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Range of Validity of the D-Brane - Black Hole Correspondence

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Abstract

For minimally coupled scalars at low frequencies, the D-brane model has the same spectrum of radiation as the Hawking radiation from a black hole. We perform a similar comparison for another type of scalar which we call an intermediate scalar. In this case, we find that there is a discrepancy between the D-brane model and the black hole even for very low frequency scalars. This suggests that the model is only valid within the moduli space approximation.

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I. INTRODUCTION

There has been much progress in the past year in understanding the microstates of black holes through D-brane physics. The Bekenstein-Hawking entropy of certain extremal and near-extremal black holes can be understood through the counting of D-brane microstates [1–5]. Furthermore, the Hawking radiation from a black hole was shown in many cases to agree with the calculation of the corresponding process calculated in the D-brane picture [6-22].

However, there is still a puzzle as to why these correspondences occur. The D-brane calculations are carried out in perturbative string theory, which requires weak coupling. The relevant coupling is actually $g_{eff} = gQ$ where Q is the charge of the black hole. As emphasized in [10], the perturbative picture is valid for $g_{eff} \ll 1$, whereas the semiclassical analysis is only valid for $g_{eff} \gg 1$. Therefore, there is no reason to expect agreement between the two calculations.

To explain why these correspondences occur, an argument was proposed in [24] based on a non-renormalisation theorem. It turns out that the objects that carry entropy are hypermultiplets on the D-brane world-volume. The hypermultiplet moduli space is not corrected at strong coupling. Based on this fact, it was argued that the interactions in the D-brane regime would be the same as those in the black hole regime, as long as one stayed in the moduli space approximation, which is equivalent to low frequencies. Thus the cross-section for Hawking radiation calculated in the D-brane picture should reproduce the semiclassical calculation for very low frequencies. This was shown to occur for minimally coupled scalars in [10], where the authors showed that even the greybody factors of the black hole could be reproduced in the D-brane picture, for low frequency emission.

Not all scalars, however, are minimally coupled. There are other scalars which couple to the charges and the background moduli. Examples of these are the 'fixed' scalars considered in [23,14], which have a different cross-section from minimally coupled scalars. There is yet a third type of scalar, which we call an 'intermediate' scalar, which is different from both minimally coupled and fixed scalars.

The emission of intermediate scalars occurs at a higher order in the hypermultiplet interactions. This interaction vertex is not protected by a nonrenormalization theorem. The arguments of [24] thus do not apply to this scalar and there is no reason to expect the semiclassical calculation to match the D-brane calculation.

In this paper, we will compute the semiclassical absorption cross-section of an intermediate scalar and compare it with the D-brane prediction of [14]. We will show that there is indeed a difference between the two cross-sections. This is indirect support for the arguments of [24]. Other discrepancies between the D-brane model and the black hole description have been pointed out in [20,22,31].

We will first review the calculation of [10] of the absorption coefficient of a minimally coupled scalar. Some technical problems with this calculation were pointed out in [22]. We will attempt to clarify these problems so that we can find the range of parameters for which the calculation is valid.

We then turn to intermediate scalars. The computation of the semiclassical cross-section is somewhat difficult technically and we will do this in detail. First we review the calculation of the D-brane cross-section which was performed in [14]. We then derive the classical equation of motion for the intermediate scalar. We then calculate the semiclassical absorption cross-section in two different parameter ranges and show that they disagree with the D-brane computation. Finally we present our conclusions.

II. THE D-BRANE MODEL

The five-dimensional black hole that we will consider is a near-extremal black hole with three charges. These correspond in ten-dimensional type IIB string theory to Q_5 five-branes, Q_1 one-branes and N units of momentum. We will take the 5-branes to be oriented along x_5, x_6, x_7, x_8, x_9, the 1-branes to be oriented along x_5, and the momentum to be along x_5.

To reduce this solution to five dimensions, we compactify x_5 on a circle of length $2\pi R$

and each of x_6, x_7, x_8, x_9 on a circle of length $2\pi V^{1/4}$. Define the new parameters

$$P_1 = \frac{gQ_1}{V_\infty} \qquad P_5 = gQ_5 \qquad P = \frac{g^2 N}{R_\infty^2 V_\infty}$$
$$r_1^2 = \sqrt{P_1^2 + r_0^2} - r_0 \qquad r_5^2 = \sqrt{P_5^2 + r_0^2} - r_0 \qquad r_n^2 = \sqrt{P^2 + r_0^2} - r_0 \tag{1}$$

Define also the functions

$$f_1 = 1 + \frac{r_1^2}{r^2} \qquad f_5 = 1 + \frac{r_5^2}{r^2} \qquad f_n = 1 + \frac{r_n^2}{r^2}$$
$$f = 1 - \frac{r_0^2}{r^2} \tag{2}$$
$$\lambda = f_1 f_5 f_n$$

In terms of these parameters, we can write the five dimensional solution in the simple form [3,8,10],

$$ds^2 = \lambda^{-2/3} f dt^2 - \lambda^{1/3}(f^{-1} dr^2 + r^2 d\Omega^2)$$
$$R^2 = \frac{f_n}{\sqrt{f_1 f_5}} R_\infty^2 \qquad V = \frac{f_1}{f_5} V_\infty \tag{3}$$

In the D-brane model, we restrict ourselves to the range $r_0, r_n \ll r_1, r_5$. This is called the dilute gas region [10] and is the region where the D-brane computation is expected to be valid. In this range, we can use the effective string description in which we ignore antibranes and nonextremality comes only from the presence of both right and left moving momenta on the string. The number of left- and right- movers is constrained by

$$N_L - N_R = N \tag{4}$$
$$E = \frac{Q_1 R_\infty}{g} + \frac{Q_5 R_\infty V_\infty}{g} + \frac{1}{R_\infty}(N_L + N_R)$$

where E is the ADM mass.

