Paper presented at the 13th Int. Conf. on General Relativity and Gravitation Cordoba, Argentina, 1992: Part 2, Workshop Summaries

Quantum field theory in curved space–time

Mario Castagnino
Instituto de Astronomía y Física del Espacio
Casilla de Correo 67- Sucursal 28, 1428 Buenos Aires- Argentina

When we began to study, in the seventies, Quantum Field Theory in Curved Spacetime (QFTCST), we thought that we were constructing a fundamental theory, encompassing General Relativity and Quantum Field Theory (QFT). It was soon evident that QFTCST is only the semiclassical approximation to Quantum Gravity (QG) a yet unknown theory. After 20 years, normally a physical theory, if it is not fundamental, decays and dies. On the contrary, QFTCST is very much alive, as it is proved by the almost 50 abstracts submitted to this workshop. This is so because many people are trying to define and develope QFTCST in very interesting and important cases, such as Gott's space or non globally hyperbolic spaces etc. (as we shall see in section 1), because QFTCST is useful to study the quantum nature of Black Holes (BH) (as we shall see in section 2) and because this formalism is an esential tool for Quantum Cosmology; like in inflationary models (as we shall see in section 3).

So let us review the main topics of the Workshop, where we will find interesting contributions to all this of these lines of research.

1. The General Theory

In his talk, David Boulware proposed a QFT in Spaces with closed time-like curves, the Gott's spacetime, whith no anomalous stress- energy tensor. In the special case of a total deficit angle of $\frac{2}{\pi}$, it is possible to find a complete orthonormal set of eigenfunctions of the wave operator. From these, the QFT is constructed. The resultant interacting QFT is not unitary, because the field operators can create real, on-shell, particles, in the acausal region, which propagate for finite proper time accumulating an arbitrary phase, before being annihilated at the same spacetime point. As a result, the effective potential within the acausal region is complex and probability is not conserved.

Wai- Mo Suen posed the following question: "are perturbative constraints necessary in semi-classical gravity?" In fact, recently Simon and Parker proposed that those solutions of the semiclassical Einstein equation, which are not perturbatively expandable in powers of the Planck's constant, should be disregarded. This would then avoid the instability of flat space, and exclude pathological solutions of the semi-classical Einstein equation. Suen argued that such a restriction might not be necessary (at least for models involving only free quantum fields in which the semi-classical theory is exact to all matter loops),based on: (i) although an exact flat space is unstable with respect to infinitesimal perturbations, Robertson-Walker spacetimes with arbitrarily small(but finite) Hubble expansion are stable in semi-classical gravity, and (ii) for a

© 1993 IOP Publishing Ltd

range of renormalization parameters, solutions of the semi-classical equation starting out dominated by the higher derivative terms in the equations (hence locally not perturbations of classical solutions) are automatically driven towards classical solutions by the backreaction from quantum fields. Hence the existence of our present universe is not necessarily in contradiction to the " unrestricted semi-classical theory".

Juan Pablo Paz raised some criticisms about this talk because, he said, Simon and Parker proposal was not well interpreted by Suen.

Sung-Won Kim introduced another question about stability: "are time machines unstable by the particle production?" Kim and Thorne tried to calculate the vacuum fluctuation of quantized fields [1]. It was shown that the vacuum fluctuations produce a renormalized stress-energy tensor (SET) that diverges as one approaches the Cauchy horizon, which will be cut off by QG. However, there is a controversy. Hawking [2] conjectures an observer-independent location for the breakdown in the semiclassical theory. In his talk, Kim studied this quantum stability problem using another method, namely particle production by an arbitrary gravitational field. When the wormhole forms in the infinit past, the result is finite, while it is divergent near the Cauchy horizon when the wormhole forms at a finite time. If we adopt Kim- Thorne's conjecture, then the divergence can be cut off by QG ; therefore, the total energy cannot prevent the formation of the closed timelike curves when one is within a Planck length.

Finally, Tevian Dray, Corinne Manogue and Robin W. Tucker studied an interesting problem both related to QFTCST and QG. They consider the metric $ds^2 = f(t)dt^2 + g(t)dx^2$ where g is everywhere positive and f has two roots, at both of which it changes sign, and where furthermore $f(t) < 0$ for $|t|$ sufficiently large. This corresponds to a spatially symmetric space-time which is initially and finally Lorentzian, with a Euclidean region in between. Since the massless scalar wave equation is conformally invariant in two dimensions, and since all 2-dimensional surfaces are conformally flat, it is easy (in principle) to solve the wave equation in any region of constant signature. The issue is how to match solutions when the signature changes. Specificaly, is there a physically reasonable prescription for matching solutions of the wave equation to solutions of Laplace's equation so that the resulting picture can be reasonably described as propagation?

