

# Self-Compensation of Astigmatism in Mode-Cleaners for Advanced Interferometers

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**Abstract.** Using a conventional mode-cleaner with the output beam taken through a diagonal mirror it is impossible to achieve a non-astigmatic output. The geometrical astigmatism of triangular mode-cleaners for gravitational wave detectors can be self-compensated by thermally induced astigmatism in the mirrors substrates. We present results from finite element modelling of the temperature distribution of the suspended mode-cleaner mirrors and the associated beam profiles. We use these results to demonstrate and present a self-compensated mode-cleaner design. We show that the total astigmatism of the output beam can be reduced to  $5 \times 10^{-3}$  for  $\pm 10\%$  variation of input power about a nominal value when using the end mirror of the cavity as output coupler.

## 1. Introduction

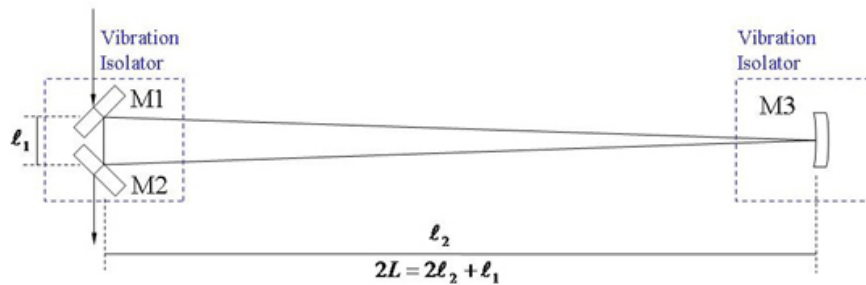
Input mode-cleaners are used in gravitational wave interferometers presently in operation. One on each LIGO detector in the USA [1], one in the French - Italian VIRGO [2], two in series at the British - German GEO600 [3] and one in TAMA300 in Japan [4]. As an important part of the input optics system mode-cleaners are used to reduce any spatial or frequency instability of the laser beam. In addition frequency fluctuations of the laser fundamental mode give rise to additional noise at the dark fringe. It provides passive stabilisation of time dependant higher order spatial modes, transmitting the fundamental mode  $TEM_{00}$  and attenuating the higher order modes.

Plans for the next generation of advanced interferometers (Advanced LIGO [5], LCGT [6] and AIGO) will also include at least one input mode-cleaner. In order to overcome photon shot noise high power ( $>100W$ ) single frequency, continuous wave Nd:YAG lasers are needed [7]. A small portion of the circulating power will remain in the mirrors due to substrate and coating absorptions. This energy will increase the temperature on the mirror causing a thermal expansion and a change in its radius of curvature [8]. Therefore the higher the circulating power the stronger the thermal effects.

It has been shown that triangular ring cavities like the mode-cleaner will always have a certain level of astigmatism due to the angles at which the beam impinges on the mirrors [10]. This creates a mild astigmatism in the beam that circulates inside the cavity, which is worsen when the input (or circulating) power is increased. This *geometrical astigmatism* is a consequence of the geometrical distribution of the mirrors in the triangular ring cavity. Whenever the beam crosses a piece of optics there is a change in path length between the centre and the outer beam [8]. This thermal lensing effect is induced by the power absorbed by the coating and the

substrate of the optics. If the beam crosses the output coupler at a relatively large angle then this effect will be stronger and even worse at high power. Similar effects were reported at the beam-splitter of the Phase Noise Interferometer at the Massachusetts Institute of Technology (MIT), where the beam crosses at  $\sim 45^\circ$  as well [9].

In this paper we first analyse the substrate deformation due to the thermal effects inside the mode-cleaner. This deformation introduces a *thermally induced astigmatism* inside the cavity. We propose a solution to this problem using the thermally induced astigmatism to advantage in order to compensate the geometrical astigmatism present in the mode-cleaner output beam. We also show the temperature distribution in the substrate when the mirror is used as an output coupler. This will be used to determine the thermal lensing effect of the output beam due to the different path between the centre and the outer parts of the beam. This will help us to quantify the astigmatism of the output beam and to select the best solution that minimise this effect.