To zeroth order, the left and right movers can be treated as independent gases at temperatures T_L, T_R. These are determined by requiring the average total momenta to be N_L and N_R respectively, and are

$$\frac{1}{T_L} = \frac{\pi r_1 r_5}{2r_0^2}(\sqrt{r_n^2 + r_0^2} - r_n) \qquad \frac{1}{T_R} = \frac{\pi r_1 r_5}{2r_0^2}(\sqrt{r_n^2 + r_0^2} + r_n) \tag{5}$$

Interactions cause open strings to combine to closed strings and escape from the brane. This is interpreted as Hawking radiation. The Hawking temperature can be calculated to be [10]

$$\frac{1}{T_H} = \frac{1}{2}\left(\frac{1}{T_L} + \frac{1}{T_R}\right) \tag{6}$$

III. MINIMALLY COUPLED SCALARS

In [10], Maldacena and Strominger calculated the absorption cross-section of a minimally coupled scalar incident on this black hole. We redo their calculation, emphasizing the questions of the validity of the approximations made.

The equation of motion of a minimally coupled scalar of frequency ω in this metric is

$$\frac{f}{r^3}\partial_r(fr^3\partial_r\Phi) + \omega^2\left(1+\frac{r_n^2}{r^2}\right)\left(1+\frac{r_1^2}{r^2}\right)\left(1+\frac{r_5^2}{r^2}\right)\Phi = 0 \tag{7}$$

We are taking $r_0, r_n \ll r_1, r_5$ and $\omega r_5 \ll 1$.

This equation is not analytically solvable. To solve it, Maldacena and Strominger used the standard method of solving the equation in two regions and matching the two solutions together smoothly on an overlap region. They called their regions the near and far regions respectively (these regions are defined more precisely below.) We will carry out their procedure of solving in the two regions and matching, paying special heed to the validity of the approximations made.

For $r \gg r_n, r_0$, the equation simplifies to

$$\frac{1}{r^3}\partial_r(r^3\partial_r\Phi) + \omega^2\left(1+\frac{r_1^2}{r^2}\right)\left(1+\frac{r_5^2}{r^2}\right)\Phi = 0 \tag{8}$$

Defining $\Phi = r^{-3/2}\Psi$, we find

$$\partial_r^2\Psi - \left(\frac{3}{4r^2}\right)\Psi + \omega^2\left(1+\frac{r_1^2+r_5^2}{r^2}+\frac{r_1^2r_5^2}{r^4}\right)\Psi = 0 \tag{9}$$

We see that the term $\omega^2\frac{r_1^2+r_5^2}{r^2}$ is always small compared to $\frac{3}{4r^2}$ and can hence be dropped for any value of r.

The far region is defined as the region where we can also drop the $\frac{1}{r^4}$ term. This requires

$$\frac{3}{4r^2} \gg \frac{\omega^2 r_1^2 r_5^2}{r^4} \Rightarrow r \gg \omega r_1 r_5.. \tag{10}$$

Thus the far approximation is valid for $r \gg \omega r_1 r_5$. In this region the equation of motion simplifies to

$$\frac{1}{r^3}\partial_r(r^3\partial_r\Phi_1) + \omega^2\Phi_1 = 0 \tag{11}$$

with the solution

$$\Phi_1 = \sqrt{\frac{\pi\omega}{2r^2}}[\alpha J_1(\omega r) + \beta N_1(\omega r)] \tag{12}$$

The near horizon region is defined as the region where we can drop the terms proportional to $\omega^2 r^0$ and $\omega^2 r^{-2}$.

As we said earlier, the term proportional to $\omega^2 r^{-2}$ can always be dropped. The term ω^2 can be dropped provided

$$\frac{3}{4r^2} \gg \omega^2 \Rightarrow r \ll \frac{1}{\omega} \tag{13}$$

This condition is definitely satisfied when $r \ll r_1, r_5$. In this region the equation of motion simplifies to

$$\frac{f}{r^3}\partial_r(fr^3\partial_r\Phi_2) + \frac{\omega^2 r_1^2 r_5^2}{r^4}\left(1 + \frac{r_n^2}{r^2}\right)\Phi_2 = 0 \tag{14}$$

with the solution [10]

$$\begin{aligned} \Phi_2 &= A(1-v)^{-i(a+b)/2}F(-ia, -ib, 1-ia-ib, 1-v) \\ v &= \frac{r_0^2}{r^2} \qquad a = \frac{\omega}{4\pi T_L} \qquad b = \frac{\omega}{4\pi T_R} \end{aligned} \tag{15}$$

We see that the near horizon approximation is definitely valid for $r \ll r_1, r_5$, whereas the far approximation is valid for $r \gg \omega r_1 r_5$. There is a large overlap region where both solutions are valid.We therefore expect a smooth matching of these two solutions without the need for an intermediate solution.