A basic property of the usual theory of the scalar field, related to unitarity, is the existence of a conserved product on solutions, namely the Klein - Gordon product. Choosing the wave equation, so that there is a conserved Klein - Gordon product, implicitly determines the junction conditions one needs to impose in order to obtain global solutions. The resulting mix of positive and negative frequencies produced by the presence of Euclidean regions depends only on the total conformal width of the regions, and not on the detailed form of the metric. Calculating the change of basis (Bogolubov relations) between in and out plane-wave basis functions and ignoring some unimportant phase factors one finds that the resulting solutions satisfy [3].

$$u_k^{out} = cosh(k\Delta\tau)u_k^{in} + i\frac{|k|}{k}sinh(k\Delta\tau)\bar{u}_{-k}^{in}$$

where $\Delta\tau$ is the difference in conformal time τ between the two roots of f. It is interesting to note that this relation extends without modification to the case where there are several Euclidean regions; $\Delta\tau$ then denotes the total conformal width of the Euclidean regions.

The authors further show [4] that solutions and their canonical momenta are well behaved at (each side of) the boundaries between regions of constant signature even though the wave equation is not, so they propose a propagation rule on matching the canonical data at each boundary. The solutions obtained earlier satisfy this condition, and therefore solve the wave equation everywhere. This problem is related with the famous De Witt problem [5] and some related examples with the turnneling models [6][7].

In the poster section M. Ale and L. P. Chimento [8] presented an exact solution of the semiclassical Einstein equations and L. Rodriguez and F. D. Mazitelli studied the Quantum instability of Minkowsky spacetime. Both can be also considered contributions to the General Theory.

2. Stress - Energy Tensor and Black Holes

Paul Anderson, William Hiscock and David Samuel developed a method which allows for the computation of the exact expectation value of the quantum stress-energy tensor for free scalar fields in static spherically symmetric spacetimes. They presented some of their results for massless scalar fields with various couplings to the scalar curvature for Schwarzschild B Hs and various Reissner-Nordstrom B Hs. The ability to compute the exact expectation value of the quantum SET in arbitrary static spherically symmetric spacetimes allows then to address the semiclassical back-reaction problem in these spacetimes. The authors present a method for solving this problem for extreme Reissner-Nordstrom B H and a Schwarzschild B H in a box.

Larry Ford and Thomas Roman talked about constraints on negative energy fluxes seen by inertial observers falling into an evaporating B H. It is known that QFT allows violations of the classical energy conditions, in the form of locally negative energy densities and fluxes, if they are produced by quantum coherence effects. They must be followed by a more than compensating positive fux. Then QFT imposes restrictions on the magnitude and duration of the negative energy flux. In flat space time they obey "uncertainty-principle" type inequalities [10] of the form: $|E|(\Delta T) \leq h$. Here $|E|$ is the magnitude of the negative energy which can be absorbed in a time ΔT. More recently it has been shown by the authors [11] that similar inequalities hold for negative energy fluxes propagating on a $Q = M$ extreme, four-dimensional, Reissner-Nordstrom B H background. These inequalities prevent the unambiguous observation of the limited duration violation of Cosmic Censorship ("Cosmic Flashing"), which would otherwise occur before the arrival of the subsequent positive energy flux. An apparent counter-example to these "quantum inequalities" is the constant negative flux seen by a stationary observer near the B H. But such observer should also see acceleration (Unruh) radiation which mask the negative energy. For this reason, the authors choose to examine the negative energy flux seen by various inertial observes falling into a two-dimensional evaporating Schwarzschild B H. If there are no constraints on these fluxes, then the observers could (in principle) use them to produce, for example, gross violations of the second law of thermodynamics. A numerical analysis of the flux as a function of proper time indicates that here there exist quantum inequality-type restrictions on the magnitude and duration of the negative fluxes seen by inertial observers. That is, $|F|\tau^2 \leq h$, where $|F|$ is the magnitude of the negative flux and τ is its typical duration in proper time. In this case at least, quantum inequalities are satisfied by inertial observes in curved spacetime.Consequently, such observers should not see gross, macroscopic effects due to negative energy.

