THE STRONGLY-INTERACTING LIGHT HIGGS

ALEX POMAROL
IFAE, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona

We present the effective low-energy lagrangian arising from theories where the electroweak symmetry breaking is triggered by a light composite Higgs, which emerges from a strongly-interacting sector as a pseudo-Goldstone boson. This lagrangian proves to be useful for LHC and ILC phenomenology that includes the study of high-energy longitudinal vector boson scattering, strong double-Higgs production and anomalous Higgs couplings.

1 Introduction

The Standard Model (SM) of elementary particles, as we know it today, is not a complete theory. As it is well known, if we calculate the amplitude of the process $W_LW_L \to W_LW_L$ we find that it grows with the energy as g^2E^2/M_W^2 violating unitarity at energies around $4\pi v \sim 1$ TeV. What unitarize this amplitude at high energy? This is the first priority question to be addressed at the LHC.

An example of a possible UV-complition of the SM can be found in QCD. The pion amplitudes are unitarized by extra resonances arising from the strongly interacting $SU(3)_c$. Nevertheless, this Higgsless approach has to face the present electroweak precision test (EWPT) and, in its simple incarnation, technicolor models, it fails to pass them. The reason is that the new resonances responsible for unitarizing the SM amplitudes have masses at around 1 TeV and give large (tree-level) contribution to electroweak observables that have not been observed.

A second option arises from the Higgs mechanism. The presence of a scalar Higgs cures the SM amplitudes from the bad high-energy behaviour and, therefore, allow the SM to be extrapolated to very high energies. It is hard to believe that nature is not using such a simple mechanism to give us a UV completed theory of electroweak interactions. Nevertheless, naturalness criteria, stop us from considering the Higgs mechanism as the last ingredient to be incorporated to the SM at the electroweak scale. Why the Higgs mass, that determines the electroweak scale, is so small compare with, for example, the Planck scale? If we want to answer this question, we must postulate new dynamics at the electroweak scale. For example supersymmetry can give an explanation to the smallness of the electroweak scale. Nevertheless, no super-partner of the SM particles have been found at the present colliders making the supersymmetic solution less and less natural.

A third possibility that has received a big boost in the recent years is the composite Higgs idea. Similarly as pions in QCD, the Higgs is assumed to be a composite particle and therefore not suffering from any naturalness problem. The role of this Higgs is, however, less ambitious than its original motivation. Since the Higgs is composite, its coupling to the SM particles will be different from those of a point-like scalar. Therefore the SM amplitudes are only partly unitarized by the composite Higgs and extra resonances must be present in these models to completely unitarize the SM amplitudes. At this point, one could ask: What are we gaining? Well, the presence of a composite Higgs makes the SM viable up to a higher energy than in models without a Higgs, implying that the extra resonances can be heavier. Hence, their effects on the EW observables are smaller and these models can be able to safely pass all the EWPT.

In the first models of composite Higgs [1] the Higgs appeared as a Pseudo-Goldstone boson (PGB) from a strong interacting theory, very similar to pions in QCD. These first proposals, however, lacked several ingredients. First, it did not incorporate a heavy top (since its mass was not known at that time). The authors of Ref. [1] had to enlarge the SM gauge group to obtain EWSB. Second, the contribution to EW observables were not calculated due, mainly, to the strong regime of the theory. Also flavor was also not successfully incorporated. Recently there has been various attempts to realize the composite Higgs idea avoiding the above problems. This includes the Little Higgs [2], Holographic Higgs [3,4] and other variations.

Here we want to study the general properties and the phenomenology of scenarios in which a light Higgs is associated with strong dynamics at a higher scale, focusing on features that are quite independent of the particular model realization [5]. We will refer to this scenario as to the Strongly-Interacting Light Higgs (SILH). Of course, in many specific models, the best experimental signals will be provided by direct production of new states, while here we concentrate on deviations from SM properties in Higgs and longitudinal gauge boson processes. Still, we believe that our model-independent approach is useful. The tests we propose here on Higgs and gauge-boson interactions will help, in case of new discoveries, to establish if the new particles indeed belong to a strongly-interacting sector ultimately responsible for electroweak symmetry breaking. If no new states are observed, or if the resonances are too broad to be identified, then our tests can be used to investigate whether the Higgs is weakly coupled or is an effective particle emerging from a strongly-interacting sector, whose discovery has been barely missed by direct searches at the LHC.

