REMARKS ON THE QUARK-HADRON TRANSITION IN THE EARLY UNIVERSE

Rémi Hakim* and Suzy Collin**

Groupe d'Astrophysique Relativiste* and
Laboratoire d'Astrophysique**
Observatoire de Meudon
92195 Meudon Principal Cedex, France

1. INTRODUCTION

These last years fascinating speculations on the consequences of phase transitions, within the context of Grand Unified Theories, in the early universe have been set forth and "predict" numerous phenomena which might occur at temperatures as high as 10^{15}GeV... a temperature far beyond present experimental possibilities. Moreover, although Grand Unified Theories do represent a natural generalization of the presently admitted theories (essentially, the Weinberg-Salam model and Quantum Chromodynamics) they unify theories whose experimental basis is not as rich as it should be (think to the Higgs'particles, for instance), whose theoretical content is far from being understood (e.g. the basic problem of quark confinement is as yet unsolved) ; where calculations are quite difficult to work out, owing to an intrinsic non-linearity, and hence to be compared with high energy experiments ; and, finally, where approximations are not undercontrol.

However, at a less - slightly less - speculative level, one may consider the state of matter at densities and/or temperatures more in accordance with what is known at our laboratory scale :

J. Audouze and J. Tran Thanh Van (eds.),

in the early universe this would involve temperatures ranging from ∿ .1 GeV to ∿ 100 GeV.

At these temperatures and/or densities the hadrons become gradually close-packed (fig.1) and next transform into a quark soup or, in more elegant words into a hot quark plasma.

This transition from hadrons to quarks (i.e. going back in time) is generally assumed to be a phase transition and more specifically a first order phase transition (fig.1). Needless to say that it could well be that there does not exist any such phase transition whatsoever : think, for instance, to the analogy with the ionization process of a neutral gas ; all thermodynamic quantities vary continuously from the atomic state (∿ hadrons) to the plasma state (∿ quarks) and there is no phase transition at all.

It should be clear that, in order to get a definite and convincing answer to this question, the basic theoretical problems of Quantum Chromodynamics must be solved. Therefore, we have to assume the existence of such a first order phase transition.

The usual way to handle such a transition consists in calculating the equation of state of the quark plasma, the equation of state of the hadron phase (with another set of theoretical assumptions) and finally link both curves via a Maxwell plateau (equality of pressures and chemical potentials in both phases ; see fig.2).

The essential interest of phase transitions in the early universe - but not the only one - lies in the fact that they are associated with violent fluctuations which might subsequently be at the origin of galaxy clusters or, more modestly, of galaxies, or even more modestly, of stars beolonging to the so-called "population III" or, at least, "seeds" for an eventual creation of galaxies (in the latter case, the mechanism for such a creation remains to be found ; see e.g. (1).

It follows that two main problems are to be considered. The first one deals both with the critical temperature at which the transition occurs (or, equivalently, at what time it occurs) and also with is duration. It is clear, indeed, that one cannot create objects more massive than the mass contained inside the horizon : hence, the size of the horizon and the energy density at the time of the transition play a basic role. Furthermore, the duration of the transition determines in part the possibilty for the fluctuations to grow sufficiently. The second problem deals with the spectrum of the fluctuations, a quantity generally put by hand at the onset of galaxy creation models.

To these two main problems we should also add the question of

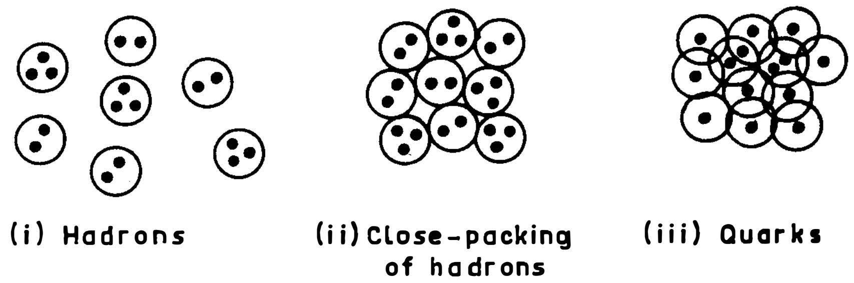


Figure 1 : Various densities of the hadron fluid.

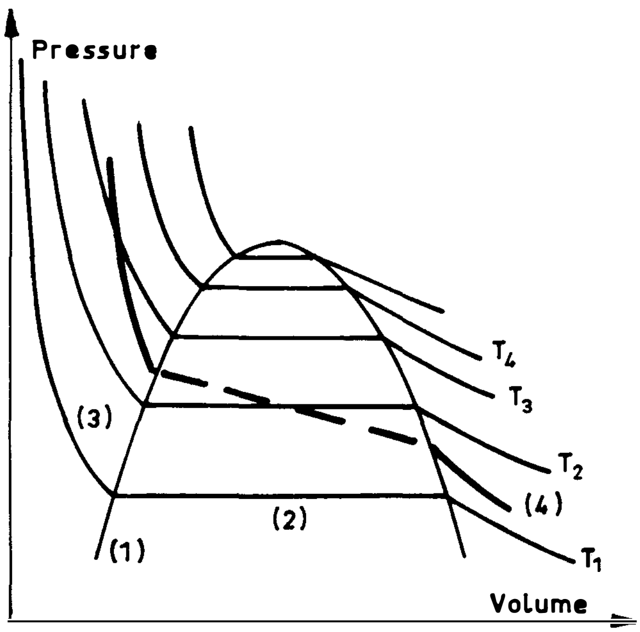


Figure 2 : Typical phase diagram for a first order phase transition; curve (1) provides the transition region while curves (2) represent the usual Maxwell plateau; curve (3) corresponds to the quark phase; curve (4) represents a typical thermodynamical path of the early universe. The intersection of curves (1) and (4) is the critical temperature we are looking for.

the nucleation of droplets of hadrons (possibly by "impurities" ; see e.g. (in another context) (27) in the quark plasma during the phase transition and also their dynamics.

Surprisingly enough the number of articles devoted to quarks in the early universe is quite limited, and particularly those which are connected with the quark/hadron phase transition. They deal either with a zero-temperature universe (3), (4)[1] or with the standard model (5) and, in the latter case the quarks may be partially confined (6), (7), (8), (9) and (10) or fully confined (11), (12), (13), (14) and (15).

In this talk, all these questions will be considered and discussed in a phenomenological way, the only reasonable one - in our opinion - when taking into account the present status of the theory. More specifically, we shall be concerned essentially with Olive's approach.

2. DYNAMICS OF QUARK SYSTEMS

Quantum chromodynamics is generally considered as being the correct theory of strong interactions and, as a matter of fact, it is quite consistent with present day experiments. However, owing to its intrinsic non-linearity calculations are very difficult to perform. Also, its vacuum structure is not yet understood and, more specifically, the confinement mechanisms of quarks and gluons are still unknown. Thence the thermodynamics of the quark/hadron plasma cannot be obtained directly from Quantum Chromodynamics as long as its theoretical problems remain unsolved.