Ted Jacobson proposed to use B H as microscopes. In fact, when we look at Hawking radiation emerging from the vicinity of a B H, we are registering quanta in outoing quantum field modes. If we assume with Hawking [12] that the field is non-interacting, then these modes can be traced backwards in time to ingoing modes far from the hole at frequency $\sim \omega exp(t/2r_s)$ in the asymptotic rest frame of the hole. Here ω is the outgoing frequency,r_s is the Schwarzschild radius, and t is roughly the time interval between formation of the hole and reception of the quantum. After a time $t \sim 2r_s ln(\omega_{Planck/\omega})$, these corresponding ingoing modes exceed Planck frequency. He presented an alternative derivation for Hawking radiation, which avoids the role played by super-Planck frequency modes , imposing, in the frame of observers freely falling from asymptotic rest into the B H, that the Planck frequency modes are in the vacuum state. No assumption is made about the state of any other modes. Unruh [13] pointed out that this condition suffices to deduce the existence of Hawking radiation. It is shown that the emitted radiation is approximately thermal. The condition that the Planck frequency modes are in their ground state cannot be derived from any assumption regarding sub-Planckian physics in the free-falling frame. Thus Hawking radiation is a descendent of very short distance physics.

Roberto Camporesi and Atsushi Higuchi computed the SET for scalar and spinor fields with arbitrary mass in anti-de Sitter spacetime using the ξ -function technique. The results agree with those obtained by Pauli-Villars regularization. Also the trace anomaly for the Wess-Zumino model is studied in this space time and takes the "convencional" value. The same authors also calculated the spectral function $\mu(\lambda)$ and the zeta function $\xi(z)$ for a field of integer spin s on a N-dimensional (simply connected) hyperbolic space.

Finally M. Dorca and E. Verdaguer studied a Quantum Field in a Colliding wave Space-time.They study the quantization of a massless scalar field on a spacetime representing the head- on collision of two plane fronted gravitational waves. They consider a vacuum solution of Einstein's equations in which the two waves focus on a non-singular Killing-Cauchy horizon. The interaction region of the two waves is locally isometric to a region of the interior of a Schwarzschild black hole, with the Killing-Cauchy horizon corresponding to the event horizon of the black hole. In this case the colliding wave solution can be maximally extended through the Cauchy horizon, provided one of the transverse coordinates is made cyclic. The resulting spacetime represents the creation of a Schwarzschild B H by the collision of two plane waves propagating on a cylindrical universe [14]. That extension is, however, not essential for quantization of the scalar field.

In the colliding region two unambiguous and physically meaningful vacua can be defined. The"in" vacuum is defined through the positive mode solutions associated to the timelike Killing field in the flat region between the two plane waves before their collision, i.e. the ordinary flat spacetime vacuum. Also an "out" vacuum can be defined at the Cauchy horizon with the modes associated to the two null Killing fields of that horizon. These are easily identified by using a kind of Kruskal-Szekeres like coordinates to describe the interaction region; in these coordinates the metric looks flat at the Cauchy horizon. A particle detector in free fall near the horizon would not react if the quantum state is in the "out"vacuum. That vacuum corresponds to the Unruh vacuum [15] defined at the past horizon of the maximally extended Schwarzschild metric.

The authors propagate the "in" modes through - out the flat, plane wave and interaction region and compare with the "out" modes in the Cauchy horizon by computing the corresponding Bogoliubov coefficients. It is found that particles are created with a thermal spectrum, with a temperature that is proportional to the inverse of the focusing time of the plane waves, which is a measure of the energy of the waves. That is, the"in" vacuum contains a thermal distribution of "out" particles . The spectrum is not exactly thermal, in the sense that it depends on the momentum of the modes in one of the transversal directions rather than the frequency of the modes. This can be understood when comparing with B H radiation, in the sense that in the Schwarzschild B H the responsible for Hawking's radiation [12] is the infinite redshift at the horizon, whereas in the colliding wave problem it is due to the collapse of one of the transversal directions of the waves. The main result is exact even though the solution for the modes cannot be found exactly.

In the poster section a result on charged non-abelian BH solutions of the Einstein-Yang-Mills equations was also presented by D. V. Gal'tsov and M. S. Volkov [16].

3. Particle Production

Esteban Calzetta and Maria Sakellariadou showed how particle creation processes, that are normally used to explain the isotropy of the Universe, and the dissipation of its inhomogeneity, make also that inflationary models can be considered more natural than their Standard counterpart [17]. While the Standard Cosmological Model provides an appealing and so far unchallenged description of the evolution of the Universe from the Big Bang to its present configuration, it also raises the issue of the origin of the highly special initial conditions required for this kind of evolution. Inflationary models of the Universe have been suggested as an explanation for these initial conditions, but they too depend upon a kind of fine tuning of the initial conditions. Concretely, recent work has established that Inflation requires initial conditions to be homogeneous on scales in excess of one horizon length. This homogeneity cannot be explained through classical physical processes, and therefore, as far as classical cosmology is concerned, Inflationary models do not seem to be more "natural" than their Standard counterpart.