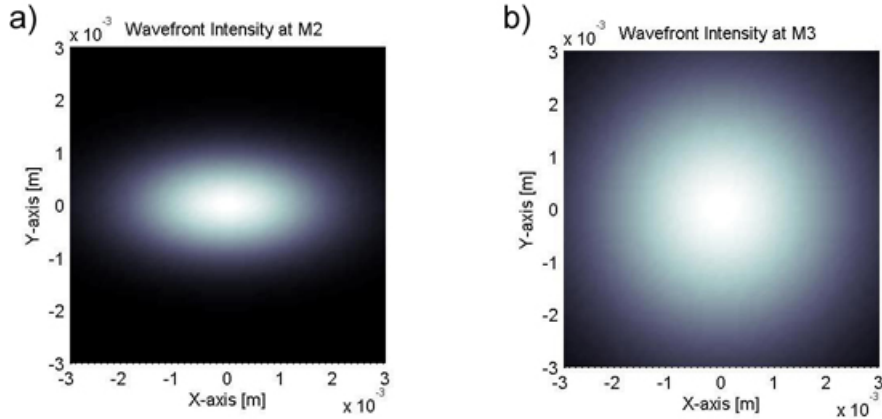
**Figure 1.** Due to its low optical feedback a triangular ring cavity using three suspended mirrors is preferred as the input mode-cleaner for interferometric gravitational wave detectors.

## 2. Substrate Deformation

Input mode-cleaners actually in use consist mainly of two flat mirrors that define the short side of the triangle and a concave mirror that forms the acute angle of an isosceles triangle as seen in figure 1. Due to this configuration and depending on the distance that separates the two flat mirrors from the concave end mirror (usually several metres) the laser spot at the flat mirrors will be strongly elliptical while at the end mirror will be closer to a circumference as can be see in figure 2. With an elliptical spot on the surface of the flat mirrors the thermal effects will be different for each axis.

We define a coordinate system where the x-axis is parallel to the plane of incidence on the cavity mirrors and the y-axis is perpendicular to it, leaving z-axis along the direction of propagation. The thermal effects due to the absorption at the dielectric coating and the deformation of the substrate as a consequence are stronger in the y-axis (tangential plane) than in the x-axis (sagittal plane). This causes an elliptical deformation of the mirror substrate producing differences in wave-front curvature between the two transverse directions or astigmatism. This effect needs to be carefully calculated since it will change the g-factor of the cavity, changing the transmission (or suppression) factor of the higher order modes.

We have shown that the astigmatism inside a triangular mode-cleaner is strongly dependant on the circulating power, which is defined by the input power and the cavity finesse [11]. Figure 3 shows the eccentricity variation of the M3 spot and cavity waist with input power. The eccentricity values depend on the radius of curvature of the mirrors that form the mode-cleaner and the cavity finesse. They also depend on the mirrors substrate and coating absorption. In our case we assume that all substrates are of fused silica with substrate absorption of  $2 \text{ ppm/cm}$ ,



**Figure 2.** a) Elliptical spot at the M2 mirror inside the cavity. The elliptical spot is the result of the  $45^\circ$  of the laser beam incident angle. b) Due to the long distance between M1/M2 to M3 the spot at the end mirror is nearly circular under the cold cavity conditions.