Of course, preliminary calculations performed within perturbative expansions[2)] have been carried out either at T=0°K (17), (18), (19), or at finite temperatures (20) as to the quark phase. At T=0°K the calculations have been pushed to the fourth-order in the coupling constant while at T ≠ 0°K only the third order has been obtained. Unfortunately, these last results are not particularly conclusive the more so since the various terms of the perturbative expansion are of the same order of magnitude, a circumstance that casts some doubts on the convergence of the expansion. Moreover, in the absence of an admitted confinement mechanism, the hadron phase cannot be properly described within this perturbative frame-work.

Let us add that non-perturbative effects have tentatively been considered (21) but the matter is still in a controversial state.

Consequently, if one wants deriving some results in view of astrophysiqcal situations - neutron stars, quark stars, quark era in the early universe - one must resort to some tractable phenomenological model. Such a model should contain both theoretical ingre-

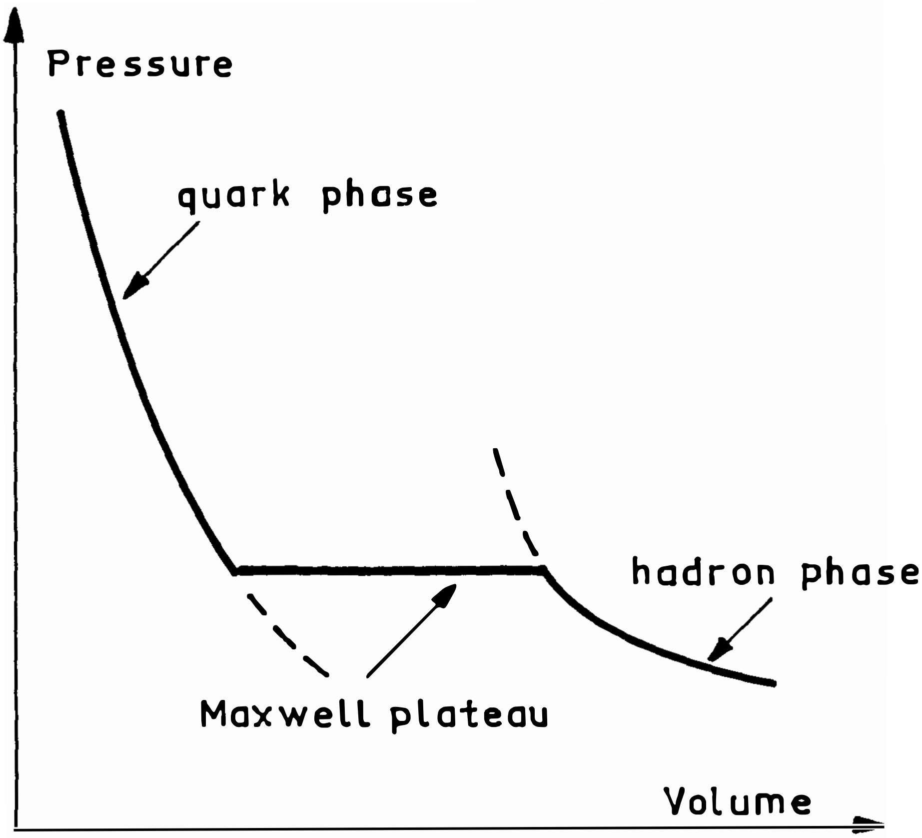


Figure 3 : The quark equation of state and the hadron one, calculated with different theories, are linked by a Maxwell plateau (equality of pressures and of chemical potentials).

dients from Quantum Chromodynamics and also some experimental input such as, for instance, a description of confinement.

Among the various phenomenological possibilities at hand, we should mention various bag models and, mainly, the MIT bag model (22). Nevertheless, they are generally as difficult to handle as the original quantum chromodynamics. However, and in spite of some attempts (23) it is extremely difficult to evaluate the density and temperature dependence of the bag because, precisely, of the absence of a convincing model of quark confinement !

2.1. The quark-quark potential

The dynamical models considered - at a phenomenological level - in this paper are based on the use of an ad hoc potential V(r). Indeed, the hadron spectroscopy is quite reasonably well described with a potential (except the pion mass, of course) inserted in a Schrödinger - like equation. Usually, the parameters of the potential are fitted so as obtain the ψ-family and the result is next checked on the Y-family and then applied to other hadrons. In this way, the phenomenological potential adopted is tested between roughly .1 fm and 1fm. Numerous potentials can be found in the litterature and some of them are listed in Appendix A.

These phenomenological potentials generally contain essentially two terms : one of them is supposed to take account of quark confinement (it is an ever increasing function of the radial coordinate r ; for instance a power law) while the second one may correspond to the one- gluon exchange in the static limit.

One of the most popular choice is

$$U(r_{12}) = - \vec{\lambda}_1 \cdot \vec{\lambda}_2 \ \{ - \frac{a}{r_{12}} + br_{12}\} \tag{1}$$

where a and b are positive constants to be fitted with the use of the charmonium spectrum and where the eight matrices $\vec{\lambda}_i$ are the well-known Gell-Mann matrices (the index i, in $\vec{\lambda}_i$ or in r_{12}, refers to the i-th quark). It is sometimes called the "QCD potential" since its Coulombian part corresponds to the one gluon exchange (in this case, a is the QCD fine structure constant) while the linearly rising term - the confining potential - seems to be suggested by lattice calculations (or simulations).

It should be noticed, however, that the confining part of this potential (or of other ones) is a little bit pathological. First, the $\vec{\lambda}_1 \cdot \vec{\lambda}_2$ matrix has opposite signs when the quarks 1 and 2 are either in a singlet or an octet state, resulting in the absence of a zero of energy in this last case (i.e. when the coefficient of r is negative). Perhaps more important is the fact that, in the singlet state case, arbitrarily high energies can be reached resulting in pair creations and hence in a softening of the potential, not so simple to handle. A supplementary drawback of the confining term is that it leads to unobserved van der Waals - like forces between hadrons (24). For instance, a linearly rising potential would lead to long range forces in 1/r in nuclear matter.

Consequently, we should be cautious while using such potentials and specially at low densities.