However, a puzzle was pointed out by the authors of [22]. The far region solution behaves for small r as

$$\Phi_1 = \sqrt{\frac{\pi\omega^3}{2}}\left(\frac{\alpha}{2} - \frac{2\beta}{\pi\omega^2 r^2}\right) \tag{16}$$

The near region solution behaves for large r as

$$\begin{aligned} \Phi_2 &= AE(1 + gv - abv\,ln(v)) \\ E &= \frac{\Gamma(1 - ia - ib)}{\Gamma(1 - ia)\Gamma(1 - ib)} \\ g &= \frac{i}{2}(a + b) + ab(1 - 2\gamma - \psi(1 - ia) - \psi(1 - ib)) \end{aligned} \tag{17}$$

where ψ is the digamma function and $\gamma = -\psi(1)$.

These two expansions seem to have different behaviours. In particular, the second expansion has a $\frac{ln(r)}{r^2}$ term which seems to dominate for small r. This is incompatible with the earlier statement that there should be a smooth matching.

For the resolution, we consider a particular case of this problem, when $r_0 = 0$. The differential equation for Φ in the range $r \ll r_1, r_5$ is then

$$\frac{1}{r^3}\partial_r(r^3\partial_r\tilde{\Phi}_2) + \frac{\omega^2 r_5^2 r_1^2}{r^4}\left(1 + \frac{r_n^2}{r^2}\right)\tilde{\Phi}_2 = 0 \tag{18}$$

with the solution

$$\begin{aligned} &\tilde{\Phi}_2 = g_1 F_0(\eta, \alpha) + g_2 G_0(\eta, \alpha) \\ &\eta = -\frac{\omega r_5 r_1}{4 r_n} \qquad \alpha = \frac{\omega r_1 r_5 r_n}{2r^2} \end{aligned} \tag{19}$$

As before, we want to match $\tilde{\Phi}_2$ to Φ_1 at large r. For large r, i.e. small α,

$$\begin{aligned} F_0(\eta, \alpha) \to C_0(\eta)\alpha \qquad & G_0(\eta, \alpha) \to \frac{1}{C_0(\eta)}(1 + 2\eta\alpha ln(2\alpha)) \sim \frac{1}{C_0(\eta)} \\ \Rightarrow \tilde{\Phi}_2 &= g_1 C_0(\eta)\frac{\omega r_1 r_5 r_n}{2r^2} + g_2\frac{1}{C_0(\eta)} \end{aligned} \tag{20}$$

In this form, the matching of $\tilde{\Phi}_2$ and Φ_1 is obvious. it is

$$g_2 = C_0(\eta)\sqrt{\frac{\pi\omega^3}{2}}\frac{\alpha}{2} \qquad g_1 = -\frac{4\beta}{\sqrt{2\pi\omega^3} r_1 r_5 r_n C_0(\eta)} \tag{21}$$

However, suppose we first impose the condition that $\tilde{\Phi}_2$ at the horizon is ingoing. Then

$$\tilde{\Phi}_2 = g_1(iF_0 - G_0) \tag{22}$$

Now the large r behaviour of $\tilde{\Phi}_2$ is

$$\tilde{\Phi}_2 = g_1\left[iC_0(\eta)\alpha - \frac{1}{C_0(\eta)}(1 + 2\eta\alpha ln(2\alpha))\right] \tag{23}$$

and we see the appearance of the $\frac{ln(r)}{r^2}$ term. The analysis above shows that this term is not to be considered since it appears in conjunction with a constant term and is therefore always small.

The same is true in the more general case when $r_0 \neq 0$. Hence the large r behaviour of Φ_2 is not as in (17), but rather

$$\Phi_2 = AE(1 + gv) \tag{24}$$

which can be matched smoothly onto the solution (16) giving the matching

$$\sqrt{\frac{\pi\omega^3}{2}}\frac{\alpha}{2} = AE \qquad -\sqrt{\frac{2}{\pi\omega}}\beta = AEgr_0^2 \tag{25}$$
$$\Rightarrow \frac{\beta}{\alpha} = -\frac{gr_0^2\pi\omega^2}{4} \ll 1$$

The absorption amplitude is then

$$|\mathcal{A}|^2 = 1 - \frac{|\alpha - i\beta|^2}{|\alpha + i\beta|^2} \tag{26}$$

which for small $\frac{\beta}{\alpha}$ can be expanded to leading order as

$$|\mathcal{A}|^2 = -4Im\left(\frac{\beta}{\alpha}\right) \tag{27}$$

The absorption cross-section is then found by

$$\sigma_{abs} = \frac{4\pi}{\omega^3}|\mathcal{A}|^2 = -\frac{16\pi}{\omega^3}Im\left(\frac{\beta}{\alpha}\right) \tag{28}$$

In this case, we find (using properties of the digamma function given in the appendix of [22]) that

$$\sigma_{abs} = \pi^3 \omega r_1^2 r_5^2 \frac{e^{\frac{\omega}{T_H}} - 1}{(e^{\frac{\omega}{2T_L}} - 1)(e^{\frac{\omega}{2T_R}} - 1)} \tag{29}$$

This is exactly the cross section obtained in [10] and agrees with the D-brane calculation of [8].

The only approximation made was that $\omega r_5 \ll 1$. Hence the D-brane calculation and the semiclassical calculation agree in this range. This is consistent with the arguments of [24] that the calculation is correct as long as we are in the moduli space approximation, which is $\omega r_5 \ll 1$.

IV. INTERMEDIATE SCALARS

A. Introduction

We now turn to a different problem of scalar scattering. This involves a new type of scalar that we will call 'intermediate' scalars.

The scalars we are considering are ultimately derived from a dimensional reduction of the fields of type IIB theory. We shall concentrate on the scalars coming from the dimensional reduction of the metric $G_{\mu\nu}$.