2 The structure of SILH

2.1 Definition of SILH

The structure of the theories we want to consider is the following. In addition to the vector bosons and fermions of the SM, there exists a new sector responsible for EW symmetry breaking, which is broadly characterized by two parameters, a coupling g_ρ and a scale m_ρ describing the mass of heavy physical states. Collectively indicating by g_{SM} the SM gauge and Yukawa couplings (basically the weak gauge coupling and the top quark Yukawa), we assume $g_{SM} \lesssim g_\rho \lesssim 4\pi$. The upper bound on g_ρ ensures that the loop expansion parameter $\sim (g_\rho/4\pi)^2$ is less than unity, while the limit $g_\rho \sim 4\pi$ corresponds to a maximally strongly-coupled theory in the spirit of naive dimensional analysis (NDA). Because of the first inequality, by a slight abuse of language, we shall refer to the new sector as "the strong sector". The Higgs multiplet is assumed to belong to the strong sector. The SM vector bosons and fermions are weakly coupled to the strong sector by means of the $SU(3)\times SU(2)\times U(1)_Y$ gauge coupling and by means of proto-Yukawa interactions, namely interactions that in the low-energy effective field theory will give rise to the SM Yukawas.

A second crucial assumption we are going to make is that in the limit $g_{SM}=0$, $g_\rho\neq 0$ the Higgs doublet H is an exact Goldstone boson, living in the $\mathcal{G}/\mathcal{H}$ coset space of a spontaneously broken symmetry of the strong sector. Two minimal possibilities in which the complex Higgs doublet spans the whole coset space are $SU(3)/SU(2)\times U(1)$ and the custodially symmetric $SO(5)/SO(4)$. The σ-model scale f will be assumed to be related to g_ρ and m_ρ by the equation

$$m_\rho = g_\rho f\,. \tag{1}$$

The gauging of $SU(2)\times U(1)_Y$ and the non-zero Yukawas explicitly break the Goldstone symmetry of the strong sector leading to terms in the (effective) action that are not invariant under the action of $\mathcal{G}$ on the coset space. In particular a mass term for the Higgs is generated at 1-loop. If the new dynamics is addressing the hierarchy problem, it should soften the sensitivity of the Higgs mass to short distances, that is to say below $1/m_\rho$. In interesting models, the Higgs mass parameter is thus expected to scale like $(\alpha_{SM}/4\pi)m_\rho^2$. Observation at the LHC of the new states with mass m_ρ will be the key signature of the various realizations of SILH. Here, as stated before, we are interested in the model-independent effects, which could be visible in processes involving the Higgs boson and/or longitudinal gauge bosons, and which would unmistakably reveal new physics in the electroweak breaking sector.

2.2 The SILH effective Lagrangian

Below m_ρ, the field content of these theories consists in the SM particles plus the Higgs. Deviations from the SM are encoded in the higher dimensional operators of the low-energy effective lagrangian. The dimension-6 operators involving the Higgs field can be separated in three parts, depending on the origin of the operators:

1. At tree-level $1/f^2$ order:

$$\frac{c_H}{2f^2}\partial^\mu\left(H^\dagger H\right)\partial_\mu\left(H^\dagger H\right)+\frac{c_T}{2f^2}\left(H^\dagger\overleftrightarrow{D^\mu}H\right)\left(H^\dagger\overleftrightarrow{D}_\mu H\right)$$
$$-\frac{c_6\lambda}{f^2}\left(H^\dagger H\right)^3+\left(\frac{c_y y_f}{f^2}H^\dagger H\bar f_L H f_R+\text{h.c.}\right) \tag{2}$$