An improvement of the potential (1), whose Fourier transform is

$$\tilde{U}(k) = - \vec{\lambda}_1 \cdot \vec{\lambda}_2 \left\{ \frac{a}{k^2} + \frac{bc}{k^4} \right\} \tag{2}$$

(c being a numerical constant), has been performed by Richardson (25) (see also (26)) who replaced the QCD fine structure constant a by the effective one obtained from the renormalization group, i.e. by

$$a \to \alpha_{eff}(k^2) \sim \frac{12\pi}{33-2n_f} \frac{1}{\ln(k^2/\Lambda^2)} , \tag{3}$$

where n_f is the number of quark flavours that come into play at momentum k and where Λ is a scale parameter to be determined by experiment (and this is not an easy task...). This procedure has the advantage of embodying the important property of asymptotic freedom, an important theoretical and phenomenological ingredient. Finally, a simple interpolation between the two regimes (low k and large k) leads to

$$\tilde{U}(k) = - \frac{4}{3} \cdot \frac{12\pi}{27} \cdot \frac{1}{k^2} \cdot \frac{1}{\ln(1+k^2/\Lambda^2)} \quad (n_f = 3) \tag{4}$$

Such a potential has been used by Schöberl (27) to obtain the hadron spectroscopy.

Once a potential is chosen and its free parameters fitted with high energy data, one assumes that it is still as good for quark-quark interactions (apart from unessential factors $\vec{\lambda}_1 \cdot \vec{\lambda}_2$) whatever their flavor (the c and b quarks used to fit the parameters are heavy).

2.2. Quark masses

The next dynamical question to be discussed is the one of the quark masses. Which one should be introduced in the Schrödinger equation ? Is it the constituent mass (most authors) ? the current mass ((11), (12), (15), etc...) ? the running mass (8) ? In fact there is no clear answer and the problem is quite delicate and complex. This is the reason why, in Appendix A, the potentials given are generally accompanied with the masses used in the fitting of the parameters.

In our opinion, in order to be consistent with e.g. the QCD potential (4) - where results arising from the group renormalization equations have been inserted - the running mass should be used, perhaps in such a way as to reduce to the current mass at high momenta and to the constituent mass at low momenta.

In fact, the renormalization group equations (see e.g. (28)) provide such a running mass ; unfortunately, they provide asymptotic forms only (exactly as for the effective coupling constant) and, moreover, they have two solutions. One of them is

$$m(p) \sim \frac{1}{\ell n^{\gamma}(p^2/\Lambda^2)} \tag{5}$$

while the other one, reads

$$m(p) \sim \frac{4m_o^3}{p^2} \ \ell n^{\gamma}(p^2/\Lambda^2) \tag{6}$$

(with $m_o \sim 300$ MeV) where

$$\gamma = \frac{12}{33-2n_f} \tag{7}$$

Theoretical arguments based on a possible spontaneous breakdown of chiral symmetry led H. Pagels et al. (29) to favour the second solution (6) which they parametrize as

$$m(p) = \frac{m(o)\Lambda^2}{\Lambda^2+p^2} \tag{8}$$

The fact that the renormalization group equations provide asymptotic forms only renders extrapolations to low momenta somewhat doubtful, owing to the fact that this corresponds to the non-perturbative region where our present knowledge fails. However, near $p \sim o$ (or a few hundred MeV) and at a phenomenological level, m(p) is practically constant, at least as a kind of average (see fig.3). This property is valid for the range of energies where the predicted spectroscopy of hadrons is correctly described. For instance, for the Richardson's potential (4), the constituent mass appears to be a valid approximation for a few hundreds MeV.

To these considerations let us also add that non-perturbative effects, such as instantons, lead to other forms for m(p) ; for

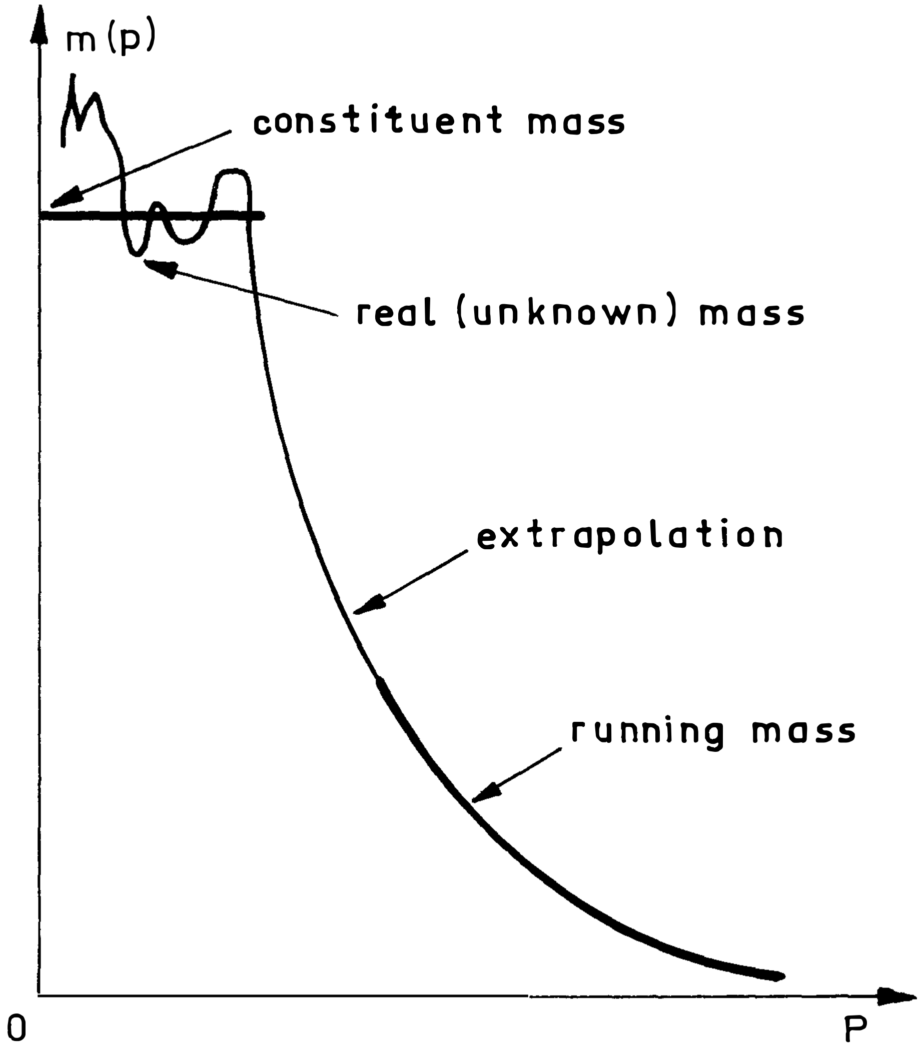


Figure 4 : Various quark masses to be (possibly) used in quark phase calculations.

instance, the use of instantons (21) provides

$$m(p) \sim 1/p^{12}. \quad (9)$$

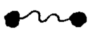
2.3. Discussion and possible improvements

We first notice that most of the potentials used in the litterature coincide more or less between .1fm and 1 fm, i.e. where they are actually tested ; more specifically, in the region where the potential is approximately linear. This might perhaps be an indication that at low densities such a phenomenology of dense matter could possibly contain some truth ...

However, at higher densities – of the order of $\sim (1/.2\text{fm})^3$ – where the various potentials deviate from each other, different results can be obtained.