Recall that the five-brane is wrapped along directions x_5, x_6, x_7, x_8, x_9 and that the one-branes are oriented along x_5 (the momentum will also be along x_5.) In that case, the dimensional reduction of the metric $G_{\mu\nu}$ provides the scalars $G_{ij}, i, j = 6, 7, 8, 9, G_{55}$ and $G_{5i}, i = 6, 7, 8, 9$.

The scalars G_{ij} with both indices in (6789) directions are minimally coupled and are examples of the scalars considered in the previous section. The scalar G_{55} is a mixture of a 'fixed' scalar and a minimally coupled scalar. Fixed scalars were considered in [23,14]. They have a cross-section that goes to zero as the frequency tends to zero, unlike the minimally coupled scalars which have a nonzero cross-section at zero frequency.

We will consider the third type of scalar, i.e. G_{5i}, with one index along the one-branes and one perpendicular to them. We shall call these scalars 'intermediate' scalars. They have an absorption cross-section which is different from both the minimally coupled and fixed scalars.

The fact that this scalar has a peculiar cross-section was already noted in [14], where the absorption cross-section for this scalar (as predicted by the D-brane model) was calculated. Our purpose here is to calculate the corresponding semiclassical cross-section and see if it matches the D-brane prediction.

The reason that this is an interesting test of the D-brane model will become clear when we review the D-brane calculation of [14]. It turns out that unlike the minimally coupled scalars, the intermediate scalars do not couple at leading order. The first relevant coupling occurs at the next order, and this coupling is not protected by a non-renormalization theorem. An agreement at this order would indicate a deeper correspondence between the two descriptions.

In fact, we find a disagreement. The semiclassical analysis at very low frequencies (the exact meaning of 'very low' is defined later) produces a cross-section which goes to zero as a power of ω. The D-brane model produces a cross-section which goes to a nonzero constant at zero frequency.

We emphasize that this is not surprising, since there is no analogue of the nonrenormalization theorem here.

We begin by reviewing the D-brane calculation performed in [14], for completeness . We then construct the classical equation of motion for the scalar h_{5i}. We will do this in some detail, since some nontrivial manipulations are required.

We then calculate the absorption cross-section from this equation of motion. It turns out that this cannot be done exactly for the whole region of parameter space. We will instead perform the calculation for two different cases, $r_0 = 0, \omega r_5^2 \gg r_n$, and $r_0 = 0, \omega r_5^2 \ll r_n$. We can calculate the cross-section analytically in the first case. For the second case, we will not be able to find an exact cross-section but we can demonstrate the scaling behaviour.

B. Intermediate scalars: the D-brane computation

We review the calculation presented in appendix A of [14].

The metric scalars couple to the D-brane world-volume through the Born-Infeld action.

$$I = -T_{eff} \int d^2\sigma e^{-\phi_{10}} \sqrt{-det\gamma_{ab}} \qquad \gamma_{ab} = G_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu \tag{30}$$

We shall use indices i, j running over $6,7,8,9$ and indices m, n running over $5,6,7,8,9$. So in this notation, the volume of the 4-torus part of the 5-torus which is orthogonal to the one branes is $V = det(h_{ij})$, while the total volume is $det(h_{mn})$.

We expand the Born-Infeld action in fluctuations around flat space $G_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. We will take only h_{5i} to be nonzero.

One also needs to specify which fields are held fixed during this variation. This was worked out in detail in [14]. It turns out that the correct scalars which one should hold fixed are the scalars G_{ij}, the 5-radius $R = \sqrt{G_{55}}$ and the six-dimensional dilaton ϕ_6.

The first few terms in the resulting expansion in powers of derivatives of X are

$$\begin{aligned} I &= -T_{eff}\int (L_K + L_1 + L_3 + ..) \\ L_K &= \frac{1}{2}(\partial_+ X^a \partial_- X_a) \\ L_1 &= \frac{1}{2} h_{5i}(\partial_+ + \partial_-)X^i \\ L_3 &= -\frac{1}{4} h_{5i}(\partial_- X^i(\partial_+ X_a)^2 + \partial_+ X^i(\partial_- X_a)^2) \end{aligned} \tag{31}$$

where we have introduced new derivatives $\partial_+ = \partial_0 + \partial_5, \partial_- = -\partial_0 + \partial_5$.

The term linear in X is not relevant since it only couples to a scalar which carries momentum in an internal direction, i.e. a charged scalar. Note that there is no term quadratic in X which couples to h_{5i}. The first relevant term is cubic in X.

Using this coupling, one calculates the absorption cross-section. The final result is (for $r_1 = r_5$)

$$\sigma_{abs}(\omega) = \frac{\pi^3 r_1^6}{8} \frac{\omega(e^{\frac{\omega}{T_H}} - 1)}{(e^{\frac{\omega}{2T_L}} - 1)(e^{\frac{\omega}{2T_R}} - 1)} (\omega^2 + 8\pi^2 T_L^2 + 8\pi^2 T_R^2) \tag{32}$$

In the extremal limit $r_0 = 0$, this reduces to

$$\sigma_{abs}(\omega) = 2\pi^2 r_n^3 \frac{\frac{\omega}{2T_L}}{1 - e^{\frac{-\omega}{2T_L}}} \left(1 + \frac{\omega^2}{8\pi^2 T_L^2}\right) \tag{33}$$

If we also take $\omega \gg T_L$, this further simplifies to

$$\sigma_{abs} = \frac{\pi^3 \omega^3 r_1^6}{8} \tag{34}$$

On the other hand, for $\omega \ll T_L$, the cross section simply becomes

$$\sigma_{abs} = 2\pi^2 r_n^3 \tag{35}$$

C. The Equation of Motion

We wish to find the classical equation of motion for an excitation of the field $h_{51} \equiv h$. To compare with the D-brane calculation, we should keep the scalars G_{ij}, G_{55}, ϕ_6 fixed.