2. At tree-level $1/m_\rho^2$ order :

$$\frac{ic_W g}{2m_\rho^2}\left(H^\dagger\sigma^i\overleftrightarrow{D^\mu}H\right)(D^\nu W_{\mu\nu})^i+\frac{ic_B g'}{2m_\rho^2}\left(H^\dagger\overleftrightarrow{D^\mu}H\right)(\partial^\nu B_{\mu\nu}) \tag{3}$$

3. At one-loop order:

$$\frac{ic_{HW}g}{16\pi^2f^2}(D^\mu H)^\dagger\sigma^i(D^\nu H)W^i_{\mu\nu}+\frac{ic_{HB}g'}{16\pi^2f^2}(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu}$$
$$+\frac{c_\gamma g'^2}{16\pi^2f^2}\frac{g^2}{g_\rho^2}H^\dagger HB_{\mu\nu}B^{\mu\nu}+\frac{c_g g_S^2}{16\pi^2f^2}\frac{y_t^2}{g_\rho^2}H^\dagger HG^a_{\mu\nu}G^{a\mu\nu}. \tag{4}$$

The coupling constants c_i are pure numbers of order unity. For phenomenological applications, we have switched to a notation in which gauge fields are canonically normalized, and gauge couplings explicitly appear in covariant derivatives. Also, we recall the definition $H^\dagger\overleftrightarrow{D}_\mu H\equiv H^\dagger D_\mu H-(D_\mu H^\dagger)H$.

The first class of operators (those of order $1/f^2$) are the most sizable. Among them only the operator proportional to c_T is constrained from the experimental data. Since it violates the custodial symmetry, it gives a contribution $\widehat{T}$ to the ρ parameter

$$\Delta\rho \equiv \widehat{T} = c_T\,\xi, \tag{5}$$

$$\xi \equiv \frac{v^2}{f^2}, \quad v = \left(\sqrt{2}G_F\right)^{-1/2} = 246\,\mathrm{GeV}. \tag{6}$$

From the SM fit of electroweak data [6], we find $-1.1\times10^{-3} < c_T\xi < 1.3\times10^{-3}$ at 95% CL (letting also $\widehat{S}$ to vary one finds instead $-1.7\times10^{-3} < c_T\xi < 1.9\times10^{-3}$ at 95% CL). This strong limit on c_T suggests that new physics relevant for electroweak breaking must be approximately custodial-invariant. In our Goldstone Higgs scenario this corresponds to assuming the coset $SO(5)/SO(4)$. When g_{SM} is turned on, c_T receives a model dependent contribution, which should be small enough to make the model acceptable. The rest of the coefficient c_H, c_y and c_6 are practically unconstrained and their implications will be discussed in the next section.

A linear combination of the operators with coefficients c_W and c_B contributes to the $\widehat{S}$ parameter of electroweak precision data:

$$\widehat{S} = (c_W + c_B)\,\frac{m_W^2}{m_\rho^2}, \tag{7}$$

where $\widehat{S}$ is defined in ref. [6]. Using the SM fit of electroweak data [6], we obtain the bound $m_\rho \gtrsim (c_W+c_B)^{1/2}$ 2.5 TeV at 95% CL. (this bound corresponds to assuming a light Higgs and $\Delta\rho \equiv \widehat{T} = 0$; by relaxing this request the bound becomes $m_\rho \gtrsim (c_W+c_B)^{1/2}$ 1.6 TeV). In terms of the parameter ξ defined in eq. (6), this bound becomes

$$\xi \lesssim \frac{1.5}{c_W + c_B}\left(\frac{g_\rho}{4\pi}\right)^2. \tag{8}$$

As we show later, new effects in Higgs physics at the LHC appear only for sizable values of ξ. Then eq. (8) requires a rather large value of g_ρ, unless $c_W + c_B$ happens to be accidentally small.