On the other hand, this approach suffers from a number of drawbacks which may be more or less cured.

(1) At high densities or equivalently, at short distances or at high momenta, relativistic effects should be taken into account and thus a Schrödinger equation approach is not adequate : a static approximation has to be corrected. This can be done either by taking account of relativistic corrections (30) or writing a two-body Dirac equation for semi-relativistic quarks (31). Among many other possibilities, it is also possible to write a Bethe-Salpeter equation (in the ladder approximation) where the gluon propagator is replaced by



$$= \frac{\alpha}{k^2} + \frac{bc}{k^4} \quad (10)$$

(where $k^2 \equiv \omega^2 - \vec{k}^2$, in equation (10)) and where the vertex is replaced by an "effective" one, i.e. where α is in fact $\alpha_{eff}(k^2)$ as given by Eq. (3) above.

(2) As important as the preceding point is a possible dependence of various physical quantities, α_{eff}, $V(r)$, $m(k)$, etc... on density and temperature.

For instance, it has been shown (32) that

$$\alpha_{eff}(k,T) \sim \frac{\alpha(0,0)}{1+\frac{33}{48\pi^2}\,\alpha\,\ln(k^2/\Lambda^2) + \frac{m^2[T]}{k^2}} \quad (11)$$

where $m^2[T] \sim T^2$; similarly, the same authors have calculated in a non-perturbative way (as a self consistent solution of the Dyson -

Schwinger equations) the static limit of the gluon propagator : a simple Fourier transform of the latter quantity gives the potential and its density/temperature dependence.

Another attempt by Zhao Wanyun (33), within the context of perturbation theory, provides

$$V(r,T) = \alpha^{5/2} \frac{3}{32\pi^3} \frac{1}{T^2 r^2} \exp. [-2m[T].r] \tag{12}$$

However, these attempts and several others are not yet completely conclusive.

(3) Another important problem linked with the passage from the non-relativistic case to the relativistic one, is the following. When one considers the Coulomb - like part of the potential (1), for instance, it should be considered as the fourth-component of the gluon field (it corresponds to the static limit of the one-gluon exchange term of a perturbative expansion).

On the other hand, it is well known that either a Lorentz - scalar potential or a four-vector potential lead to the same kind of non relativistic limit. Consequently, if the potential is to be considered as the fourth component of the gluon field A^μ, it must enter the relativistic equation under consideration through the gauge invariant combination

$$p^\mu \to p^\mu + ig\,\vec{\lambda} \,.\, \vec{A}^\mu \tag{13}$$

while if it is a scalar, it must appear in the mass term as

$$m \to m + gV, \tag{14}$$

assuming as usual a Yukawa - like coupling with the quark field.

To these two possibilities, one should add a third one, which is intermediate : the confining potential might be a Lorentz-scalar to be inserted in the quark mass as indicated in Eq. (14) while the "interacting" part - the gluonic one - should be considered as the fourth component of a four-vector.

These three cases may be summarized in the following three expressions for the relativistic energy :

$$\text{(i)} \quad \xi\,(\vec{p}) = \{\vec{p} + m^2(\vec{p})\}^{1/2} + V(r) \tag{15}$$

$(V \equiv A^\circ)$

$$\text{(ii)} \quad \xi\,(\vec{p}) = \{\vec{p}^{\,2} + [m(\vec{p}) + V(r)]^2\}^{1/2} \tag{16}$$

(V = Lorentz scalar)

$$\text{(iii)} \quad \xi\,(\vec{p}) = \{\vec{p}^{\,2} + m(\vec{p}) + V_{conf.}(r)]^2\}^{1/2} + V_{gluon}(r) \qquad (17)$$

(intermediate case).

As to this last point - considered by Boal et al. (15) - it should be noticed that a scalar field is introduced ab initio in the SLAC bag model (34) or in a simplified version (35) and that its treatment is certainly simplier than the use of the hamiltonians (16) or (17). However, despite the pseudo-Thomas-Fermi calculation of Boal et al. (15), it leads to partial confinement only (36), (37) ; see also (38) : quarks appear to be heavy at low densities and light at higher ones. In any case, if a scalar field is introduced so as to mimic confinement, it should be considered as the result of a complex process (interactions with scalar gluonic modes ?) steming from Quantum Chromodynamics.

3. STATISTICAL DESCRIPTION OF THE QUARK PLASMA

If the dynamical approach considered above were completely correct and trustworthy, we should eventually find the thermodynamical parameters that characterize the (possible) phase transition from hadrons to quarks. This would demand a proper treatment of two-body and three-body correlations. Usually two-body correlations can be handled although with technical difficulties while three-body correlations can be treated - in a non quantum framework - only in quite particular cases. Needless to say that this is an almost impossible task in this context. However, one might perhaps think of these three-body correlations as already included, at least by part, in the phenomenological confining potential ? Furthermore, while three-body forces appear to play an extremely small role in hadron spectroscopy, at the temperatures where the transition is supposed to take place (∿ 280 MeV) only few baryons (∿ 3% according to Olive's calculations (11) and (12) are produced.

Therefore we are led - as other - to deal with the simplest tractable statistical approximations ; essentially mean field approximations such as Thomas-Fermi's, Hartree's or Hartree-Fock (see e.g. (39)) which are quite easy to deal with. Fortunately, these approximations are generally valid at high densities (in the Quantum Chromodynamic case, the validity of the Hartree approximation at high densities has been shown by E. Alvarez (40), leading thereby to a more or less correct description of the quark phase. As the density is lowered, this kind of approximation is less and less valid especially in the physically interesting region of the quark-hadron phase transition (assumed to exist, of course).

It follows that the hadron phase presumably cannot be described from these quark dynamics and statistical approximations and another model must be used. This is at least as difficult as for the quark phase (!) so that we shall use, for the sake of comparison, the very simple model considered by Olive (11), (12). (Thomas-Fermi approximation for hadrons (mainly π-mesons) interacting via the exchange of various particles (ω-mesons as to the baryons)).

In this paper, where mean field approximations are used, a quark (or an antiquark) is moving freely in the average potential of the other quarks and antiquarks. As a consequence, one can get rid off all the λ-matrices algebra since

$$< U(r) > \equiv < \vec{\lambda}_1 \cdot \vec{\lambda}_2 V(r) > \quad (18)$$

simply reduces to the average $q-\bar{q}$ potential, owing to the fact that the system as a whole is in a color singlet state.

3.1. Statistical description of the quarks

Let us first consider the statistical state of the quarks, the gluons being dealt with later on. There are only five known flavors (u, d, s, c, b) and the corresponding quarks differ only by their masses (at the temperatures considered, only the u, d, s quarks play a role since $m_\mu \sim m_d \sim 350$ MeV and $m_s \sim 550$ MeV, while $m_c \sim 1.5$ GeV and $m_b \sim 4.5$ GeV).