To simplify the calculation a little, we will instead take the scalars G_{ij}, G^{55}, ϕ_6 fixed while h is excited. Usually $G^{55} = \frac{1}{G_{55}}$ and so the two choices are identical. This is no longer true if off diagonal elements G_{5i} are nonzero.

We find that the perturbed metric is of the form

$$\begin{aligned} G_{55} = R^2 + \frac{h^2}{\sqrt{V}} \qquad & G_{51} = G_{15} = h \\ G_{22} = G_{33} = G_{44} = & G_{11} = \sqrt{V} \end{aligned} \tag{36}$$

with the inverse metric

$$\begin{aligned} G^{55} = \frac{1}{R^2} \qquad & G^{51} = G^{15} = -\frac{h}{R^2\sqrt{V}} \\ G^{22} = G^{33} = G^{44} = \frac{1}{\sqrt{V}} \qquad & G^{11} = \frac{1}{\sqrt{V}} + \frac{h^2}{R^2 V} \end{aligned} \tag{37}$$

As we wished, G^{55} is fixed. We are however forced to vary G_{55}.

The point is that whether we take (G_{ij}, G_{55}, ϕ_6) to be constant, or whether we take (G_{ij}, G^{55}, ϕ_6) to be constant, we get the same equation of motion for the field h. This is because G_{55} is varying by an amount of order h^2. The equation of motion for h is linear and of the general form

$$\Box h + f(G_{ij}, G_{55}, \phi_6)h = 0 \tag{38}$$

If we vary G_{55} by an amount of order h^2, this leads to a change in the equation of motion which is second order in h and can thus be neglected. We can therefore allow $G_{55} \to G_{55} + \frac{h^2}{\sqrt{V}}$. But as we have seen, this is equivalent to keeping G^{55} fixed.

Note that if G_{ij}, G^{55}, ϕ_6 are fixed, then so is the four-volume $V_4 = det(G_{ij})$, the five volume $V_5 = det(G_{mn})$ and the five-dimensional dilaton $\phi_5 = \phi_6 \frac{V_4}{V_5}$.

Let us now vary the Lagrangian to find the effective action for h. We will simplify by going to the extremal limit $r_0 = 0$ and also taking $r_1 = r_5$. As before $r_n \ll r_1$. The moduli fields then take the form

$$R^2 = \frac{f_n}{f_1} R_\infty^2 \qquad V = V_\infty \tag{39}$$

Since the dilaton is constant, we can work with the string metric. The Lagrangian in string metric was derived by Maharana and Schwarz [25] and contains the relevant terms

$$\mathcal{L} = R - \frac{1}{4}\partial G_{mn}\partial G^{mn} - G_{pq}(F_{\mu\nu}^{(KK)p})(F_{\mu\nu}^{(KK)q}) - G^{pq}H_{\mu\nu p}H_{\mu\nu q} - V_5(F_{\mu\nu}^{(5-brane)})^2 \tag{40}$$

First we have the kinetic terms $-\frac{1}{4}\partial G_{mn}\partial G^{mn}$. Using $\partial(V) = 0$, we find

$$-\frac{1}{4}\partial G_{mn}\partial G^{mn} = -\frac{1}{4}\left[\partial\left(\frac{h^2}{R^2}\right)\partial\left(\frac{1}{R^2}\right) - 2\partial h \partial\left(\frac{h}{R^2\sqrt{V}}\right)\right] \tag{41}$$

$$= \frac{1}{2}\frac{(\partial h)^2}{\sqrt{V}R^2} \tag{42}$$

Due to the presence of cross terms like $G^{51}(H_{\mu\nu 5})(H_{\mu\nu 1})$, we must first integrate out $H_{\mu\nu 1}$ †. After doing so, we also integrate out $H_{\mu\nu 5}$. We also perform a similar operation for the KK-gauge fields. In the end, we find a potential term [26]

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† I would like to thank A.Tseytlin for pointing this out to me.

$$-\left(\frac{1}{2}\frac{(\partial_i f_1)^2}{f_1^2}\right)\frac{h^2}{R^2\sqrt{V}} \tag{43}$$

The full effective action for h is thus

$$\mathcal{L}_h = \frac{1}{2}\frac{(\partial h)^2}{R^2\sqrt{V}} - \left(\frac{1}{2}\frac{(\partial_i f_1)^2}{f_1^2}\right)\frac{h^2}{R^2\sqrt{V}} \tag{44}$$

The equation of motion is [26]

$$-\lambda\omega^2 h - \frac{R^2}{r^3}\partial_r\left(\frac{r^3}{R^2}\partial_r h\right) + \frac{(\partial_i f_1)^2}{f_1^2}h = 0 \tag{45}$$

We want to solve this equation and find the absorption cross-section. Since an analytical solution is not available, one must use the standard procedure of solving the equation in several regions and then matching these solutions together.

In this case, even this procedure does not work for all the parameter range. Specifically, if $\omega r_1^2 \ll r_n$, there are ranges of r where even the simplified equation is not analytically solvable. We can, however, find out how the cross-section behaves as a function of the various parameters r_n, r_1, r_0 although the exact numerical factors are unknown. (These can however be found numerically).