The operators with coefficients c_{HW} and c_{HB} originate from the 1-loop action $\mathcal{L}^{(1)}$, under our assumption of minimal coupling for the classical action. Although they are H^2D^4 terms, like c_W, c_B, they cannot be enhanced above their 1-loop size by the exchange of any spin 0 or 1 massive field. The operators proportional to c_γ and c_g are suppressed by an extra power $(g_{SM}/g_\rho)^p$ with respect to those proportional to c_{HW} and c_{HB}. Moreover, while c_H and c_y indirectly correct the physical Higgs coupling to gluons and quarks by $O(v^2/f^2)$ with respect to the SM, the direct contribution of c_γ and c_g is of order $(v^2/f^2)(g_{SM}/g_\rho)^p$. Their effect is then important only in the weakly coupled limit $g_\rho \sim g_{SM}$. Notice that from the point of view of the Goldstone symmetry, O_{BB} and O_g are like a Higgs mass term with extra field strength insertions. In the simplest models $m_H^2 \sim (g_{SM}^2/16\pi^2)m_\rho^2$. We have here assumed this simplest possibility, which accounts for the extra g_{SM}^2/g_ρ^2 appearing in eq. (4). More precisely, for phenomenological purposes, we have chosen g_{SM} as the coupling of the largest contribution in the corresponding SM loop, *i.e.*, $g_{SM}^2 = g^2$ (y_t^2) for the operator involving photons (gluons), respectively.

3 Phenomenology of SILH

In this section we analyze the effects of the SILH interactions and study how they can be tested at future colliders. Let us start by considering the new interaction terms involving the physical Higgs boson. For simplicity, we work in the unitary gauge and write the SILH effective Lagrangian in eqs. (2)-(4) only for the real Higgs field h (shifted such that $\langle h\rangle = 0$). We reabsorb the contributions from c_f and c_6 to the SM input parameters (fermion masses m_f, Higgs mass m_H, and vacuum expectation value $v = 246\,\mathrm{GeV}$). Similarly, we redefine the gauge fields and the gauge coupling constants and we make the gauge kinetic terms canonical. In this way, the SILH effective Lagrangian is composed by the usual SM part, written in terms of the usual SM input parameters (physical masses and gauge couplings), by new Higgs interactions ($\mathcal{L}_h$), and new interactions involving only gauge bosons ($\mathcal{L}_V$) which, at leading order, are given by

$$\mathcal{L}_h = \xi\,\Bigg\{\frac{c_H}{2}\left(1+\frac{h}{v}\right)^2\partial^\mu h\partial_\mu h - c_6\frac{m_H^2}{2v^2}\left(vh^3+\frac{3h^4}{2}+\dots\right)+c_y\frac{m_f}{v}\bar f f\left(h+\frac{3h^2}{2v}+\dots\right)$$
$$+\left(\frac{h}{v}+\frac{h^2}{2v^2}\right)\Bigg[\frac{g^2}{2g_\rho^2}\left(\hat c_W W_\mu^- \mathcal{D}^{\mu\nu}W_\nu^+ + \mathrm{h.c.}\right)+\frac{g^2}{2g_\rho^2}Z_\mu\mathcal{D}^{\mu\nu}\left[\hat c_Z Z_\nu+\left(\frac{2\hat c_W}{\sin 2\theta_W}-\frac{\hat c_Z}{\tan\theta_W}\right)A_\nu\right]$$
$$-\frac{g^2}{(4\pi)^2}\left(\frac{c_{HW}}{2}W^{+\mu\nu}W^-_{\mu\nu}+\frac{c_{HW}+\tan^2\theta_W c_{HB}}{4}Z^{\mu\nu}Z_{\mu\nu}-2\sin^2\theta_W\, c_{\gamma Z}F^{\mu\nu}Z_{\mu\nu}\right)+\dots$$
$$+\frac{\alpha g^2 c_\gamma}{4\pi g_\rho^2}F^{\mu\nu}F_{\mu\nu}+\frac{\alpha_s y_t^2 c_g}{4\pi g_\rho^2}G^{a\mu\nu}G^a_{\mu\nu}\Bigg]\Bigg\} \tag{9}$$