Let us consider the species i ; its average occupation number is

$$< n_i(p) > = \frac{1}{\exp.[\beta(\xi_i(p)-\mu)] +1} , \quad (19)$$

where $\beta \equiv (k_B T)^{-1}$; where μ is the chemical potential, which we take to be zero in the primordial universe, due to the fact that, at high temperatures, the quarks coming from the $B \neq 0$ matter constitute only a small "impurity" compared with the ones produced by the background blackbody radiation. In Eq. (19) $\xi_i(p)$ is the excitation spectrum of the quasi-quarks within the medium.

(1) In the Thomas-Fermi approximation, first considered by Olive in this context, it reads

$$\xi_i(p) = \{\vec{p}^2 + m^2(\vec{p})\}^{1/2} + V(d), \ i = 1,2,...,5 \quad (20)$$

where d is the average interquark distance, given by

$$\frac{4\pi}{3} d^3 = n_Q^{-1} \quad (21)$$

where n_Q is the total quark density, i.e. including both all species, all colors and also antiquarks :

$$n_Q = \sum_{i=1}^{i=5} n_i \tag{22}$$

with

$$n_i = g_Q \; d^3p \; < n_i(p) > , \tag{23}$$

where g_Q is the degeneracy factor

$$g_Q = 2 \text{ (particle/antiparticle) } \times 2 \text{ (spin) } \times 3 \text{ (color)} = 12. \tag{24}$$

Here two points are worth mentioning.

First and unlike what was done by (11) and (12), d should not necessarily be given by Eq. (21). For instance, at extremely high temperatures, the quark density is simply proportional to the photon density (there are only free particles in the medium and each of them contribute with the number of degrees of freedom they carry), i.e. by

$$n_Q \sim [g_Q/16] \cdot n_\gamma , \tag{25}$$

where the photon density is given by

$$n_\gamma = a\, T^3 \tag{26}$$

(a being the Stefan constant), so that we could take d as being

$$d = \left(\frac{12}{g_Q a \pi} \right)^{1/3} \frac{1}{T} . \tag{27}$$

In fact, Eqs. (21) and (27) represent two extreme cases : the low temperature case and the high temperature one, respectively. In fact, the correct screening factor d should be calculated from a study of the proper oscillation modes of the quark plasma. This is evaluated below.

Next, in Eq. (22) or (25) only quark densities come into play: this is due to the fact that a given quark is sensible only to color irrespective of the species considered in the interaction. However, this has been criticized by Boal et al. (15) who argued that gluon contributions (and hence gluon density) should be included in the evaluation of d. Their argument essentially amounts to saying that since quarks feel colored states, gluons should necessarily be taken into account. In fact, this argument rests also on their statistical treatment of gluons. Although it is perfectly right to argue that way, we shall take the opposite view for consistency reasons which we explain below.

Finally, Eqs. (19), (20), (22) and (23) lead to the following self-consistent equation

$$n_Q = \sum_{i=1}^{i=5} \frac{g_Q}{2\pi^2} \quad dp \, \frac{p^2}{\exp.[\beta\, \xi_i(p)] + 1} , \tag{28}$$

where <u>a priori</u> $\xi_i(p)$ depends on n_Q through d. Its solution provides the total quark density as a function of β which, once inserted into $< n_i(p) >$, allows the determination of the various thermodynamical quantities, such as the <u>energy density</u>

$$\rho = \sum_{i=1}^{i=5} \rho_i \tag{29}$$

with

$$\rho_i = \frac{g_Q}{2\;^2} \int dp \, \frac{p^2.[p^2 + m_i^2(p)]^{1/2}}{\exp.[\beta\, \xi_i(p)] + 1} + \frac{1}{2} n_i V(d) \tag{30}$$

where the factor 1/2 occuring in the last term is due to the need of avoiding a double counting of the interaction (note that in the mean field approximations considered in this paper, it appears in a natural manner). Similarly, the pressure is obtained as

$$P = \sum_{i=1}^{i=5} P_i \tag{31}$$

where

$$P_i = \frac{1}{3} < \vec{v} . \vec{p} > \tag{32}$$

the Hamiltonian used shows that

$$\vec{v} = \frac{\partial H}{\partial \vec{p}} \tag{33}$$

$$\vec{v} = \frac{\partial}{\partial \vec{p}} [\vec{p}^2 + m_i^2(\vec{p})]^{1/2} , \tag{34}$$

which, in the case where $m_i(p)$ = const., reduces as usual to

$$\vec{v} = \vec{p}/[\vec{p}^2 + m_i^2]^{1/2} \tag{35}$$

or

$$P_i = \frac{g_Q}{6\pi^2} \int dp \, \frac{p^3(\partial/\partial p)[p^2 + m_i^2(p)]^{1/2}}{\exp.\,[\beta\, \xi_i(p)] + 1} \tag{36}$$

Finally the whole thermodynamics of the quarks is completly determined in this approximation and it remains to evaluate the contributions of the gluons.

(2) In the true Hartree approximation (see e.g. (39)), the average potential is given by

$$< V(r) > = \int d^3\vec{r}'\; n_Q(\vec{r}').\; V(\vec{r}-\vec{r}') \quad , \tag{37}$$

which reduces to

$$< V(r) > = n_Q \int d^3r \; V(r) \tag{38}$$

in the case of a homogeneous thermal equilibrium, as considered here. In this case, the quasi-quark excitation spectrum is simply

$$\xi_i(p) = [p^2 + m_i^2(p)]^{1/2} + 4\pi\, n_Q \int dr\; r^2\; V(r) \tag{39}$$

In fact, the integral in this last equation does not extend to infinity but to the screening length d to be taken either from Eq. (21) as Olive or from Eq. (27) as Wagnoner *et al*. (7) or rather in an intermediate way as is done below.

When one takes Eq. (21) for d and in the high density limit $n_Q >> 1$, the last term of Eq. (39) can be written as

$$\begin{aligned} 4\pi\, n_Q \int dr\; r^2\; V(r) &\sim 4\pi\; n_Q\; d^3\; V(d) \\ &\sim 4\pi\; n_Q \frac{3}{4\pi n_Q}\; V(d) \\ &\sim 3\; V\; [(3/4\pi n_Q)^{1/3}]. \end{aligned} \tag{40}$$

Up to the factor 3, this is Olive's assumption (actually, if one takes his assumption seriously, his potential energy is underestimated by a factor 12, corresponding to the number of quarks felt by a given quark (i.e. 12 is roughly the average number of spheres tangent to a given one in an assembly of equal radius close-packed spheres)).