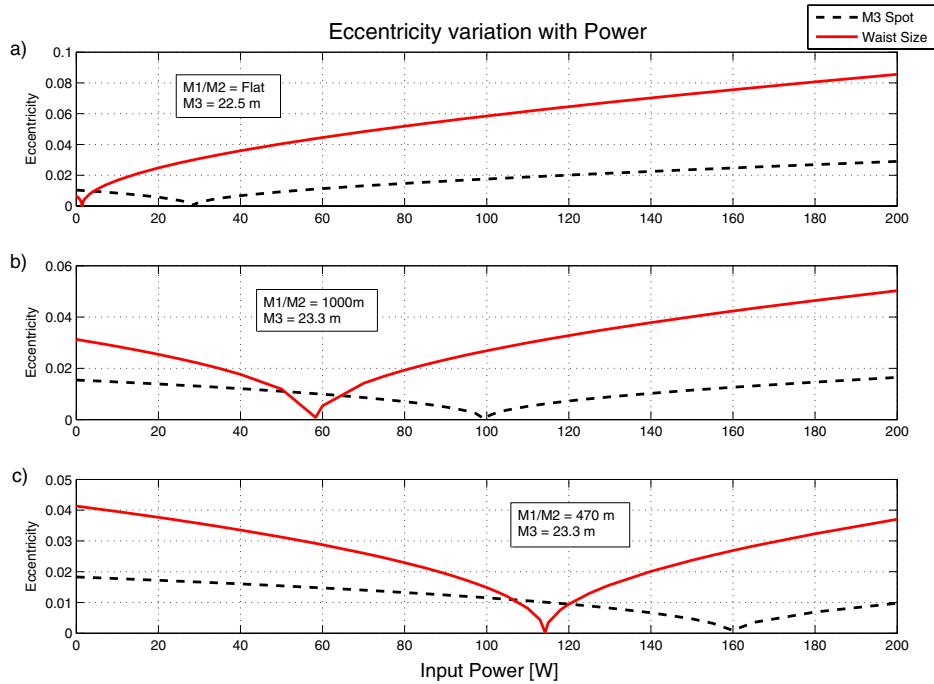
heat conductivity  $k = 1.38 \text{ W m}^{-1} \text{ K}^{-1}$ , thermal expansion coefficient  $\alpha = 0.51 \times 10^{-6} \text{ K}^{-1}$  and refractive index temperature dependence  $\beta = 8.7 \times 10^{-6} \text{ K}^{-1}$ .

Input mode-cleaners at the operational gravitational wave interferometers are all designed with two flat mirrors and one end concave mirror at the acute end of it. In this case when operating at low power the cavity waist will have an almost circular profile. As soon as the circulating power is increased the waist gets strongly elliptical and therefore with high eccentricity, as shown in figure 3a. This figure also shows that operating a standard mode-cleaner at a 100 W of input power we have a relatively high eccentricity of  $\sim 0.06$  at the waist, even if we use M3 as an output coupler we still get an eccentricity of  $\sim 0.02$ .

When using the design proposed in figure 3b, we obtain a beam free of astigmatism at mirror M3. This result is very difficult to obtain in reality due to power variations and the fact that coating and substrates are never exactly the ones predicted. However we notice that a variation of  $\pm 10\%$  in the input power only increases the eccentricity from nearly 0 to 0.005 when using M3 as an output coupler. A similar case is presented in figure 3c. With a different design, this time for 160 W of input power. In this case a variation of  $\pm 10\%$  in the input the input power increases the eccentricity up to 0.006 when using M3 as the output coupler.

By choosing different radius of curvature for the mirrors and combining them with lower absorption coatings it is possible to design a mode cleaner free of astigmatism. When doing the design it is important to maintain the intended cavity g-factor, in order to keep the higher order modes non-degenerated in the cavity. Graphs at figure 3 only consider the thermal deformation of the substrate. However we note that with the right combination it is possible to design a mode-cleaner free of astigmatism for different input power levels. It is interesting to see that at the waist and at M3 there is also a change in the ellipse major axis. The thermal effects inside the cavity will stretch the y-axis to the point that crosses a circular profile. It even gets larger than the x-axis when the input power (and therefore the circulating power) is increased. The geometrical astigmatism has the opposite sign from the thermally induced astigmatism from the mirror absorption. Using the parameters for the mirrors we show this thermally induced astigmatism can be used to correct the intrinsic geometrical astigmatism of an isosceles triangular ring cavity mode-cleaner. The balance of these two effects can lead to a self compensated mode-cleaner.