In the parameter range where $\omega r_1^2 \gg r_n$, the procedure for finding an approximate solution can be carried through and we can find an analytical expression for the cross-section.

V. THE SEMICLASSICAL CALCULATION

A. High frequencies; $\omega \gg T_L$

The equation of motion is

$$-\lambda\omega^2 h - \frac{R^2}{r^3}\partial_r\left(\frac{r^3}{R^2}\partial_r h\right) + \frac{4r_1^4}{r^2(r^2+r_1^2)^2}h = 0 \tag{46}$$

Recall that $R^2 = \frac{f_n}{f_1}R_\infty^2$.

We now assume $\omega \gg T_L$ i.e. $\omega r_1^2 \gg r_n$.

For $r \sim r_n$ the equation of motion reduces to

$$\left(1+\frac{r_n^2}{r^2}\right)\frac{\omega^2 r_1^4}{r^4}h_1+\left(\frac{r^2+r_n^2}{r^3}\right)\partial_r\left(\frac{r^3}{r^2+r_n^2}\partial_r h_1\right)=0 \tag{47}$$

Defining $h_1=\sqrt{r^2+r_n^2}\Psi$, we find

$$\left(1+\frac{r_n^2}{r^2}\right)\frac{\omega^2 r_1^4}{r^4}\Psi+\frac{1}{r^3}\partial_r(r^3\partial_r\Psi)+\Psi\frac{\sqrt{r^2+r_n^2}}{r^3}\partial_r\left(\frac{r^3}{r^2+r_n^2}\partial_r\sqrt{r^2+r_n^2}\right)=0 \tag{48}$$

The last term is always of order $\frac{\Psi}{r_n^2}$ and can be dropped relative to the first term. The resulting equation is

$$\left(1+\frac{r_n^2}{r^2}\right)\frac{r_1^4}{r^4}\omega^2\Psi+\frac{1}{r^3}\partial_r(r^3\partial_r\Psi)=0 \tag{49}$$

which has the solution

$$\begin{aligned}\Psi &= c_1F_0(\eta,\frac{r_1^2\omega r_n}{2r^2})+c_2G_0(\eta,\frac{r_1^2\omega r_n}{2r^2})\\ \eta &= -\frac{r_1^2\omega}{4r_n}\end{aligned} \tag{50}$$

Since we require an ingoing wave at the horizon, we require

$$c_1=-ic_2 \tag{51}$$

For $r_n \ll r \ll \omega r_1^2$, the equation of motion is

$$\frac{\omega^2 r_1^4}{r^4}h_2+\frac{1}{r}\partial_r(r\partial_r h_2)-\frac{4}{r^2}h_2=0 \tag{52}$$

with the solution

$$h_2=c_3J_2\left(\frac{\omega r_1^2}{r}\right)+c_4N_2\left(\frac{\omega r_1^2}{r}\right) \tag{53}$$

To match these solutions, we first introduce an auxiliary function ξ satisfying

$$\frac{\omega^2 r_1^4}{r^4}\xi+\frac{1}{r^3}\partial_r(r^3\partial_r\xi)=0 \tag{54}$$

with the solution

$$\xi = \frac{1}{r}\left[c_5 J_1\left(\frac{r_1^2\omega}{r}\right) + c_6 N_1\left(\frac{r_1^2\omega}{r}\right)\right] \tag{55}$$

For $r > r_n$, the differential equation (49) for Ψ approaches the differential equation (54) for ξ. Hence the solutions should match smoothly, in particular, the large r expansions should be the same. For large r [30],

$$\Psi \to c_1 C_0(\eta)(\frac{r_1^2\omega r_n}{2r^2}) + c_2\frac{1}{C_0(\eta)} = c_1\sqrt{\frac{\pi r_1^2\omega}{2r_n}}\left(\frac{r_1^2\omega r_n}{2r^2}\right) + c_2\left(\sqrt{\frac{2r_n}{\pi r_1^2\omega}}\right) \tag{56}$$

where we have used $\eta \ll 0$ to write an approximate expression for $C_0(\eta)$.

For large r,

$$\xi \to \frac{1}{r}\left[c_5\frac{r_1^2\omega}{2r} + c_6\frac{2r}{\pi r_1^2\omega}\right] \tag{57}$$

Matching the two solutions, we find

$$c_5 = c_1\sqrt{\frac{\pi\omega r_1^2 r_n}{2}} \qquad c_6 = c_2\sqrt{\frac{\pi\omega r_1^2 r_n}{2}} \tag{58}$$

Similarly for small r, define $\xi = \frac{\phi}{r}$. Then we find

$$\frac{\omega^2 r_1^4}{r^4}\phi + \frac{1}{r}\partial_r(r\partial_r\phi) - \frac{\phi}{r^2} = 0 \tag{59}$$

For $r \ll \omega r_1^2$, we can drop the last term. For small r, the differential equation (52) for h_2 then coincides with this equation.. Hence ϕ should approach h_2. In particular, their small r limits should be the same.

For small r,

$$\phi = [c_5 J_1(\frac{r_1^2\omega}{r}) + c_6 N_1(\frac{r_1^2\omega}{r})] \to c_5\sqrt{\frac{2r}{\pi r_1^2\omega}}cos\left(\frac{r_1^2\omega}{r}\right) + c_6\sqrt{\frac{2r}{\pi r_1^2\omega}}sin\left(\frac{r_1^2\omega}{r}\right) \tag{60}$$
$$h_2 \to c_3\sqrt{\frac{2r}{\pi r_1^2\omega}}cos\left(\frac{r_1^2\omega}{r}\right) + c_4\sqrt{\frac{2r}{\pi r_1^2\omega}}sin\left(\frac{r_1^2\omega}{r}\right)$$

giving $c_5 = c_3, c_6 = c_4$.

