IMPLICATIONS OF RECENT RESULTS FROM THE SOLAR NEUTRINO EXPERIMENTS(*)

A. Dar and S. Nussinov

Abstract

The recent results from the Kamiokande II and Baksan solar neutrino experiments, if correct, imply that lepton flavour is not conserved. The Mikheyev–Smirnov–Wolfenstein (MSW) solution to the solar neutrino problem, which was first exposed by the Homestake Cl Experiment, fully explains also these results if the electron neutrino is mixed with the muon neutrino or the tau neutrino with mixing parameters $\Delta m^2 \sim 10^{-6}$ eV^2 and $\sin^2 2\theta \sim 4 \times 10^{-2}$. The MSW solution will be tested by the new generation of solar neutrino experiments that will be able to detect the "missing" solar ν_e as ν_μ or ν_τ. Further evidence may be obtained from the neutronization burst from the birth of a neutron star in a nearby supernova. Moreover, the MSW solution combined with the seesaw mechanism for generating neutrino masses further suggests $m_{\nu_e} \sim 10^{-8}$ eV, $m_{\nu_\mu} \sim 10^{-3}$ eV, $m_{\nu_\tau} \sim 10$ eV, and $\sin^2 2\theta \sim 4 \times 10^{-2}$ for $\nu_\mu \nu_\tau$ mixing. These predictions can be tested by previously proposed neutrino oscillation experiments at accelerators and by detecting neutrinos from a nearby supernova explosion. A ~ 10 eV tau neutrino can account for most of the dark matter in the Universe and is a viable candidate for the hot dark matter scenario of the formation of large scale structure in the Universe.

1. Introduction

The recent results from the Large Electron Positron (LEP) collider at the European Organization for Nuclear Research (CERN) show that there are only three generations of light neutrinos [1]. Their interactions are well described by the standard minimal electroweak theory [2–4] where neutrinos are usually assumed to be massless (and therefore also unmixed). However, no established symmetry require this. In fact, massive neutrinos are expected in most extensions of the minimal model. In particular, the seesaw mechanism relates the masses of the neutrinos to corresponding Dirac fermions [5, 6]. If we identify the latter with the up quarks, then

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} \approx m_u^2 : m_c^2 : m_t^2 \approx 0.005^2 : 1.5^2 : 135^2 , \quad (1)$$

where we used a recent estimate for the mass of the top quark [1], $m_t = (135 \pm 20)$ GeV.

The many attempts to determine neutrino masses and mixings, involving terrestrial experiments as well as astrophysical and cosmological observations, so far have yielded only upper bounds. The only positive indication came [7] from the Cl solar neutrino experiment of Davis et al. [8,9]. During the past twenty years the observed production rate of ^{37}Ar in the Cl experiment [10] has been much smaller than that predicted by Bahcall [11] from the Standard Solar Model (SSM), with an average value

$$R_{exp} = (2.1 \pm 0.3)\ \text{SNU} = (0.27 \pm 0.04)\ R_{SSM} , \quad (2)$$

for neutrino above the 0.81 MeV threshold energy. This has become known as "the solar neutrino problem" but, in spite of the insistence of Bahcall, it was not widely accepted as an evidence for physics beyond the minimal electroweak model. However, the recent results from the Kamiokande II (KII) solar neutrino experiment [12] and the preliminary results from the Soviet–American Gallium Experiment (SAGE) [13], if confirmed by future measurements, may change this picture dramatically.

The KII Experiment has detected [12] electron recoils from scattering of neutrinos from the direction of the sun with energies above 9.3 MeV during 450 days and above 7.5 MeV during another 590 days, with a rate

$$R_{exp} = (0.46 \pm 0.05\,[stat] \pm 0.06\,[syst])\ R_{SSM} , \quad (3)$$

where the quoted errors are one standard deviation.

The SAGE reported [13] seeing no decays of ^{71}Ge atoms, during the first 11.43 days half-lifetime of ^{71}Ge, after extraction of the Ge atoms from four runs (January, February, March and April 1990), while they expected to see ~ 14 events corresponding to 132 SNU (Solar Neutrino Unit) predicted [11] by the SSM. From these results their best estimate was

$$R_{exp} = (0.00 \pm 0.52)\ R_{SSM} , \quad (4)$$

for neutrinos with energies above the 0.23 MeV threshold energy.

