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Numerical relativity

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In GR13 we heard many reports on recent progress as well as future plans of detection of gravitational waves. According to these reports (see the report of the workshop on the detection of gravitational waves by Paik in this volume), it is highly probable that the sensitivity of detectors such as laser interferometers and ultra low temperature resonant bars will reach the level of $h \sim 10^{-21}$ by 1998. In this level we may expect the detection of the gravitational waves from astrophysical sources such as coalescing binary neutron stars once a year or so. Therefore the progress in numerical relativity is urgently required to predict the wave pattern and amplitude of the gravitational waves from realistic astrophysical sources. The time left for numerical relativists is only six years or so although there are so many difficulties in principle as well as in practice.

Apart from detection of gravitational waves, numerical relativity itself has a final goal:

Solve the Einstein equations numerically for *any* initial data as accurately as possible and clarify physics in strong gravity.

In GR13 there were six oral presentations and 11 poster papers on recent progress in numerical relativity. I will make a brief review of six oral presentations. The Regge calculus is one of methods to investigate spacetimes numerically. Brewin from Monash University Australia presented a paper *Particle Paths in a Schwarzshild Spacetime via the Regge Calculus.* One of the merits in the Regge Calculus is that the metric interior to each pair of adjacent blocks is Minkowskian so that the computations of particle or photon orbits can be performed using only the rules of special relativity. Williams and Ellis[1] formulated how to compute

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particle paths in the Regge spacetime. Unfortunately the value they obtained for the precession of the perihelia of Mercury was at best 1800 times larger than the correct one. Brewin argued how to recover the correct value by developing the approach in the context of the Regge spacetime. The first point is that he uses the continuous time 3+1 approach, that is, he discritizes space while retaining a continuous time. Next he argues the convergence properties for several Schwarzschild geodesics in his method. The question here is : if the number of blocks (N) is increased (i.e. $N \to \infty$), will we obtain the correct geodesics? The answer is NO. If the error in each block is of order $O(h)$ where h is a linear size of a block the global error is of order unity because the total number of blocks is proportional to $1/h$. It became clear from this that the only way in which accurate paths could be obtained from the Regge calculus is to modify the Regge equations. He used a linear interpolation of the original equations so that the error in each block becomes $O(h^2)$. This guarantees that a global error is $O(h)$ which implies the convergence. As for advance of the periherion of Mercury he obtained the correct value.

Clarke from Southampton UK presented a paper entitled *Numerical Relativity in a Transputer Array* by A C W Garret, R A d'Inverno and C J S Clarke. They are now using a parallel processor in characteristic initial value problem with compactified equations in which $r = \infty$ corresponds to the finite value of a new coordinate z ,where r is the luminosity distance. For an axisymmetric problem all the quantity q is a function of $q(u, z, y = \cos\theta)$ where u is a time coordinate. Each processor corresponds to each value of y and they are linked by fast communication link. At present they are using 32 commercial (Parsy) array. The performance for vacuum problem with 500 time levels is 52 sec and in near future they will have 10 times faster array. They are also testing a code with matter. In this case they are working with Bishop[2] from University of South Africa who explained the method. The characteristic initial value problem is appropriate in vacuum but in the presence of matter it loses the advantage because the characteristics of matter do not coincide with those of the gravitational fields. So he uses usual 3+1 Cauchy problem within some distance R_+ which is larger than R_m where matter exists only for $R \leq R_m$. For the development of the gravitational fields outside R_+ the characteristic method will be adopted. So one must (1) construct the coordinate and metric for $R \geq R_+$ from Cauchy data and (2) obtain the boundary condition at R_+ for the Cauchy evolution from the results of the characteristic initial value problem. The scheme has difficulty at the start up stage. To calculate the null geodesics from $R = 0$ to $R = R_+$, Cauchy data at t=0 is not enough. The numerical implementation of the scheme is now developing.