$$\hat c_W = c_W + \left(\frac{g_\rho}{4\pi}\right)^2 c_{HW}\,,\;\hat c_Z = \hat c_W + \tan^2\theta_W\left[c_B+\left(\frac{g_\rho}{4\pi}\right)^2 c_{HB}\right]\,,\; c_{\gamma Z}=\frac{c_{HB}-c_{HW}}{4\sin 2\theta_W} \tag{10}$$

$$\mathcal{L}_V = -\frac{\tan\theta_W}{2}\widehat S\, W^{(3)}_{\mu\nu}B^{\mu\nu} - ig\cos\theta_W\, g_1^Z Z^\mu\left(W^{+\nu}W^-_{\mu\nu}-W^{-\nu}W^+_{\mu\nu}\right)$$
$$-ig\left(\cos\theta_W\,\kappa_Z Z^{\mu\nu}+\sin\theta_W\,\kappa_\gamma A^{\mu\nu}\right)W_\mu^+W_\nu^- \tag{11}$$

$$\widehat S = \frac{m_W^2}{m_\rho^2}(c_W+c_B)\,,\; g_1^Z=\frac{m_Z^2}{m_\rho^2}\,\hat c_W\,,\;\kappa_\gamma=\frac{m_W^2}{m_\rho^2}\left(\frac{g_\rho}{4\pi}\right)^2(c_{HW}+c_{HB})\,,\;\kappa_Z = g_1^Z-\tan^2\theta_W\,\kappa_\gamma. \tag{12}$$

In $\mathcal{L}_V$ we have included only trilinear terms in gauge bosons and dropped the effects of O_{2W}, O_{2B}, O_{3W}. In $\mathcal{L}_h$ we have kept only the first powers in the Higgs field h and the gauge fields. We have defined $W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu$ (and similarly for the Z_μ and the photon A_μ) and $\mathcal{D}_{\mu\nu} = \partial_\mu\partial_\nu - \Box g_{\mu\nu}$. Notice that for on-shell gauge bosons $\mathcal{D}_{\mu\nu}A^{\mu i} = M_i^2 A^i_\nu$. Therefore $\hat c_W$ and $\hat c_B$ generate a Higgs coupling to gauge bosons which is proportional to mass, as in the SM, and do not generate any Higgs coupling to photons.

The new interactions in $\mathcal{L}_h$, see eq. (9), modify the SM predictions for Higgs production and decay. At quadratic order in h, the coefficient c_H generates an extra contribution to the Higgs kinetic term. This can be reabsorbed by redefining the Higgs field according to $h\to h/\sqrt{1+\xi c_H}$. The effect of c_H is then to renormalize by a factor $1-\xi c_H/2$, the couplings of the canonical field h to all other fields. We can express the modified Higgs couplings in terms of the decay widths in units of the SM prediction, expressed in terms of physical pole masses,

$$\Gamma\left(h\to f\bar f\right)_{\mathrm{SILH}} = \Gamma\left(h\to f\bar f\right)_{\mathrm{SM}}\left[1-\xi\left(2c_y+c_H\right)\right] \tag{13}$$

$$\Gamma\left(h\to W^+W^-\right)_{\mathrm{SILH}} = \Gamma\left(h\to W^+W^{(*)-}\right)_{\mathrm{SM}}\left[1-\xi\left(c_H-\frac{g^2}{g_\rho^2}\hat c_W\right)\right] \tag{14}$$

$$\Gamma\left(h\to ZZ\right)_{\mathrm{SILH}} = \Gamma\left(h\to ZZ^{(*)}\right)_{\mathrm{SM}}\left[1-\xi\left(c_H-\frac{g^2}{g_\rho^2}\hat c_Z\right)\right] \tag{15}$$

$$\Gamma\left(h\to gg\right)_{\mathrm{SILH}} = \Gamma\left(h\to gg\right)_{\mathrm{SM}}\left[1-\xi\,\mathrm{Re}\left(2c_y+c_H+\frac{4y_t^2c_g}{g_\rho^2 I_g}\right)\right] \tag{16}$$