The thermodynamical quantities are calculated in a similar way as previously except for the energy density which reads (for the i-th species) (see (39)).

$$\rho_i = \frac{g_Q}{2\pi^2} \int dp\, \vec{p}^{\,2} \langle n_i(p)\rangle \{\xi_i(p) - \frac{1}{2} n_Q \int d^3r\, V(r)\} \quad (41)$$

$$= \frac{g_Q}{2\pi^2} \int dp\, \vec{p}^{\,2} \langle n_i(p)\rangle [\vec{p}^{\,2} + m_i^2(p)]^{1/2}$$

$$+ 2\pi\, n_Q \cdot n_i \int_0^d dr\, r^2\, V(r) \quad (42)$$

so that

$$\rho = \sum_{i=1}^{i=5} \frac{g_Q}{2\pi^2} \int dp\, \vec{p}^{\,2} \langle n_i(p)\rangle [\vec{p}^{\,2} + m_i^2(\vec{p})]^{1/2}$$

$$+ 2\pi\, n_Q^2 \int_0^d dr\, r^2\, V(r)\ . \quad (43)$$

These last equations show that in the Hartree approximation there is no need of any particular technique to deal with the double counting of the average potential energy.

(3) The next approximation which we consider in this paper is the Hartree-Fock one, because of the particular importance of exchange correlations at relatively low densities and/or temperatures. Also it has been proposed to deal with nuclear forces as the result of exchange forces between quarks (41).

In this case, the excitation spectrum is given by (39) the following integral equation

$$\xi_i(p) = \xi_i^{\mathrm{Hartree}}(p) - \frac{1}{(2\pi)^3} \int d^3p'\, \tilde{V}(\vec{p}\,'-\vec{p}) . \langle n_i(\vec{p}\,')\rangle , \quad (44)$$

where $\tilde{V}$ is the Fourier transform of the potential under consideration. This self-consistent equation can be solved by iteration and the result is used in the calculation of the pressure and of the energy density.

3.2. Statistical treatment of the gluons

The statistical analysis given above for the quarks can be repeated mutatis mutandis for the gluons. In particular, in our mean field approximations, their occupation number $\langle n_g(\vec{k})\rangle$ is given by

$$\langle n_g(\vec{k})\rangle = - \frac{1}{\exp.\,[\beta\omega(\vec{k})] - 1} \quad (45)$$

where $\omega(\vec{k})$ is their excitation spectrum. From Eq. (45) the contributions to pressure, energy density and gluon density are found via standard formulae (one should also remember that there are eight kinds of gluons).

The essential problem is thus the one of the derivation of the excitation spectrum $\omega(\vec{k})$. Olive (11) and (12), and also Boal et al. (15), chase

$$\omega(\vec{k}) = |\vec{k}| + V(d) \quad , \tag{46}$$

which embodies the facts that (i) the gluons are massless and (ii) they are interacting via the potential V (the same as the quark-quark potential and with the same Thomas-Fermi approximation, in Olive's article ; note that V = bd, the confining potential. in his paper) which therefore contributes to the gluon energy $\omega(\vec{k})$ by the factor V(d).

This approach can be questioned on several points. First, when considered inside matter, the gluons become massive, although one could distinguish (see e.g. (32)) between an "electric" and a "magnetic" mass (in fact, they are of the same order of magnitude). Next, the gluons do not appear as such within the medium but only as modes (or quasi-gluons or plasmons) propagating in the quark plasma.

We are thus faced with two problems : (i) how to confine the gluons and (ii) how to calculate the modes (i.e. the dispersion relation) of the quark plasma ?

In fact, as to the first problem, there is no particular need to confine the gluons since they are just modes propagating in the quark plasma : if the quarks are confined (below a critical temperature) there is no longer any possible modes ! Hence, no quarks implies no gluons.

The second problem is much more difficult to deal with essentially because of the lack of control of the approximations used. For instance, the Hartree approximation and the use of color singlet states as physical states lead to an excitation spectrum (37) similar to the one of a quantum elctrodynamical plasma (42) and (43). Other non-perturbative approachs provide other results (32). In the spirit of the approach used by Olive or by Boal et al. (15) as well as in this article, we should actually use a spectrum derived from our dynamical model ; this would be consistent with what was done previously and does not present any technical difficulties.

However, a common characteristic of most of the spectra obtained is that they are of the form

$$\omega^2(\vec{k}) \sim \omega_p^2 + \vec{k}^2 , \qquad (47)$$

where the plasma frequency ω_p^2 may differ slightly from model to model and also for "electric" and for "magnetic" modes. Eq. (47) may sometimes be a good approximation in the two limiting cases of long and short wavelengths. For the plasma frequency ω_p, instead of the relativistic quantum plasma frequency (42) and (43).

$$\omega_{pi}^2 = \frac{4\pi\alpha}{m} \int \frac{d^3p}{\xi_i(p)} < n_i(p) > , \qquad (48)$$

a natural generalization including both the results of the renormalization group equations and the excitation spectrum adopted for the quarks, can be used ; it reads

$$\omega_{pi}^2 = 4\pi \int \frac{d^3p}{\xi_i(p)} \frac{\alpha(p)}{m_i(p)} < n_i(p) > . \qquad (49)$$

Note also that there exists as many gluonic modes as quark flavors and, usually, with a third order degeneracy.

Finally, we are in position to come back to the screening length d which should be derived directly from the quasi-gluons excitation spectrum. In fact, K. Kajantie et al. (32) found in their non perturbative QCD approach

$$\left\{ \begin{array}{l} d_{elect.} \sim (1/\alpha T^2)^{1/2} \\ \\ d_{mag.} \sim \{ \frac{8\pi}{3T^2} \cdot (\frac{1}{\alpha^2})^{3/2} \}^{1/2} , \end{array} \right. \qquad (50)$$

which both behave as T^{-1}, showing thereby that Eq. (27) is not a so bad approximation. A more phenomenological approach for cases intermediate between the cold and the hot case is, more conventionally,

$$d \sim \frac{\omega_p}{V_{th}} \qquad (51)$$

where V_{th} is a relativistic thermal velocity (see e.g. J.L. Synge (44) or R. Hakim et al. (45)).

4. FLUCTUATIONS

The calculation of fluctuations for various physical quantities of importance in cosmology, such as density or energy density, can be performed in a particularly simple way within the framework of our mean field approximations. In particular, their spectrum can be obtained quite easily and also the typical coherence length associated with them.