For the simulations here presented it was assumed a perfectly mode matched gaussian beam,



**Figure 3.** Waist and M3 spot eccentricity variation with input power. The upper graph shows two flat mirrors and an end concave mirror assuming coating absorption of 1ppm (best result for M3 at 28.5 W). Figure 3.b shows the cavity tuned for 100 W input power, while 3.c shows an Advanced LIGO type of mode-cleaner tuned for 160 W of input power. For the last two cases we assumed coating absorption of 0.5 ppm.

however the effects of the substrate in the input beam when entering the cavity were considered. In reality the input beam quality is such that the input performance of the mode-cleaner is dominated by the poor quality of the input beam. Therefore mode matching losses are generally significantly smaller than the mode cleaning losses, and as a consequence negligible.

### 3. Thermal Lensing

The thermal lensing effects as described by Hello and Vinet will cause a deformation in the curvature of the wave-front of the transmitted beam [12]. This change mainly depends on the effect produced by the power absorbed by the dielectric coating due to the high power circulating inside the cavity. In our case with a finesse of  $\sim 1500$  and  $\sim 100$  W of input power there will be  $\sim 45$  kW of circulating power. There is also a small contribution from the power absorbed by the substrate during the transmission of the output beam.

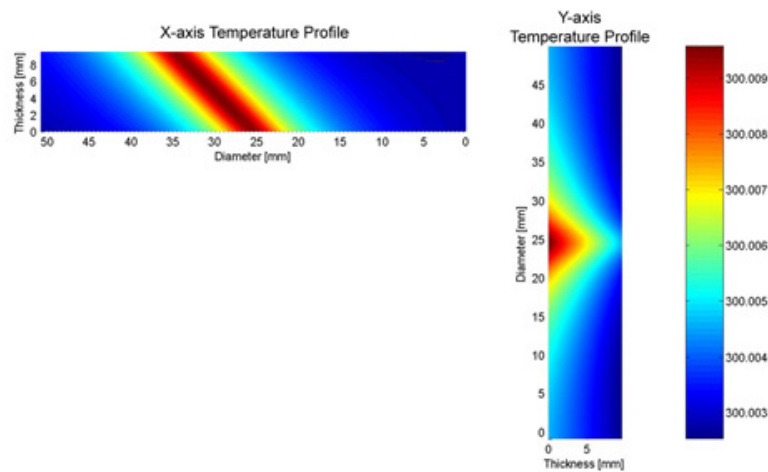
In most cases the two flat mirrors are used as input and output couplers for the mode-cleaner cavity. This can be a problem in high optical power mode-cleaners, not only due to the deformation of the mirror's substrate, but mainly due to the thermal lensing effects when transmitting the beam through the output coupler.

Figure 4 shows the steady state temperature distribution of a flat mirror when used as the output coupler. By crossing the substrate at almost  $45^\circ$  the laser beam will cross a larger section of substrate, which will increase the thermal lensing effect produced by the temperature distribution compared to a perpendicular transmission. The main contribution to the thermal lensing from the power absorbed by the coating, is even larger as can be inferred from the higher

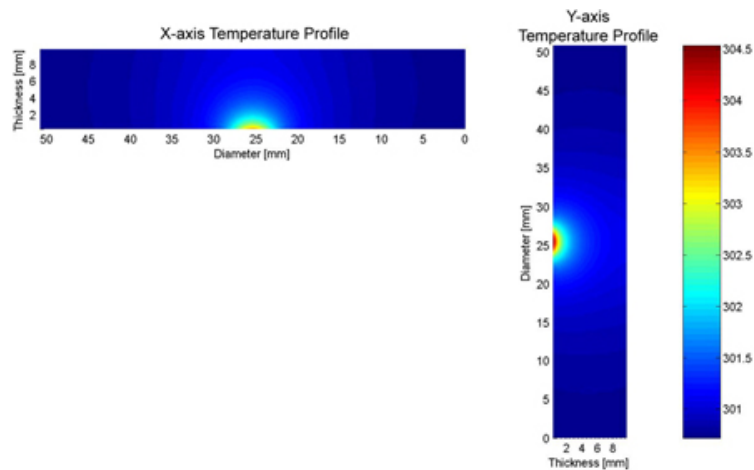
temperatures in figure 5.