Combining with (58), we find

$$c_3 = c_1\sqrt{\frac{\pi\omega r_1^2 r_n}{2}} \qquad c_4 = c_2\sqrt{\frac{\pi\omega r_1^2 r_n}{2}} \tag{61}$$

For $r_1 \sim r \gg \omega r_1^2$, we drop the term with ω.

$$\frac{1}{r(r^2+r_1^2)}\partial_r[r(r^2+r_1^2)\partial_r h_3] - \frac{4r_1^4}{r^2(r^2+r_1^2)^2}h_3 = 0 \tag{62}$$

with the solution

$$h_3 = c_7\left(1+\frac{r_1^2}{r^2}\right) + c_8\left(1+\frac{r_1^2}{r^2}\right)^{-1} \tag{63}$$

We match the small r behaviour of h_3 to the large r behaviour of h_2.

For small r

$$h_3 \to c_7\left(\frac{r_1^2}{r^2}\right) + c_8\left(\frac{r^2}{r_1^2}\right) \tag{64}$$

For large r,

$$h_2 \to c_3\left(\frac{\omega^2 r_1^4}{8r^2}\right) - c_4\left(\frac{4r^2}{\pi\omega^2 r_1^4}\right) \tag{65}$$

Hence

$$c_7 = \frac{c_3}{8}\omega^2 r_1^2 \qquad c_8 = -\frac{4c_4}{\pi\omega^2 r_1^2} \tag{66}$$

For $r \gg r_1$, the equation of motion is

$$\omega^2 h_4 + \frac{1}{r^3}\partial_r(r^3\partial_r h_4) = 0 \tag{67}$$

with the solution

$$h_4 = \sqrt{\frac{\pi\omega}{2r^2}}[\alpha J_1(\omega r) + \beta N_1(\omega r)] \tag{68}$$

For small r

$$h_4 = \sqrt{\frac{\pi\omega^3}{2}}\left(\frac{\alpha}{2} - \frac{2\beta}{\pi\omega^2 r^2}\right) \tag{69}$$

We match this to the large r behaviour of h_3

$$h_3 \to (c_7+c_8) + (c_7-c_8)\frac{r_1^2}{r^2} \tag{70}$$

which gives

$$c_7 + c_8 = \sqrt{\frac{\pi\omega^3}{2}}\frac{\alpha}{2} \qquad c_7 - c_8 = -\beta\sqrt{\frac{2}{\pi\omega r_1^4}} \tag{71}$$

Combining the various matching conditions, we find

$$Im\left(\frac{\beta}{\alpha}\right) = -\frac{\pi^2\omega^6 r_1^6}{64} \tag{72}$$

The absorption cross section is then

$$\sigma_{abs} = \frac{\pi^3\omega^3 r_1^6}{4} \tag{73}$$

which differs from the D-brane calculation (34), though only by a factor of 2.

B. Low frequencies; $\omega \ll T_L$

We will now consider the case $\omega \ll T_L$, i.e. $\omega r_1^2 \ll r_n$. In this case we have the full differential equation

$$-\left(1+\frac{r_n^2}{r^2}\right)\left(1+\frac{r_1^2}{r^2}\right)^2\omega^2 h - \frac{(r^2+r_n^2)}{r^3(r^2+r_1^2)}\partial_r\left(\frac{r^3(r^2+r_1^2)}{(r^2+r_n^2)}\partial_r h\right) + \left(\frac{4r_1^4}{r^2(r^2+r_1^2)^2}\right)h = 0 \tag{74}$$

We can divide the space into several regions as usual.

For $r \gg r_1$, we get

$$\omega^2 h_4 + \frac{1}{r^3}\partial_r[r^3\partial_r h_4] = 0 \tag{75}$$

with the solution

$$h_4 = \sqrt{\frac{\pi\omega}{2r^2}}[\alpha J_1(\omega r) + \beta N_1(\omega r)] \tag{76}$$

For $r_1 \sim r \gg \omega r_1^2$, we find

$$\frac{1}{r(r^2+r_1^2)}\partial_r[r(r^2+r_1^2)\partial_r h_3] - \frac{4r_1^4}{r^2(r^2+r_1^2)^2}h_3 = 0 \tag{77}$$

with the solution

$$h_3 = c_7 \left(1 + \frac{r_1^2}{r^2}\right) + c_8 \left(1 + \frac{r_1^2}{r^2}\right)^{-1} \tag{78}$$

As before the matching gives

$$c_7 + c_8 = \sqrt{\frac{\pi\omega^3}{2}\frac{\alpha}{2}} \qquad c_7 - c_8 = -\beta\sqrt{\frac{2}{\pi\omega r_1^4}} \tag{79}$$

For small r, the second solution h_3 behaves as

$$h_3 \to c_7 \left(\frac{r_1^2}{r^2}\right) + c_8 \left(\frac{r^2}{r_1^2}\right) \tag{80}$$

In the near horizon region $r \ll r_n$, the equation becomes

$$\frac{\omega^2 r_1^4 r_n^2}{r^6} h_1 + \frac{1}{r^3}\partial_r[r^3 \partial_r h_1] - \frac{4}{r^2} h_1 = 0 \tag{81}$$

with the solution

$$\begin{aligned} h_1 &= \frac{1}{r}\left[c_1 J_\nu(w) + c_2 N_\nu(w)\right] \\ w &= \left(\frac{r_1^2 r_n \omega}{2r^2}\right) \qquad \nu = \sqrt{\frac{5}{4}} \end{aligned} \tag{82}$$