Let us focuse our attention on the case of baryon number fluctuations or, equivalently, on the fluctuations of the quark density. Their spectrum $<\delta n^2>_{\omega,k}$ can be obtained from the spectrum of the quasi-quarks, $<\delta n^2>^0_{\omega,k}$, propagating within the plasma and from its "dielectric" constant $\varepsilon\ (\omega,\vec{k})$ (see e.g. (46) and (39) ; the relativistic case has been considered by H. Sivak (47)) through

$$<\delta n^2>_{\omega,k} = \frac{<\delta n^2>^0_{\omega,k}}{|\varepsilon(\omega,k)|} \tag{52}$$

while the k-spectrum is given by

$$\begin{aligned} <\delta n^2>_k &= \int d\omega \quad <\delta n^2>_{\omega,k} \\ &= \int d\omega \quad \frac{<\delta n^2>^0_{\omega,k}}{|\varepsilon(\omega,k)|} \end{aligned} \tag{53}$$

It should be noticed that this last equation actually contains two terms : one of them represents the coherent fluctuations, i.e. of the plasma waves, corresponding to the implicit pole terms in the denominator $|\varepsilon(\omega,k)|$ while the other one is nothing but the thermal fluctuations. The quasi-quark spectrum $<\delta n^2>^0_{\omega,k}$ is essentially a quasi-free fermian spectrum and has been calculated by H. Sivak (47) in the relativistic case as

$$\begin{aligned} <\delta n^2>^0_{\omega,k} &= \frac{1}{e^{\beta\omega}-1}\ \frac{1}{(2\pi)^2} \sum_{a,\ell=\pm 1} \int \frac{d^3p}{E_p . E_p^a} \times \\ &\times(\delta_{\ell 1} - <n(p)>)\ .\ \delta[E_p + \ell E_p^a + a\omega] \times \\ &\times\left[\frac{\omega^2 - \vec{k}^2}{2} + 2E_p^2 + 2\ a\omega E_p\right] \end{aligned} \tag{54}$$

In this equation, one has set

$$\begin{cases} E_p \equiv E(\vec{p}) \\ E_p^a \equiv E(\vec{p}+a\vec{k}). \end{cases} \tag{55}$$

Moreover, the $\delta_{\ell 1}$ - term represents a vacuum term which has to be dropped in our phenomenological approach. It can also be remarked that the term corresponding to $\ell=1$ being a high frequency term can be dropped in our cosmological context since we are essentially interested in more or less static modes and/or low wave lengths.

As to the calculation of the "dielectric" constant $\varepsilon(\omega,k)$ there is no particular problem in this approach, and only relativistic generalization (36), (49), of the usual calculation (46), (48), (39) must be considered.

As a final remark we may notice that, if we admit T.D. Lee's (50) suggestions as to the confinement mechanism of a color dielectric constant smaller than one in the hadron phase, then an enhancement of the density fluctuations should result (as shown an Eq.52)) in the transition region. This might be considered as an indication that the potential actually used leads to a confining phase transition, eventhough higher correlations were not dealt with.

5. CONCLUSION

The above discussion gives idea of numerous sources of uncertainties occuring in the dynamics and the statistical treatment of the quark-gluon plasma : uncertainties on the "correct" quark-quark potential (if any) ; uncertainties on the quark masses ; difficulties of a treatment of three-body correlations, etc. It should also be noticed that the hadron phase is as uncertain as the quark phase !

Consequently - still assuming the existence of a first order quark-hadron transition - it is not surprising that various calculations, resting on different assumptions, give rise to critical temperatures ranging from $\sim$ 150 MeV to 600 MeV ! On figure 5 we have plotted the pressure of the quark-gluon plasma versus the temperature and, for the sake of comparison the ideal gas case has been drawn. The "error bars" indicate ranges of pressure within which pass curves corresponding to different assumptions. Incidentally, some of our curves stop at a given temperature (of the order of 180 MeV to 250 MeV) where the self-consistent equation for n has no solution : they are indicated by dots and a hached region in the figure. These temperatures should not be interpreted as the critical temperatures we are looking for but rather as the indication of the breakdown of our statistical approximations.

Finally, even this phenomenological approach does not give credible answers to our question (as far as cosmology is concerned): this will be our (pessimistic) conclusion.

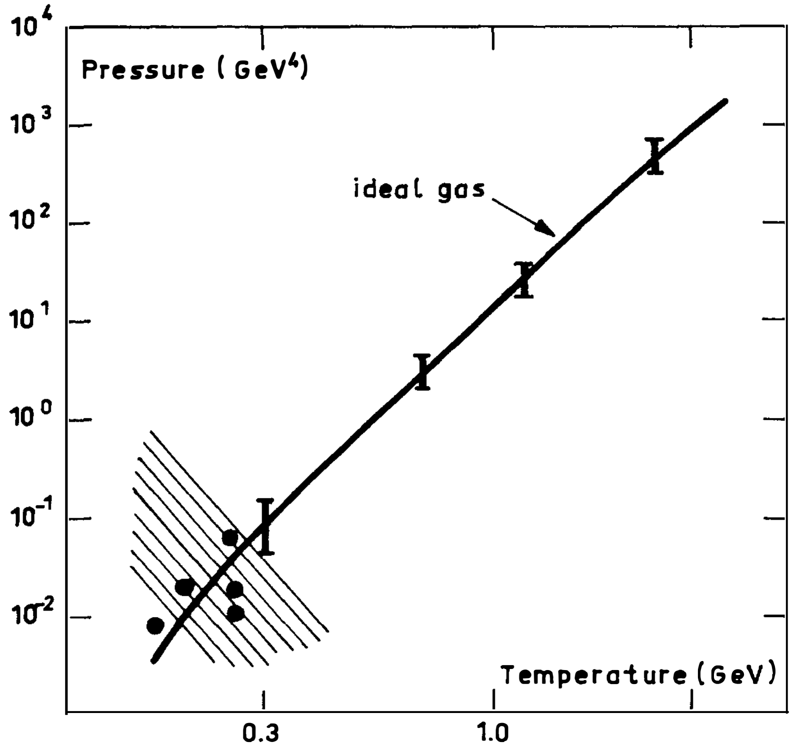


Figure 5 : The pressure-temperature diagram of the quark phase of the early universe; the ideal gas case has been plotted for the sake of comparison. The "error bars" indicate limits within which pass various curves corresponding to several assumptions discussed in the text. The dots indicate some points where the self-consistent equation for the density has no longer any solution, for various assumptions.

ACKNOWLEDGEMENTS

We are quite indebted to Drs. A. Le Yaouank, L. Oliver, O. Pène and J.C. Raynal for their patient explanations on quarks. We are also indebted to Dr. H. Sivak for communicating his results prior to publication.

APPPENDIX : Some phenomenological potentials

Ther are, at the present moment, dozens of more or less satisfactory phenomenological quark-antiquark potentials and we mention only a few among them.

(1) Pure confinement

$$v(r) = br$$

(2) "QCD" potential

$$v(r) = -\frac{a}{r} + br$$

(3) Improved "QCD" potential (25)

$$v(r) = \frac{8\pi}{27} \Lambda \left[\Lambda r - \frac{f(\lambda r)}{\lambda r}\right]$$

where,

$$f(t) = 1-4 \int dq \frac{\sin qt}{q} \left\{\frac{1}{\ln(1+q^2)} - \frac{1}{q^2}\right\}$$

with (27) $\Lambda = 430$ MeV, $m_u = m_d = 410$ MeV, $m_s = 625$ MeV.