The results reported by KII and the preliminary results reported by SAGE, if confirmed, imply that lepton flavour is not conserved. This can be seen in sect. 2.

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2. Violation of lepton flavour conservation

The general success of stellar evolution theory suggests that hydrogen fusion, via the *pp* and carbon–nitrogen–oxygen (CNO) nuclear reaction chains, powers the stars like the sun. The ^{8}B solar neutrino flux is very sensitive to the conditions in the sun [7]. Therefore, the fact that KII detects solar neutrinos with an energy spectrum consistent with that of ^{8}B and a flux which agrees within a factor ~ 2 with the SSM predictions [7,14] is a strong indication that the sun does derive its energy from fusion of hydrogen into helium. Baryon number, charge and lepton flavour conservation then require that the net reaction is

$$4p + 2e^- \rightarrow {}^4\mathrm{He} + 2\nu_e + Q \ , \qquad (5)$$

and energy conservation requires that $Q \approx 26.78$ MeV. Consequently, for a steady sun that derives its energy from fusion of hydrogen into helium, the total solar neutrino flux at earth is given by(*)

$$\phi_\nu = \frac{2L_\odot}{Q - 2\bar{E}_\nu}\frac{1}{4\pi D^2} \geq \frac{2L_\odot}{Q}\frac{1}{4\pi D^2} \approx 6.4\times10^{10}\mathrm{cm}^{-2}\mathrm{s}^{-1} \ , \qquad (6)$$

where $L_\odot \approx 3.86 \times 10^{33}$ erg/s is the solar luminosity, $\bar{E}_\nu$ is the average neutrino energy and $D \approx 1.496 \times 10^{13}$ cm is the distance to the sun. Among all conceivable neutrino producing reactions in the sun the *pp* reaction has the smallest $\bar{E}_\nu$ (~ 0.27 MeV). With this value substituted in eq. (6), the predicted (minimal) solar neutrino flux is $\phi_\nu \approx 6.6 \times 10^{10}$ cm^{-2}s^{-1}, in excellent agreement with the **total** flux obtained from the SSM [11] (the agreement is expected because the KII and ^{37}Cl experiments observe much smaller fluxes of more energetic neutrinos and consequently $\bar{E}_\nu \approx \bar{E}_\nu(pp)$). Since this minimal solar flux is larger than the SSM *pp* flux [11] by ~ 10%, one obtains that the capture rate of solar neutrinos in gallium must be larger than 1.1 × 70.8 SNU ≈ 78 SNU, where 70.8 SNU is the SSM *pp* capture rate [7]. If the capture rate in the gallium experiment is significantly smaller, as suggested by the preliminary results of SAGE, it implies that the solar neutrino flux that reaches earth is different from that produced by the sun. This may be due to [11] decay in flight, and/or helicity flip, and/or flavour change of the solar neutrinos. The neutrino decay solution has been ruled out [15] by the arrival of neutrinos from SN1987A at a distance of ~ 165 000 light-years. Helicity flip via neutrino magnetic interaction (that may explain on the one hand the reduced capture rate and its variation with time in the ^{37}Cl Experiment [16], and on the other hand the observed solar neutrino flux and the absence of any significant time variation in the KII Experiment [17]), requires neutrino magnetic moment which is inconsistent with bounds on the neutrino magnetic moment that were derived from the cooling rate of He stars [18] and from big bang nucleosynthesis [19]. Therefore, we conclude that the preliminary SAGE results, if correct, imply that **lepton flavour is not conserved**. This conclusion has far reaching implications for particle physics, astrophysics and cosmology.

(*) One of us (A. Dar) has learned this argument from a private discussion with S. Weinberg and V.F. Weisskopf in 1967.

3. The Mikheyev–Smirnov–Wolfenstein solution

A simple manifestation of lepton flavour violation is neutrino oscillations [20,21] and the MSW effect [22–26]. In fact, the experimental results of all three solar neutrino experiments agree well with the MSW solution to the solar neutrino problem [27–29] provided

$$\Delta m^2 \sim 10^{-6}\,\mathrm{eV}^2 \ \ \mathit{and} \ \ \sin^2 2\theta \sim 4\times10^{-2} \ . \qquad (7)$$

This can be seen directly from the following simple considerations [30, 31].

