing with X (i.e. $g - g'y > 0$), then $c_s^2 > 0$ requires $g'' > 0$. Thus g is a monotonously decreasing convex function of y.

Tracker solutions require $w = w_B$, a condition that fixes y as the solution y_T of $g/(yg') = -w_B$. One can show [27] that this implies the following form for $K(\phi)$:

$$K(\phi) = \frac{\text{cst}}{\phi^2} \ . \tag{65}$$

Then

$$\frac{\rho_\phi}{\rho_\phi + \rho_B} = -Cg'(y_T)y_T^2 \ , \tag{66}$$

where the constant C is 2 for a radiation background, and 9/8 for a matter background.

Accelerating solutions ($w < 0$) are obtained on the other hand for negative values of g, that is for $y > y_0$ where y_0 is the value where g vanishes. It is possible to find functions g such that such accelerating solutions appear only at the onset of matter domination. Then, matter domination triggers the dark energy component: a nice solution to the "Why now?" problem.

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2.4. The problems of scalar field models of dark energy

However appealing, the quintessence idea is difficult to implement in the context of realistic models [30, 31]. As we have already mentionned, the main problem lies in the fact that the quintessence field must be extremely weakly coupled to ordinary matter. This problem can take several forms:

- the quintessence field must be very light. If we return to our example of Ratra-Peebles potential in (34), $V''(m_P)$ provides an order of magnitude for the mass-squared of the quintessence component:

$$m_\phi \sim M \left(\frac{M}{m_P}\right)^{1+\alpha/2} \sim H_0 \sim 10^{-33} \text{ eV}. \tag{67}$$

  using (41). This might argue for a pseudo-Goldstone boson nature of the scalar field that plays the rôle of quintessence. This field must in any case be very weakly coupled to ordinary matter; otherwise its exchange would generate observable long range forces. Eötvös-type experiments put very severe constraints on such couplings.
- it is difficult to find a symmetry that would prevent any coupling of the form $\beta(\phi/m_P)^n F^{\mu\nu}F_{\mu\nu}$ to the gauge field kinetic term. Since the quintessence behavior is associated with time-dependent values of the field of order m_P, this would generate, in the absence of fine tuning, corrections of order one to the gauge coupling. But the time dependence of the fine structure constant for example is very strongly constrained: $|\dot{\alpha}/\alpha| < 5 \times 10^{-17}\text{yr}^{-1}$. This yields a limit [30]:

$$|\beta| \leq 10^{-6} \frac{m_P H_0}{< \dot{\phi} >}, \tag{68}$$

  where $< \dot{\phi} >$ is the average over the last 2×10^9 years.

All the preceding shows that there is extreme fine tuning in the couplings of the quintessence field to ordinary matter, unless they are forbidden by some symmetry. This is due to the lightness of the field which induces a new long range force. The most stringent constraints come from tests of the equivalence principle. However these constraints apply to ordinary matter: no experiment has tested the equivalence principle with neutrinos or with dark matter. This leaves thus the possibility that dark energy is coupled in a non-negligible way to neutrinos or dark matter. We consider such possibilities in the next section.

3. Can dark energy be coupled to some form of matter?

3.1. Mass varying sterile neutrinos

Let us consider a sterile neutrino with a mass $m_\nu(\phi)$ which depends on a scalar field ϕ [32, 33, 34]. The effective coupling between the neutrino and the scalar field may be described by the function:

$$\beta(\phi) = \frac{d \log m_\nu(\phi)}{d\phi} \ . \tag{69}$$

In the case of a nonrelativistic uniform neutrino background, the neutrino energy density is simply $\rho_\nu(\phi) = n_\nu m_\nu(\phi)$ (n_ν is the neutrino number density), while the pressure $p_\nu(\phi)$ is negligible. The effective potential for the scalar field thus reads

$$V_{\text{eff}}(\phi) = V(\phi) + \rho_\nu(\phi) = V(\phi) + n_\nu m_\nu(\phi) \ , \tag{70}$$

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where $V(\phi)$ is the original scalar potential. The minimum is obtained for

$$V'(\phi) + n_\nu m'_\nu(\phi) = 0 \ , \tag{71}$$

and the mass of the scalar field is given by $V''_{\rm eff} = V'' + \rho_\nu(\beta^2 + \beta')$. In this model, the scalar mass is not as small as for quintessence. Indeed, under the condition

$$V''_{\rm eff} = V'' + \rho_\nu(\beta^2 + \beta') \gg H^2 \tag{72}$$

the scalar field evolves adiabatically and tracks the minimum of (71) as neutrinos are diluted by the expansion ($n_\nu \sim a^{-3}$).