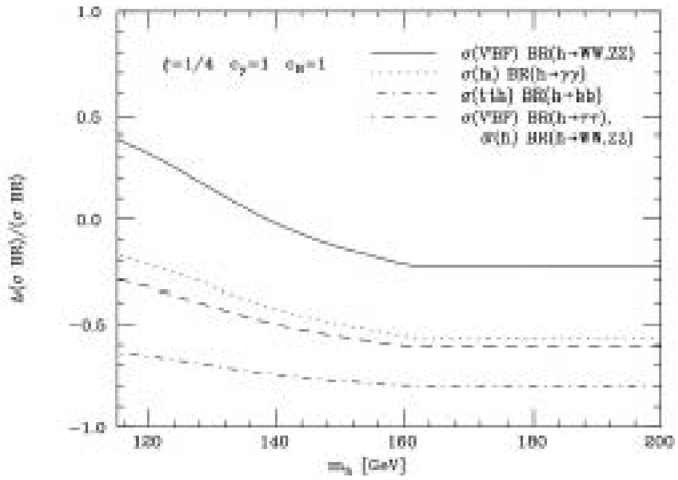

Figure 1: The deviations from the SM predictions of Higgs production cross sections (σ) and decay branching ratios (BR) defined as $\Delta(\sigma\ BR)/(\sigma\ BR) = (\sigma\ BR)_{\rm SILH}/(\sigma\ BR)_{\rm SM} - 1$. The predictions are shown for some of the main Higgs discovery channels at the LHC with production via vector-boson fusion (VBF), gluon fusion (h), and topstrahlung (tth). The SILH Lagrangian parameters are set by $c_H\xi = 1/4$, $c_y/c_H = 1$.

$$\Gamma\left(h\to\gamma\gamma\right)_{\rm SILH} = \Gamma\left(h\to\gamma\gamma\right)_{\rm SM}\left[1-\xi\ {\rm Re}\left(\frac{2c_y+c_H}{1+J_\gamma/I_\gamma}+\frac{c_H-\frac{g^2}{g_\rho^2}\hat{c}_W}{1+I_\gamma/J_\gamma}+\frac{\frac{4g^2}{g_\rho^2}c_\gamma}{I_\gamma+J_\gamma}\right)\right] \tag{17}$$

$$\Gamma\left(h\to\gamma Z\right)_{\rm SILH} = \Gamma\left(h\to\gamma Z\right)_{\rm SM}\left[1-\xi\ {\rm Re}\left(\frac{2c_y+c_H}{1+J_Z/I_Z}+\frac{c_H-\frac{g^2}{g_\rho^2}\hat{c}_W}{1+I_Z/J_Z}+\frac{4c_{\gamma Z}}{I_Z+J_Z}\right)\right]. \tag{18}$$

Here we have neglected in $\Gamma(h\to W^+W^-, ZZ)_{\rm SILH}$ the subleading effects from c_{HW} and c_{HB}, which are parametrically smaller than a SM one-loop contribution. The loop functions I and J are given in Ref. [5].

The leading effects on Higgs physics, relative to the SM, come from the three coefficients c_H, c_y, $c_{\gamma Z}$, although $c_{\gamma Z}$ has less phenomenological relevance since it affects only the decay $h\to\gamma Z$. Therefore, we believe that an important experimental task to understand the nature of the Higgs boson will be the extraction of c_H and c_y from precise measurements of the Higgs production rate (σ_h) and branching ratios (BR_h). The contribution from c_H is universal for all Higgs couplings and therefore it does not affect the Higgs branching ratios, but only the total decay width and the production cross section. The measure of the Higgs decay width at the LHC is very difficult and it can be reasonably done only for rather heavy Higgs bosons, well above the two gauge boson threshold, while the spirit of our analysis is to consider the Higgs as a pseudo-Goldstone boson, and therefore relatively light. However, for a light Higgs, LHC experiments can measure the product $\sigma_h \times BR_h$ in many different channels: production through gluon, gauge-boson fusion, and top-strahlung; decay into b, τ, γ and (virtual) weak gauge bosons. At the LHC with about 300 fb^{-1}, it is possible to measure Higgs production rate times branching ratio in the various channels with 20–40 % precision [7], although a determination of the b coupling is quite challenging [8]. This will translate into a sensitivity on $|c_H\xi|$ and $|c_y\xi|$ up to 0.2–0.4.