(4) "Gluon condensate" (51)

$$v(r) = -\frac{4\alpha}{3r} + \frac{8}{5} \left\{ \left[\left(\frac{3\alpha}{2r}\right) + \frac{5\pi^2 M_0^2 r^2}{72} \right]^{1/2} - \frac{3\alpha}{2r} \right\}$$

with $\alpha = .4$ and $M_0 = 330$ MeV

(5) Martin's potential (52)

$$v(r) = -\ 8.064 \text{ GeV} + 6.870\ r^{.1}\ (\text{GeV})^{1.1}.$$

FOOTNOTES

1) Although this article deals with the transition to a pion-condensed state, most (if not all) the results apply to the quark-

hadron phase transition as well.
2) Let us note, however, some Hartree-Fock calculations (16).

REFERENCES

(1) Jones, B.J.T., Jones, J.E., 1983, ed. The origin and Evolution of Galaxies, (D. Reidel, Dordrecht, Holland).
(2) Hosotany, Y., 1983, Phys. Rev. D27, p. 789.
(3) Lascher, G., 1979, Phys. Rev. Letts. 42, p. 1646
(4) Hogan, C.J., 1982, Astrophys. J. 252, p. 418.
(5) Weinberg, S., 1972, "Gravitation and Cosmology" (Wiley ; New York).
(6) Fowler, G.N., Raha, S., Weiner, R.M., 1980, Preprint.
(7) Wagoner, R.V., Steigman, G., 1979, Phys. Rev. D20, p. 825.
(8) Dicus, D.A., Pati, J.C., Teplitz, V.L., 1980, Phys. Rev. D21 p. 922.
(9) Alvarez, E., Hakim, R., 1979, Phys. Rev. D19, p. 1696.
(10) Alvarez, E., Hakim, R., 1979, Astron. Astrophys. 80, p. 71.
(11) Olive, K., 1981, Nuclear Phys. B 190, p. 483.
(12) Olive, K., 1982, Nuclear Phys. B198, p. 461.
(13) Källmann, C.G., 1982, Phys. Letts. B112, p. 213.
(14) Källmann, C.G., Montonen, C., 1982, Phys. Letts. B115, p. 473.
(15) Boal, D., Schächter, J., Woloshyn, R., 1982, Phys. Rev. D26, p. 3245.
(16) Alvarez, E., 1981, Phys. Rev. D23, p. 1715.
(17) Kislinger, M.B., Morley, P.D., 1979, Phys. Reports 51, p. 63.
(18) Freedman, B.A., Mc Lerran, L.D., 1977, Phys. Rev. D16, p. 1130.
(19) Baluni, V., 1978, Phys. Rev. D17, p. 2092.
(20) Kapusta, J.I., 1979, Nuclear Phys. B148, p. 461.
(21) Shuryak, E.V., 1980, Phys. Reports 61, p. 71.
(22) Chodos, A., Jaffe, R.L., Johnson, K., Thorn, C.B., Weisskopf, V.F., 1974, Phys. Rev. D9, p. 3771.
(23) Müller, B., Rafelski, J., 1980, CERN preprint TH. 2928.
(24) Gavela, M.B., Le Yaouank, A., Oliver, L., Raynal, J.C., Sood, S., 1979, Phys. Letts. B82, 431.
(25) Richardson, J.R., 1979, Phys. Letts. 82B, 272.
(26) Rafelski, J., Viollier, R.D., 1979, CERN Report TH. 2673.
(27) Schöberl, F., 1982, CERN Report TH. 3287.
(28) Politzer, H.D., 1974, Phys. Reports 14C, 129.
(29) Pagels, H., Stokar, S., 1979, Phys. Rev. D20, 2947.
(30) Miller, K.J., Olsson, M.G., 1982, Preprint MAD/PH/57.
(31) Childers, R.W., 1982, Phys. Rev. D26, 2902.
(32) Kajantie, K., Kapusta, J.I., 1982, CERN Report TH. 3284.
(33) Wanyun, Zha, 1982, ISAS Preprint 63/82/ E.P.
(34) Bardeen, W.A., Chanowitz, M.S., Drell, S.D., Weinstein, M., Yan, T.M., 1975, Phys. Rev. D11, 1094.
(35) Rafelski, J., 1976, Phys. Rev. D14, 2358.
(36) Hakim, R., 1978, Riv. Nuovo Cim. 1, N°6.
(37) Hakim, R., 1981, Plasma Physics Techniques for QED and QCD

Systems in Statistical Mechanics of Quarks and Hadrons, ed. H. Satz, North Holland.
(38) Diaz-Alonso, J., Hakim, R., 1978, Phys. Letts. 66A, 466.
(39) Kadanoff, L.P., Baum, G., 1962, Quantum Statistical Mechanics, Benjamin, New York.
(40) Alvarez, E., 1982, Phys. Letts. 110B, 315.
(41) Barry, G.W., 1977, Phys. Rev. D16, 2886.
(42) Hakim, R., Heyvaerts, J., 1978, Phys. Rev. A18, 1250.
(43) Hakim, R., Heyvaerts, J., 1980, J. Phys. A13, 2001.
(44) Synge, J.L., 1957, The Relativistic Gas, Amsterdam, N. Holland.
(45) Hakim, R., Mangeney, A., 1971, Phys. Fluids 14, 2751.
(46) Sitenko, A.G., 1967, Electromagnetic Fluctuations in Plasma, Academic Press, New York.
(47) Sivak, 1982, submitted to Ann. Phys.
(48) Ichimaru, S., 1973, Basic Principles of Plasma Physics, Benjamin, Reading Mass.
(49) Carruthers, P., 1983, Rev. Mod. Phys. 55, 245
(50) Lee, T.D., 1981, Particle Physics and Introduction to Field Theory, Harwood Acad. Publ., New York.
(51) Flory, C.A., 1982, SLAC-PUB 2889.
(52) Martin, A., 1980, Phys. Letts. 93B, 338.
(53) Klinkhamer, F.R., 1982, "Quark Liberation at High Temperature", in "The Birth of the Universe", 17-th Moriond Meeting ; J. Audouze and J. Tran Than Van Eds.
(54) Schramm, D.N., Crawford, M., Planetary Mass Black Holes and the Quark-Hadron Transition in "The Birth of the Universe ; 17-th Moriond Meeting, 1982, ed. J. Audouze and J. Tran Than Van (Editions Frontières).
(55) Schramm, D.N., Crawford, M., 1982, Nature 298, 538.

NOTE ADDED :

Although they are not directly related to the quark-hadron transition in the early universe, we would like to mention two interesting papers connected with the subject. In the first one (53) emphasis is put on results from Monte Carlo simulations of gauge theories on a lattice which, indeed, do exhibit first order phase transition interpreted as a quark-hadron transition. In the second article (54) (55) simulations with classical quarks interacting via a simplified Richardson (25) potential are used in a study of primeval fluctuations.