The MSW effect implies that if a ν_e on its way out of the sun encounters a resonant electron density

$$n_e = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}\, G_F E_\nu} \ , \qquad (8)$$

then it may change into ν_μ (or ν_τ) with a considerable probability. The probability P_r that the ν_e retains its flavour [32–35] is given by the Landau–Zenner formula

$$P_r \approx e^{-\epsilon/E_\nu}, \ \ \text{where} \ \ \epsilon \equiv \frac{\pi H_r \Delta m^2 \sin^2 2\theta}{4\cos 2\theta} \qquad (9)$$

and $H_r \equiv |(n_e/(dn_e/dr))|$ is the scale height in the sun at the resonance density. Due to further vacuum oscillations the overall probability of the solar neutrinos to retain their flavour until they reach earth is

$$P \approx \cos^2\theta\, P_r + \sin^2\theta(1 - P_r) \ . \qquad (10)$$

The solar neutrino flux at earth is therefore given by

$$\Phi_\nu = \Phi_{\nu_e} + \Phi_{\nu_\mu} \ ; \ \Phi_{\nu_e} \approx P\Phi_\nu \ ; \ \Phi_{\nu_\mu} \approx (1-P)\Phi_\nu \ , \qquad (11)$$

where ϕ_ν is the flux predicted by the SSM.

The Cl detector is blind to ν_μ, but has a relatively low, 0.81 MeV, energy threshold and can see the ν_e both from ^{8}B and from CNO, ^{7}Be and *pep*. The KII Experiment can detect both ν_e and ν_μ, (with a relative cross section $\sigma(\nu_\mu e) \approx (1/6)\,\sigma(\nu_e e)$), but cannot distinguish between them. Because of its 7.5 MeV energy threshold it can detect only the ^{8}B flux. If only ^{8}B neutrinos contribute to the Cl capture rate, eq. (11) implies that

$P \approx 2.1/6.1 \approx 0.35$, where 6.1 SNU is the SSM ^{8}B contribution [11] to the Cl capture rate. In that case the KII detector sees an effective ν_e flux ~ 35% + (1/6)65% ≈ 46% of the SSM flux, in excellent agreement with the observed flux (eq. (3)). We shall show later that small mixing angles are required if the gallium capture rate is indeed very small. For small mixing angles eq. (10) implies that $P_r \approx P \approx 0.35$. Since $\sigma\phi$, the capture rate of ^{8}B neutrinos in ^{37}Cl, peaks around 10 MeV, it follows from eq. (9) that $\in \approx 10.7$ MeV, and consequently

$$\Delta m^2 \sin^2 2\theta \approx 3.8\times 10^{-9}(R_\odot / H_r)\,\mathrm{eV}^2 \approx 4\times 10^{-8}\,\mathrm{eV}^2 , \qquad (12)$$

where $R_\odot/H_r \approx 10.5$ near the centre of the sun [7]. Essentially this solution was found for the Cl and KII experiments by Rosen and Gelb [36, 37].

From eqs (9) and (10) it follows that the CNO, ^{7}Be and *pep* solar ν_e fluxes are strongly suppressed by the MSW effect while the solar ν_e flux from ^{8}B is reduced only by a factor $P \sim 0.35$. Consequently, we expect the counting rate in the gallium experiments to be above 0.35 × 14 SNU ≈ 4.9 SNU, where 14 SNU is the SSM ^{8}B contribution to the gallium capture rate [7]. If $R_{exp} \approx 4.9$ SNU then all the solar neutrinos with energy > 0.23 MeV (the gallium threshold energy), must encounter a resonance density on their way out of the sun. Since the production region of *pp* neutrinos extends to $r \approx 0.25\, R_\odot$, eq. (8) predicts that $\Delta m^2 < 7\times 10^{-7}$ eV^2. In fact, because of vacuum oscillations the capture rate (in SNU) must be above 4.9 SNU and satisfy

$$\sin^2\theta \approx \frac{R_{exp} - 4.9}{132 - 14} . \qquad (13)$$

If the ^{71}Ge production rate is indeed of the order of 7 SNU$^{(*)}$ then eqs (12) and (13) yield

$$\Delta m^2 \sim 6\times 10^{-7}\,\mathrm{eV}^2 \; ; \; \sin^2 2\theta \sim 7\times 10^{-2} . \qquad (14)$$

4. Implications of the MSW solution

Testable implications of the MSW solution include:

The appearance of the missing solar ν_e flux as a ν_μ (or ν_τ) flux. If this ν_μ flux will be detected by the new generation of solar neutrino detectors [11], which are sensitive also to the neutral current interactions of the ^{8}B neutrinos (for example, the Sudbury Neutrino Observatory (SNO) heavy water Cherenkov detector or the Super Kamiokande light water Cherenkov detector [38]), it will provide a convincing proof for the MSW solution.