The wave-front change in the light path can be quantified as a change in the focal length of the beam. In this case there is higher temperature along the y-axis (vertical cross-section) due to a smaller spot size along the y-axis. This will produce a stronger deformation of the wave-front that will in consequence produce a stronger astigmatism in the output beam. Since the distortion is different for each axis we will study both separately.

By calculating the wave-front deformation including bulk and coating absorptions it is possible to determine the best lens fit for it. Using a MATLAB<sup>®</sup> code written by the authors



**Figure 4.** Steady state solution for the bulk absorption case. If M2 is used as output coupler the diagonally transmitted beam produces strong astigmatic thermal lensing. (90.5 W transmitted power, 0.6ppm coating absorption, fused silica substrate absorption of 2ppm/cm).



**Figure 5.** Steady state temperature distributions for coating absorption due to the circulating power inside the mode-cleaner. M2 high reflectivity coating absorption produces astigmatic thermal lensing. The spot ellipticity produces different distribution between X and Y axis. (45 kW of circulating power, 0.6 ppm coating absorption, fused silica substrate absorption of 2 ppm/cm).

it was possible to calculate the new focal length for the output beam. This method was later compared to FFT simulations obtaining very close agreement.

The main contributors to the thermally induced optical path change are the dependence of the refractive index on temperature, the strain and the thermal expansion. Different authors [13, 14] have already examined these effects, which are summarised in equation (1).

$$\delta s = 1.32 \frac{p_a}{4\pi k} \left( \frac{\partial n}{\partial T} - \frac{n^3}{2} \rho_{12} \alpha + 2\alpha n \frac{\omega}{d} \right) d. \quad (1)$$

Here  $\delta s$  corresponds to the path change,  $p_a$  to the power absorbed per unit length,  $k$  to the heat conductivity,  $\partial n/\partial T = \beta$  to the refractive index change with temperature,  $n$  the fused silica refractive index,  $\rho_{12}$  the fused silica photo elastic coefficient,  $\alpha$  the thermo elastic coefficient,  $\omega$  the beam radius of the intensity profile and  $d$  is the substrate thickness.

If we use M2 as the output coupler the output beam will be strongly astigmatic. This astigmatism according to our simulations presented in table I will lead to a focal length difference between the x-axis and the y-axis of  $\sim 133$  m.

If instead of using M2 we use M3 as the output coupler the astigmatism will be much smaller due to the small ellipticity of the spot, producing similar thermal effects in both axes. Even though at table I the results for M3 look the same they have a difference in focal length of about  $1 \mu\text{m}$ .

**Table 1.** Output beam thermal lensing focal length. Best result is obtained when using M3 as output coupler. Input power of 99.6 W gives a focal length difference between X and Y axis of  $\sim 10^{-6}m$ .

	X - axis (m)	Y - axis (m)	
M2 output coupler	269.61	136.28	Strong astigmatism
M3 output coupler	253.82	253.82	Mild astigmatism

#### 4. Conclusions

The current mode-cleaner designs in use for interferometric gravitational wave detectors have mild astigmatism at low input power due only to the geometrical distribution of the mirrors. The same design used with higher laser power leads to significant astigmatism of the output beam due to thermal effects in the mode-cleaner mirrors. We have shown that the balance of the two effects can lead to a self-compensated high optical power mode-cleaner. Our proposed solution for the AIGO mode-cleaner is to use fused silica substrates, M1 and M2 with a radius of curvature of 1000 m and 23.3 m for the end mirror, assuming coating absorption losses of 0.5 ppm. The need for an adaptive optic element for astigmatism control and mode-matching can be avoided, since using M3 as output coupler contributes negligible additional astigmatism in the output beam. The design presented keeps the astigmatism below 0.5% over a  $\pm 10\%$  power range for 100 W of input power. Self compensation can be adjusted to any input power level using an additional single fixed astigmatic corrector at the output.

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