To get an ingoing wave at the horizon, we require

$$c_1 = -i c_2 \tag{83}$$

For large r, this solution behaves as

$$h_1 = \frac{1}{r}\left[c_1 \left(\frac{w}{2}\right)^\nu \frac{1}{\Gamma(\nu+1)} - c_2 \left(\frac{2}{w}\right)^\nu \frac{\Gamma(\nu)}{\pi}\right] \tag{84}$$

In the intermediate region $r \sim r_n$, the equation becomes

$$-\frac{(r^2 + r_n^2)}{r^3}\partial_r \left(\frac{r^3}{r^2 + r_n^2}\partial_r h_2\right) + \frac{4}{r^2} h_2 = 0 \tag{85}$$

Finding an analytical form for h_2 however seems to be impossible. We will instead use a weaker scaling argument.

Defining $\rho = \frac{r}{r_n}$, the equation for h_2 can be rewritten

$$\frac{\rho^2 + 1}{\rho^3}\partial_\rho \left(\frac{\rho^3}{\rho^2 + 1}\partial_\rho h_2\right) - \frac{4 h_2}{\rho^2} = 0 \tag{86}$$

There are two independent solutions which we call $F_i(\rho), i = 1, 2$. Then $h_2 = c_3F_1 + c_4F_2$.

For small ρ, the equation becomes

$$\frac{1}{\rho^3}\partial_\rho[\rho^3\partial_\rho F_i] - \frac{4F_i}{\rho^2} = 0 \tag{87}$$

Hence $F_i \sim \rho^n$ where $n = -1 \pm \sqrt{5}$.

Let us choose F_1 to be the solution that behaves as $\rho^{-1+\sqrt{5}}$, and F_2 to be the solution that behaves as $\rho^{-1-\sqrt{5}}$. Then matching with the near solution h_1, we find

$$\begin{aligned} c_4 r_n^{1+\sqrt{5}} &= \frac{1}{\Gamma\left(1+\frac{\sqrt{5}}{2}\right)}\left(\frac{r_1^2 r_n \omega}{4}\right)^{\frac{\sqrt{5}}{2}} c_1 \\ c_3 r_n^{1-\sqrt{5}} &= -\frac{\Gamma\left(\frac{\sqrt{5}}{2}\right)}{\pi}\left(\frac{4}{r_1^2 r_n \omega}\right)^{\frac{\sqrt{5}}{2}} c_2 \end{aligned} \tag{88}$$

For large ρ the equation simplifies to

$$\frac{1}{\rho}\partial_\rho[\rho\partial_\rho F_i] - \frac{4F_i}{\rho^2} = 0 \tag{89}$$

with linearly independent solutions ρ^2, ρ^{-2}.

So for large ρ,

$$F_1 = x_{11}\rho^2 + x_{12}\rho^{-2} \qquad F_2 = x_{21}\rho^2 + x_{22}\rho^{-2} \tag{90}$$

where $x_{11}, x_{12}, x_{21}, x_{22}$ are undetermined constants of order 1.

Matching to the intermediate solution h_3, we find

$$c_3 x_{12} + c_4 x_{22} = c_7 \frac{r_1^2}{r_n^2} \tag{91}$$

$$c_3 x_{11} + c_4 x_{21} = c_8 \frac{r_n^2}{r_1^2} \tag{92}$$

We will henceforth forget about factors of order 1. To this accuracy, we have

$$Im\left(\frac{c_3x_{12} + c_4x_{22}}{c_3x_{11} + c_4x_{21}}\right) \sim \left(\frac{r_1^2\omega}{r_n}\right)^{\sqrt{5}} \tag{93}$$

It is now straightforward to calculate the cross section. Upto factors of order 1, we find

$$Im\left(\frac{\beta}{\alpha}\right) \sim \frac{\pi\omega^2 r_n^4}{r_1^2}\left(\frac{r_1^2\omega}{r_n}\right)^{\sqrt{5}} \tag{94}$$

and so the cross section is

$$\sigma_{abs} \sim r_n^3\left(\frac{r_1^2\omega}{r_n}\right)^{\sqrt{5}-1} \tag{95}$$

which disagrees with the D-brane prediction (35). In particular, the expression above goes to zero for zero frequency, unlike the D-brane prediction which goes to a nonzero constant.

VI. CONCLUSIONS

We have found that for certain scalars, the D-brane model does not reproduce the semi-classical calculation. This was also consistent with the nonrenormalization arguments of [24]. This suggests that the D-brane model breaks down beyond the moduli space approximation.

On the other hand, there is at least one case where the D-brane model works beyond the moduli space approximation. This is the analysis of [14], where a fourth order hypermultiplet interaction was used to calculate the absorption cross-section for a fixed scalar. This was shown to agree with the semiclassical analysis, at least for $r_1 = r_5$. A calculation of higher angular momentum processes [19], which again goes beyond the moduli space approximation, also yielded results which were correct upto unknown constant factors.These results suggest that there might be a deeper reason for the D-brane-black hole correspondence. If so, there may be a way to reconcile the disagreement pointed out in this paper.

We emphasize that if the disagreement cannot be resolved, then one can distinguish a black hole from a D-brane even at very low frequencies. This may have implications for the information paradox in these models.

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