In fig. 1, we show our prediction for the relative deviation from the SM expectation in the main channels for Higgs discovery at the LHC, in the case $c_H\xi = 1/4$ and $c_y/c_H = 1$ (as in the Holographic Higgs). For $c_y/c_H = 0$, the deviation is universal in every production channel and is given by $\Delta(\sigma\ BR)/(\sigma\ BR) = -c_H\xi$.

Cleaner experimental information can be extracted from ratios between the rates of processes with the same Higgs production mechanism, but different decay modes. In measurements of these ratios of decay rates, many systematic uncertainties drop out. Our leading-order ($g_\rho \gg g_{SM}$) prediction is that $\Delta[\Gamma(h \to ZZ)/\Gamma(h \to W^+W^-)] = 0$, $\Delta[\Gamma(h \to f\bar{f})/\Gamma(h \to W^+W^-)] = -2\xi c_y$, $\Delta[\Gamma(h \to \gamma\gamma)/\Gamma(h \to W^+W^-)] = -2\xi c_y(1 + J_\gamma/I_\gamma)^{-1}$. However, the Higgs coupling determinations at the LHC will still be limited by statistics, and therefore they can benefit from a luminosity upgrading, like the SLHC. At a linear collider, like the ILC, precisions on $\sigma_h \times BR_h$ can reach the percent level [9], providing a very sensitive probe on the new-physics scale. Moreover, a linear collider can test the existence of c_6, since the triple Higgs coupling can be measured with an accuracy of about 10% for $\sqrt{s} = 500$ GeV and an integrated luminosity of 1000 fb^{-1} [10].

Deviations from the SM predictions of Higgs production and decay rates, could be a hint towards models with strong dynamics, especially if no new light particles are discovered at the LHC. However, they do not unambiguously imply the existence of a new strong interaction. The most characteristic signals of a SILH have to be found in the very high-energy regime. Indeed, a peculiarity of SILH is that, in spite of the light Higgs, longitudinal gauge-boson scattering amplitudes grow with energy and the corresponding interaction becomes strong, eventually violating tree-level unitarity at the cutoff scale. Indeed, the extra Higgs kinetic term proportional to $c_H\xi$ in eq. (9) prevents Higgs exchange diagrams from accomplishing the exact cancellation, present in the SM, of the terms growing with energy in the amplitudes. Therefore, although the Higgs is light, we obtain strong WW scattering at high energies.

From the operator $O_H \equiv \partial^\mu(H^\dagger H)\partial_\mu(H^\dagger H)$ in eq. (2), using the equivalence theorem [11], it is easy to derive the following high-energy limit of the scattering amplitudes for longitudinal gauge bosons

$$\mathcal{A}\left(Z_L^0 Z_L^0 \to W_L^+ W_L^-\right) = \mathcal{A}\left(W_L^+ W_L^- \to Z_L^0 Z_L^0\right) = -\mathcal{A}\left(W_L^\pm W_L^\pm \to W_L^\pm W_L^\pm\right) = \frac{c_H s}{f^2}, \qquad (19)$$

$$\mathcal{A}\left(W^\pm Z_L^0 \to W^\pm Z_L^0\right) = \frac{c_H t}{f^2}, \qquad \mathcal{A}\left(W_L^+ W_L^- \to W_L^+ W_L^-\right) = \frac{c_H (s+t)}{f^2}, \qquad (20)$$