Dark energy is in this case the coupled scalar field-neutrino fluid. Its energy density is simply $\rho_X = V_{\rm eff}$. Using (6), we have

$$\dot{\rho}_X = -3\frac{\dot{a}}{a}\rho_X(1 + w_X) \ . \tag{73}$$

Hence

$$\begin{aligned} 1 + w_X &= -\frac{a}{3V_{\rm eff}}\frac{\partial V_{\rm eff}}{\partial a} \\ &= -\frac{a}{3V_{\rm eff}}\left[m_\nu \frac{\partial n_\nu}{\partial a} + n_\nu \frac{\partial m_\nu}{\partial a} + \frac{\partial V}{\partial a}\right] \\ &= -\frac{a}{3V_{\rm eff}}\left[-\frac{3}{a}m_\nu n_\nu + \frac{\partial \phi}{\partial a}\left(n_\nu m'_\nu + V'\right)\right] = -\frac{m_\nu V'(\phi)}{m'_\nu(\phi) V_{\rm eff}(\phi)} \ , \end{aligned} \tag{74}$$

where we have used (71). Thus w_X turns out to be close to -1 in two cases: a flat scalar potential $V(\phi)$ or a steep dependence of the neutrino mass $m_\nu(\phi)$ (which triggers a motion of the ϕ field following (71)).

If we put numbers, we note that the cosmological bound $m_\nu < 1$ eV induces an upper bound on the neutrino energy density $\Omega_\nu < 0.02$. Thus $\Omega_X \sim 0.70$ gives $V_\phi/\rho_\nu \sim 35$. Hence most of the dark energy density is still in the scalar field.

The difficulty in this type of scenario is that the exchange of the scalar field induces a force between the neutrinos. They thus feel an effective Newton's constant [35]

$$G_{\rm eff} \sim G_N \left(1 + 2\beta^2 m_P^2\right) \ . \tag{75}$$

This may lead to the formation of neutrino nuggets which are not observed. The condition for avoiding such nuggets is roughly: $2\beta^2 m_P^2 \Omega_\nu < \Omega_M$, that is $\beta^2 m_P^2 < 15$.

3.2. Environmental coupling

We mentionned above that the possibility that masses may depend on the ultralight quintessence field, leads to stringent experimental constraints. Interestingly enough, one can twist the problem around and use the couplings of the quintessence field to make its mass dependent on the environment, or more precisely on the local matter density: within ordinary matter, it is a massive field and its interacting range remains within the experimental bounds (in the submillimeter region); without matter, i.e. in outer space, it is almost massless and has a very large interaction range. This is the basic idea of the so-called chameleon cosmology [36, 37].

Let us illustrate on the example of a scalar-tensor theory

$$S = \int d^4x \sqrt{-g}\left[\frac{m_P^2}{2}R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + S_m\left(\psi, A^2(\phi) g_{\mu\nu}\right) \ , \tag{76}$$

where S_m is the matter action and ψ a generic matter field. The Euler equation for the scalar field is then

$$\Box\phi = -\frac{dV_{\text{eff}}}{d\phi}\ , \quad V_{\text{eff}}(\phi) = V(\phi) + A(\phi)\rho_{_M}\ . \tag{77}$$

If $V(\phi)$ is a runaway potential and $A(\phi)$ is a monotonically increasing function, then the effective has a ground state which evolves with $\rho_{_M} \propto a^{-3}$. The coupling between matter and the scalar field is measured by the field-dependent function

$$\beta(\phi) = \frac{\partial \log A(\phi)}{\partial\phi}\ . \tag{78}$$

Taking for example a constant $\beta(\phi) = M^{-1}$ i.e. $A(\phi) = e^{\phi/M}$ and

$$V(\phi) = \Lambda^4 e^{(\alpha\Lambda/\phi)^n} \sim \Lambda^4 + \frac{\alpha^n\Lambda^{4+n}}{\phi^n} \text{ for } \phi \gg \alpha\Lambda\ , \tag{79}$$

we have at the ground state, under the assumption $\alpha\Lambda \ll \phi_0 \ll M$,

$$\phi_0 = \left(\frac{n\alpha^n M\Lambda^{4+n}}{\rho_{_M}}\right)^{1/(n+1)}\ . \tag{80}$$

and the scalar field mass is

$$m^2 = (n+1)\frac{\rho_{_M}}{M^2}\left(\frac{\rho_{_M} M^n}{n\alpha^n\Lambda^{4+n}}\right)^{1/(n+1)}\ . \tag{81}$$