Suen from Washington St.Louis reported the paper *Horizon Boundary Conditions in Numerical Relativity* with Seidel . One of the major problems in numerical relativity has been how to avoid the space-time singularities which exist from the beginning like in collisions of black holes or which are formed from the non- singular initial data. Many different types of singularity avoiding slicings have been proposed since in general relativity we have a freedom to choose time coordinate. However the question to the capability of long time integration of the space-time has not been answered. Suen and Seidel proposed a horizon locking coordinate to answer this problem. They choose a spatial coordinate in spherical symmetric space time such that the location of the apparent horizon has the constant coordinate value in time. After the horizon is locked all grid points are tied to it by requiring the radial metric function to be constant in time. In this case the problem is that the shift vector becomes so large that x^i=constant line may not be time like in an extreme case, which causes numerical instabilities. To avoid this instability they propose causal finite difference such that 1) Return to the zero shift coordinate and make the finite difference. 2) Transform the finite difference version of equations to coordinates with large shift vector. The final results are similar to the up-wind finite difference method in hydrodynamics which is numerically stable. They demonstrated the ability of long time integration of their method by evolving the Schwarzschild geometry with initial data of Einstein-Rosen bridge.

Numerical simulations of dynamical black hole systems (oscillating black holes and head-on two black hole collisions) were presented by D. Hobill and E. Seidel[3]
Both simulations are axi-symmetric and utilize maximal slicing conditions to

calculate the lapse function. A two component shift vector is introduced to control axis instabilities. The conditions imposed on the shift vector are: i) the three-metric is diagonal and ii) the shift vector components are determined from partial derivatives of a scalar function that is obtained from solving a linear second-order elliptic equation. The evolution is calculated using a leapfrog (with half time step extrapolation) finite difference technique and the elliptic equations for the lapse and shift are solved with a multigrid method. Essentially the codes are the same modulo initial data and boundary conditions, although a number of different gauge and coordinate choices have been tried for the 2 black hole collision. For the oscillating black hole, a Brill wave is superposed with a black hole and for the two black hole collision, Misner initial data is used.

The codes have been run on Cray-YMP, Cray-2 and NEC SX-3 supercomputers. The standard resolution involves 200 radial zones and 56 angular zones to cover one quadrant (equatorial symmetry is maintained in addition to axial symmetry). Various methods for analyzing these spacetimes have been developed. The Zerilli function (for the $\ell = 2$ and $\ell = 4$ modes) can be extracted and its propagation compared to analytic perturbation theory for quasi-normal mode generation. For low amplitude Brill waves the agreement is good to a percent or better. All of the Newman-Penrose spin coefficients and Weyl tensor components can be constructed as can various Bel-Robinson quantities. The mass loss rates can be calculated from the above quantities and they all agree to within numerical errors. The (quasi-local) ADM mass is measured at the outer boundary of the grid and the black hole mass can be measured by calculating the area of the apparent horizon or alternatively from measurements of the wavelength and damping factor of the quasi-normal mode wave functions. These black hole mass measurements are consistent with each other to within a few per cent. Furthermore, the difference between the ADM mass and the black hole mass can be accounted for in the energy loss associated with the emission of gravitational waves.

Computer graphics animations were presented for the simulations. For both large and small amplitude distortions of a single black hole the values of Ψ_0 and

Ψ_4 were tracked until late times (of order $100M$). The standard quadrupole waves were seen to dominate the perturbation case where a pure $\ell = 2$ Brill wave was introduced in the initial data. Nonlinear interactions between the $\ell = 2$ and $\ell = 4$ modes were evident in the high amplitude wave case. The two metric induced on the apparent horizon was embedded into a 3D flat space and a color map showing the local Gaussian curvature of the surface was used to track the incoming waves. The geometry of the horizon surface oscillates with the quasinormal frequency of the black hole. In the two black hole collision, it was shown where and when a second outer most trapped surface forms as the holes collide. This surface surrounds the original separated trapped surfaces associated with each hole, and then oscillates as in the case of a single, distorted black hole. It was also shown that for the parameters studied in the two black hole collision, that the nonlinearities were not as strong as those generated from highly distorted black holes. In fact all simulations to date seem to have gravitational waveforms that are clearly dominated by quasi-normal mode waveforms.