The possibility remains, that the picture changes when quantum cosmological effects are taken into account. For example, it is known that quantum particle creation processes could afford an explanation of the isotropy of the Universe, and in general would tend to favour dissipation of inhomogeneities. Thefore the authors explore the relevance of quantum effects to the onset of inflation, focussing on inhomogeneous but isotropic models with respect to a preferred point, and showed that these effects relax the requirements for succesful Inflation to the point where they become compatible with generic cosmic initial conditions.

Several contibutions were related to particle production in the poster section: L. P. Chimento, A.E. Cossarini and F. G. Pensa [18] studied a Spin 1 Field in Rindler Space and found a non-Planckian spectrum for the Rindler observer, A.Higuchi, A. Matsas and D. Sudarsky [19] presented a contribution about the Bremsstrahlung and Zero-energy Rindler Photons and U.Percoco, V.M.Villalba and P.Pujol [20] analyzed the vacuum effects asociated with the scalar and Dirac particles in non uniformly accelerated frames of reference.

Finally C. Loustó, J. J. Wheeler and G. Domenech, M. Levinas and N. Umérez presented interesting results in their posters [21], [22] and [23].

4. Conclusion

Unfortunately many authors that sent abstracts were absent at the meeting. Anyhow interesting subjects were analized and many interesting contributions were presented. But we were particulary impressed by three contributions, that we consider rather important: the study of David Boulware, about QFT in Gott's space time, was an excellent exploration of a new, difficult, and promising subject. The contribution of Paul Anderson, William Hiscock and David Samuel, about the exact computation of the expectation value of the SET, was a solution of an old and well known problem. The report of Esteban Calzetta and Maria Sakellariadou was an outstanding example of how QFTCST can help us to improve the inflationary model in the best tradition of this theory.

References

[1] Kim S. W., Thorne K. S., Phys Rev. D, **43**, 3929 (1991).
[2] Hawking S. W., Phys Rev. D, to be published.
[3] Dray T., Manogue C. A., Tucker R. W., Gen. Rel. Grav., **23**, 967 (1991).
[4] Dray T., Manogue C.A., Tucker R. W., The Effect of Signature Change on Scalar Field Propagation (in preparation)
[5] Anderson A., De Witt B., Found of Phys. **16**, 91 (1986)
[6] Hartle J. B., Hawking S. W., Phys Rev. D **28**, 2960 (1983)
[7] Vilenkin A., Phys. Lett. B **117**,25, (1982) and Phys. Rev., **27**, 2848 (1983)
[8] Ale M., Chimento L. P., Abstracts GR, 13, 575 (1992)
[9] Rodriguez L., Mazzitelli F. D., Abstracts GR 13, 612 (1992)
[10] Ford L. H., Phys. Rev. D., **43**, 3972 (1991)
[11] Ford L. H., Roman T. A., Phys. Rev. D, **41**, 3662 (1990)
[12] Hawking S. W., Nature **248**, 30 (1974) and Comm. Math. Phys., **43**, 199 (1975)
[13] Unruh W. G., Phys. Rev. D, **15**, 365 (1977)
[14] Yurtsever U., Phys. Rev. D, **40**, 360 (1989)
[15] Unruh W. G., Phys. Rev. D, **14**, 870 (1976)
[16] Gal'tsov D. V., Volkov M. S., Abstracts G 13, 596 (1992)
[17] Calzetta E., Sakellariadou M., Phys. Rev. D, **45**, 2802 (1992)
[18] Chimento L. P., Corssarini A. E., Pensa F. G., Abstracts GR 13, 588 (1992)
[19] Higuchi A., Matsas G. E. A., Sudarsky D., Abstracts GR 13, 601 (1992)
[20] Percoco U., Villalba V. M., Pujol P., Abstracts GR 13, 611 (1992)
[21] Loustó C. O., Abstracts GR 13, 607 (1992)
[22] Wheeler J. T., Abstracts GR 13, 614 (1992)
[23] Domenech G., Levinas M., Umérez N., "The ideal fluid as the classical limit of free quantum field and a model for an interacting fluid in Curved Space- Time", Preprint IAFE (1992)