One may wander whether the presence of this scalar field modifies the standard tests of gravity. Let us take for example a large celestial body such as a planet, which we identify with a sphere of radius R and mass M. In the center of the planet, the matter density fixes the value of the scalar field at the minimum to be ϕ_{in} and its mass m_{in}, whereas outside the matter energy density is much smaller and correspondingly the scalar field value is different; we note it ϕ_{out} and the corresponding mass m_{out}. Solving the equation for the ϕ field in this static spherically symmetric situation

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{\partial V_{\text{eff}}}{\partial\phi} \tag{82}$$

allows to determine the profile $\phi(r)$. The transition between the values ϕ_{in} and ϕ_{out} occurs in a region of thickness of order m_{in}^{-1}, a thin shell if $m_{\text{in}}R \gg 1$. One can easily deduce the force felt by a test particle of mass m placed at a distance $r > R$ of the center [38]:

$$F = \frac{G_{_N} Mm}{r^2}\left[1 + \frac{\beta(\phi_{\text{out}} - \phi_{\text{in}})}{\Phi_{_N}}\right]\ , \tag{83}$$

where $\Phi_{_N} = G_{_N} M/R$ is the Newtonian potential at the surface of the body. One recognizes the gravitational force in the first term and in the second term the force $F(\phi) = -m\beta d\phi/dr$ induced by the exchange of the scalar field. We must therefore impose

$$\frac{\beta(\phi_{\text{out}} - \phi_{\text{in}})}{\Phi_{_N}} \ll 1\ . \tag{84}$$

3.3. Dark matter

The stringent constraint that exists on the coupling of dark energy to observable matter does not apply to dark matter. It could thus be envisaged that dark energy and dark matter are coupled [39, 40, 41, 42, 43, 44, 45]. Since dark matter is the main matter component, this coupling might help to solve the "Why now?" question: the onset of matter domination may have some influence on dark energy density.

Let us for example [44] consider the case where the dark matter particle χ depends on the value of the dark energy scalar ϕ:

$$m_\chi(\phi) = m_0 \exp(-\lambda\phi) \ . \tag{85}$$

If the scalar potential is exponential, i.e. $V(\phi) = V_0 \exp(\beta\phi)$, there is an attractor solution corresponding to

$$\phi = -\frac{3}{\lambda+\beta}\log(a/a_0) \ , \quad \Omega_\phi \sim 1 - \Omega_\chi = \frac{3+\lambda(\lambda+\beta)}{(\lambda+\beta)^2} \ . \tag{86}$$

In fact, we have

$$2\frac{a\ddot{a}}{\dot{a}^2} = -(1+3W) \ , \quad W \equiv -\frac{\lambda}{\beta+\lambda} \ , \tag{87}$$

and, as a result of the coupling between dark matter and dark energy, the following scaling behavior for the energy densities

$$\rho_\chi \sim \rho_\phi \sim a^{-3(1+W)} \ . \tag{88}$$

The scaling behavior of dark matter may not seem so surprising when one realizes that

$$\rho_\chi = m_\chi(\phi) n_\chi \sim a^{-3(1+W)} \ , \tag{89}$$

where, as usual, the matter number density scales as $a{-}3$.

It might be worth noting that, in this kind of scenario, the onset of acceleration may be at higher redshift than in the ordinary case [46].

Such scenarios suffer from the same kind of instability as the mass varying neutrino scenario of section 3.1 [39, 47].

4. Back to the cosmological constant

Given the difficulties inherent to most dark energy scenarios, as we have sen, as well as the observatinal results which concord towards a value of the equation of state parameter w_X close to -1, it might be advisable to return to the issue of the vacuum energy.

From the point of view of high energy physics, it is difficult to imagine a rationale for a pure cosmological constant, especially if it is nonzero but small compared to the typical fundamental scales (electroweak, strong, grand unified or Planck scale). There should be dynamics associated with this form of energy.