Schutz from Cardiff UK reported papers *An ADI Scheme for a Black Hole Problem* with Allen and *Time-Symmetric ADI and Causal Reconnection* with Alcubierre. He first pointed out various difficulties in calculating coalescing binary black hole in 3D. 1)One needs a quasi-rectangular 3D grid which can remain fixed at infinity and through which the holes move. This makes stringent requirement on the gauge and slicing conditions. 2) If black holes move through the grid, then grid points go down into a hole and then pop back out the other side. This popping out requires that the grid move faster than light, which causes numerical instability. 3)Coalescing black hole may begin with the holes relatively far apart so that it will take much longer time-steps compared with dynamical time. Since conventional explicit integration schemes are restricted by the Courant condition, one may like to use implicit method, which is less well known.

As for 2) they propose causal reconnection of grids. When a grid is moving faster than light, the grid point at the desired time-steps is outside the light cones of grid points at previous time steps. For a simple wave equation, the standard numerical schemes go unstable in such a situation. Their remedy for this is that they simply reformulate the computational grid so that its members are within each others's light cones. As for 3) they argue ADI(Alternating Direction Implicit) method on moving grids. They have found that all the standard ADI schemes for the simple wave equation go unstable on grids that move. They also noticed that none of the standard methods preserve the fundamental time-symmetry of the original equation. Requiring that the time-symmetry be maintained, they found a stable scheme for all grid speeds up to the speed of light. For grids moving faster than light, one can perform causal reconnection of the grids.

Nakamura reported the present status of the construction of a 3D code in Kyoto University, Japan. To construct fully GR 3D code is highly difficult. So his group divided the problem into easier problems. 1)In 1988 they constructed a fully 3D GR code in which only metric part of the Einstein equations is included (see T. Nakamura and K Oohara in Proceedings of GR12 p 61.). They used the Cartesian grids of 80^3 and showed that they could trace propagation of the $l=m=2$ quadrupole linear localized gravitational wave within a few percent error. 2) In 1989 Oohara and Nakamura[4] succeeded in determining initial data for coalescing binary neutron stars using the Cartesian Coordinate and ICCG (Incomplete Cholesky decomposition and Conjugate Gradient) method to solve coupled Poisson equations. 3) From 1989, Oohara, Shibata and Nakamura started the Post-Newtonian 3D simulations of coalescing binary neutron stars including radiation reaction by gravitational waves for various initial data.[5] If one can combine all these three numerical codes, a fully GR code will be completed. For this purpose we need the good gauge condition and the time slice. For the time being, they are using the *quasi -minimal* shear conditions and the conformal time slicing. In the minimal shear condition all the equations are coupled so that one need too much computing time. In the *quasi-minimal* shear conditions one uses the shift vector one time step before or changes the basic equations so that this coupling is resolved. In the conformal time slicing[6], the lapse function is determined as a function of conformal factor of the 3-space metric. They are now using 47^3 grids and developing their code for coalescing binary neutron stars running on FACOM VP2600.

There were so many good papers in the workshop but the space is limited. So I just show the list of papers that I could not mention.

Bishop, N. T.*The Numerical Calculation of Gravitational Radiation on a Bondi Sphere*

Dubal M.R., Oliveira S. R. and Matzner R. A., *Three Dimensional Initial Data for the Two-Black-Hole collision problem*

Gorgoulhon E., Bonazzola S. and Marck J. A. *A High Precision Numerical code for Spherically Symmetrically Gravitational Collapse Based on a Chebyshev Spectral Method*

Harleston, L.H. *Numerical Solution of the Einstein-Boltzmann Equations in Spherical Symmetry: Results and Perspectives*

Shinkai H. and Maeda K. *Gravitational Waves in a Planar Universe with Cosmological Constant*

REFERENCES

1. Williams R M and Ellis G F R 1981 Gen.Rel.Grav.**13** 361, 1984 Gen.Rel.Grav.**16** 1003.
2. Bishop N T ,Preprint of Univ. South Africa, 116/92(2).
3. A. Abrahams, D. Bernstein, D. Hobill, E. Seidel and L. Smarr, 1992 *Phys. Rev.*, **D45**, 3544.
4. Oohara K and Nakamura T 1989 Prog. Theor. Phys. **81** 360.
5. Oohara K and Nakamura T 1992 Prog. Theor.Phys. **88** 307. and references therein.
6. Shibata M and Nakamura T 1992 Prog. Theor.Phys. **88** 317.