$$\mathcal{A}\left(Z_L^0 Z_L^0 \to Z_L^0 Z_L^0\right) = 0. \qquad (21)$$

This result is correct to leading order in s/f^2, and to all orders in ξ in the limit $g_{SM} = 0$, when the σ-model is exact. The absence of corrections in ξ follows from the non-linear symmetry of the σ-model, corresponding to the action of the generator $T_{\hat{h}}$, associated with the neutral Higgs, under which v shifts. Therefore we expect that corrections can arise only at $\mathcal{O}(s/m_\rho^2)$. The growth with energy of the amplitudes in eqs. (19)–(21) is strictly valid only up to the maximum energy of our effective theory, namely m_ρ. The behaviour above m_ρ depends on the specific model realization. In the case of the Little Higgs, we expect that the amplitudes continue to grow with s up to the cut-off scale Λ. In 5D models, like the Holographic Goldstone, the growth of the elastic amplitude is softened by KK exchange, but the inelastic channel dominate and strong coupling is reached at a scale $\sim 4\pi m_\rho/g_\rho$. Notice that the result in eqs. (19)–(21) is exactly proportional to the scattering amplitudes obtained in a Higgsless SM [11]. Therefore, in theories with a SILH, the cross section at the LHC for producing longitudinal gauge bosons with large invariant masses can be written as

$$\sigma\left(pp \to V_L V_L' X\right)_{c_H} = \left(c_H \xi\right)^2 \sigma\left(pp \to V_L V_L' X\right)_{\not{H}}, \qquad (22)$$

where $\sigma(pp \to V_L V_L' X)_{\not{H}}$ is the cross section in the SM without Higgs, at the leading order in $s/(4\pi v)^2$. With 200 fb^{-1} of integrated luminosity, it should be possible to identify the signal of a Higgsless SM with about 30–50% accuracy [12,13], i.e., to a sensitivity up to $c_H\xi \simeq 0.5$–0.7.

In the SILH framework, the Higgs is viewed as a pseudo-Goldstone boson and therefore its properties are directly related to those of the exact (eaten) Goldstones, corresponding to the longitudinal gauge bosons. Thus, a generic prediction of SILH is that the strong gauge boson scattering is accompanied by strong production of Higgs pairs. Indeed we find that, as a consequence of the $O(4)$ symmetry of the H multiplet, the amplitudes for Higgs pair-production grow with the center-of-mass energy as eq. (19),

$$\mathcal{A}\left(Z_L^0 Z_L^0 \to hh\right) = \mathcal{A}\left(W_L^+ W_L^- \to hh\right) = \frac{c_H s}{f^2}. \tag{23}$$

Notice that scattering amplitudes involving longitudinal gauge bosons and a single Higgs vanish. This is a consequence of the Z_2^4 parity embedded in the $O(4)$ symmetry of the operator O_H, under which each Goldstone change sign. Non-vanishing amplitudes necessarily involve an even number of each species of Goldstones.

Using eqs. (19), (20) and (23), we can relate the Higgs pair production rate at the LHC to the longitudinal gauge boson cross sections

$$2\sigma_{\delta,M}\left(pp \to hhX\right)_{c_H} = \sigma_{\delta,M}\left(pp \to W_L^+ W_L^- X\right)_{c_H} + \frac{1}{6}\left(9 - \tanh^2\frac{\delta}{2}\right)\sigma_{\delta,M}\left(pp \to Z_L^0 Z_L^0 X\right)_{c_H}. \tag{24}$$

Here all cross sections $\sigma_{\delta,M}$ are computed with a cut on the pseudorapidity separation between the two final-state particles (a boost-invariant quantity) of $|\Delta\eta| < \delta$, and with a cut on the two-particle invariant mass $\hat{s} > M^2$. The sum rule in eq. (24) is a characteristic of SILH. However, the signal from Higgs-pair production at the LHC is not so prominent. It was suggested that, for a light Higgs, this process is best studied in the channel $b\bar{b}\gamma\gamma$ [14], but the small branching ratio of $h \to \gamma\gamma$ makes the SILH rate unobservable. However, in SILH, one can take advantage of the growth of the cross section with energy. Although we do not perform here a detailed study, it may be possible that, with sufficient luminosity, the signal of $b\bar{b}b\bar{b}$ with high invariant masses could be distinguished from the SM background.

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