For example, in the context of string models, any dimensionful parameter is expressed in terms of the fundamental string scale M_S and of vacuum expectation values of scalar fields. The physics of the cosmological constant would then the physics of the corresponding scalar fields.

Indeed, it was difficult from the start to envisage string theory in the context of a true cosmological constant. The corresponding spacetime is known as de Sitter spacetime and has an event horizon. This is difficult to reconcile with the S-matrix approach of string theory in the context of conformal invariance. More precisely, in the S-matrix approach, states are asymptotically (i.e. at times $t \to \pm\infty$) free and interact only at finite times: the S-matrix element between an incoming set of free states and an outgoing set yields the probability associated with such a transition.

Steven Weinberg [48] has constrained the possible mechanisms for the relaxation of the cosmological constant by proving the following "no-go" theorem: *it is not possible to obtain a vanishing cosmological constant as a consequence of the equations of motion of a finite number of fields.*

Indeed, let us consider N such fields φ_n, $n = 1, \cdots, N$. In the equilibrium configuration these fields are constant and their equations of motion simply read

$$\frac{\delta \mathcal{L}}{\delta \varphi_n} = 0 \quad . \tag{90}$$

Remembering that $\lambda_{\text{eff}} \sim \langle T^\mu{}_\mu \rangle$ where the energy-momentum tensor may be obtained from varying the metric ($T^{\mu\nu} = \delta \mathcal{L}/\delta g_{\mu\nu}$), we see that the vanishing of the cosmological constant is a consequence of the equations (90) if we can find N functions $f_n(\varphi)$ such that

$$2 g_{\mu\nu} \frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} = \sum_n \frac{\delta \mathcal{L}}{\delta \varphi_n} f_n(\varphi) \quad . \tag{91}$$

This amounts to a symmetry condition, the invariance of the Lagrangian $\mathcal{L}$ under

$$\delta g_{\mu\nu} = 2\alpha g_{\mu\nu} \quad , \quad \delta \varphi_n = -\alpha f_n(\varphi) \quad . \tag{92}$$

However, one can redefine the fields φ_n, $n = 1, \cdots, N$ into σ_a, $a = 1, \cdots, N-1$ and φ in such a way that the invariance reads

$$\delta g_{\mu\nu} = 2\alpha g_{\mu\nu} \quad , \quad \delta \sigma_a = 0 \quad , \quad \delta \varphi = -\alpha \quad . \tag{93}$$

The Lagrangian which satisfies this invariance is written

$$\mathcal{L} = \sqrt{\text{Det }\ (e^{2\varphi} g_{\mu\nu})} \mathcal{L}_0(\sigma) = e^{4\varphi} \sqrt{|g|}\ \mathcal{L}_0(\sigma) \quad , \tag{94}$$

which does not provide a solution to the relaxation of the cosmological constant, as can be seen by redefining the metric: $\hat{g}_{\mu\nu} = e^{2\varphi} g_{\mu\nu}$ (in the new metric, the field φ has only derivative couplings).

Obviously, Weinberg's no-go theorem relies on a series of assumptions: Lorentz invariance, *finite* number of *constant* fields, possibility of globally redefining these fields... All attempts to propose a relaxation mechanism have tried to avoid the conclusions of the theorem by relaxing one of these asumptions.

4.1. Emergent gravity

Padmanabhan [49] makes an interesting remark regarding the geometric mean formula of equation (26). Consider a 3-dimensional macroscopic region of size L and divide this region into N microscopic cells of size ℓ_P each having a Poissonian fluctuation in energy of amount $\epsilon \sim 1/\ell_P$. The mean square energy fluctuation in the macroscopic region is $(\Delta E)^2 \sim N/\ell_P^2$ and the corresponding energy density $\rho = \Delta E/L^3 = N^{1/2}/(\ell_P L^3)$. If we assume that the degrees of freedom scale as the surface enclosing the region, then $N \sim (L/\ell_P)^2$ and

$$\rho \sim \frac{1}{L^2 \ell_P^2} \ . \tag{95}$$

On the other hand, if one makes the more standard assumption that the degrees of freedom scale as the volume of the bulk region, then $N \sim (L/\ell_P)^3$ and $\rho \sim 1/(L^{3/2}\ell_P^{5/2})$.

We see that (26) corresponds to the first case, that is equation (95) with $L = \ell_{H_0}$. This may be a simple numerical coincidence or have a deeper meaning.

Padmanabhan [49] proposes to construct the gravitational interaction as an emergent long wavelength phenomenon, described in terms of an effective theory using vector fields. The corresponding action is invariant under the shift $T_{\mu\nu} \rightarrow T_{\mu\nu} + \lambda/(8\pi G_N) g_{\mu\nu}$: the bulk cosmological constant can be gauged away. It is thus only the degrees of freedom located in the boundary of the spacetime region considered, which participate to the observed value of the cosmological constant. This leads naturally to the value (26) or (95).

A simlar route has been followed by C. Hogan [50], who proposes a holographic quantum geometry of spacetime with only two spatial dimensions. The basic effect can be understood on the basis of the Rayleigh criterion of wave optics. If one ray of light (wavelength λ), emitted by an aperture of size d, travels a distance L, one observes a diffraction spot of size $\lambda L/d$. The endpoints of the ray are uncertain by an amount $\lambda^{1/2}L^{1/2}$, which corresponds to an aperture of the same size as the diffraction spot ($d \sim \lambda L/d$). Setting $\lambda = \ell_P$ and $L = \ell_{H_0}$, the resemblance of this uncertainty distance with (26) is striking.

4.2. Fluxes and the landscape

In the context of string and brane theory, an interesting proposal has been put forward, which makes use of the many nontrivial fluxes present in semi-realistic models [51].

The inspiring example was provided by the Brown-Teitelboim mechanism [52, 53] where the quantum creation of closed membranes leads to a reduction of the vacuum energy inside. This is easier to understand on a toy model with a single spatial dimension.

Let us thus consider a line and establish along it a constant electric field $E_0 > 0$: the corresponding (vacuum) energy is $E_0^2/2$. Quantum creation of a pair of $\pm q$-charged particles ($q > 0$) leads to the formation of a region (between the two charges) where the electric field is partially screened to the value $E_0 - q$ and thus the vacuum energy is decreased to the value $(E_0 - q)^2/2$. Quantum creation of pairs in the new region will subsequently decrease the value of the vacuum energy. The process ends in flat space when the electric field reaches the value $E \leq q/2$ because it then becomes insufficient to separate the pairs created.

In a truly three-dimensional universe, the quantum creation of pairs is replaced by the quantum creation of membranes and the one-dimensional electric field is replaced by a tensor field $A_{\mu\nu\rho}$.There are two potential problems with such a relaxation of the cosmological constant.

First, since the region of small cosmological constant originates from regions with large vacuum energies, hence exponential expansion, it is virtually empty: matter has to be produced through some mechanism yet to be specified. The second problem has to do with the multiplicity of regions with different vacuum energies: why should we be in the region with the smallest value? Such questions are crying for an anthropic type of answer: some regions of spacetime are preferred because they allow the existence of observers.

The anthropic principle approach can be sketched as follows [48]. We consider regions of spacetime with different values of t_G (time of galaxy formation) and t_Λ, the time when the cosmological constant starts to dominate i.e. when the Universe enters a de Sitter phase of exponential expansion. Clearly galaxy formation must precede this phase otherwise no observer (similar to us) would be able to witness it. Thus $t_G \leq t_\Lambda$. On the other hand, regions with $t_\Lambda \gg t_G$ have not yet undergone any de Sitter phase of reacceleration and are thus "phase-space suppressed" compared with regions with $t_\Lambda \sim t_G$. Hence the regions favoured have $t_\Lambda \gtrsim t_G$ and thus $\rho_\Lambda \sim \rho_M$.

5. Conclusion: testing dark energy

An ambitious observational program is now set up at the international level to unravel the mysteries associated with dark energy. After all, if the acceleration of the expansion is to be explained in terms of dark energy, it is the dominant contribution to the energy density of the Universe that one is searching for. Cosmological observations associated with large scale surveys, whether ground-based or space-based, will presumably tell us in the future whether this dark energy is the energy of the vacuum or a more dynamical form of energy. But, in the latter case, provided this new form of energy is explained by one or several scalar fields, it is most probable that tests of fundamental physics such as the equivalence principle, the constancy of constants,... will provide key information for selecting the right theory.

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