The day-night effect [34–35, 39–40]. The MSW solution predicts a considerable flavour conversion of ^{7}B*e* neutrinos and part of the ^{8}B neutrinos when they cross earth before reaching the detectors during nighttime. However, the predicted effect is small and difficult to detect with radiochemical detectors [35].

A short ν_μ neutronization burst from Type II supernova explosions. Because of the relatively large scale height in the stellar envelope of a giant star, $H >> H_\odot$, eq. (9) implies that practically all the ν_e from the expected short (few milliseconds) neutronization burst from a Type II supernova explosion [41] will be converted via the MSW effect in the stellar envelope into a ν_μ (or ν_τ) burst. Such a short ν_μ neutronization burst from a nearby Type II supernova explosion could be detected by the new generation of neutrino telescopes such as the SNO heavy water Cherenkov detector. **The short neutronization burst of ν_μ can be used to measure m_{ν_μ} or to limit it to below ~ 0.1 eV if the burst lasts few milliseconds.**

Non detectable $\nu_e \leftrightarrow \nu_\mu$ oscillations of accelerator, reactor or atmospheric neutrinos. The tiny Δm^2 and the small mixing precludes seeing any $\nu_e \leftrightarrow \nu_\mu$ oscillations of accelerator, reactor or atmospheric neutrinos.

5. The MSW effect and the seesaw mechanism

Let us now consider the implications of the MSW solution combined with the seesaw mechanism [5], which is the only scheme presently known for naturally generating small, non-vanishing, neutrino masses. The Dirac masses of the quarks and leptons in the ith generation and a heavy right-handed ν_R of a mass M_R fix the mass of the ordinary ν_i via

$$m_{\nu_i} \approx m_{F_i}^2 / M_R . \qquad (15)$$

We implicitly assumed a generation independent M_R usually associated with a new scale of left-right [42] or grand-unified symmetry breaking [43]. While the actual value of M_R is unknown (and ranges between 10–100 M_W and M_{Planck}) it has been emphasized [44] that the pattern of ν_i mass ratios is extremely useful. To fix these ratios we assume that the Dirac masses of the upper members of the lepton and quark doublets in each generation are proportional, yielding eq. (1). Adopting the value $\Delta m^2 \approx 10^{-6}$ eV^2 suggested by the MSW solution to the solar neutrino problem we then find

$$m_{\nu_e} \sim 10^{-8}\,\mathrm{eV} \; ; \; m_{\nu_\mu} \sim 10^{-3}\,\mathrm{eV} \; ; \; m_{\nu_\tau} \sim 10\,\mathrm{eV} . \qquad (16)$$

Equation (15) also implies a rather high, $M_R \sim 10^{12}$ GeV, right-handed mass scale.

(*) The best fits to the ^{71}Ge production rate during the first four months that were reported by SAGE in ref. [14] were 0, 29, 0 and 0 SNU, respectively, corresponding to an average production rate of ~ 7 SNU.

Let us turn to the issue of the neutrino mixing angles. In two generations the seesaw mechanism prescribes the diagonalization of the 4 × 4 matrix.

$$\begin{pmatrix} 0 & 0 & m_1 & m_{12} \\ 0 & 0 & m_{12} & m_2 \\ m_1 & m_{12} & M_R & 0 \\ m_{12} & m_2 & 0 & M_R \end{pmatrix} \underset{\text{Diag in 1-2 subspace}}{\rightarrow} \begin{pmatrix} 0 & 0 & m_1' & 0 \\ 0 & 0 & 0 & m_2' \\ m_1' & 0 & M_R & 0 \\ 0 & m_2' & 0 & M_R \end{pmatrix} . \quad (17)$$

To get $m_1' << m_2'$ we can take $m_{12} \approx \sqrt{m_1 m_2}$. In this case we find a 1–2 mixing angle

$$\theta_{12} \approx \sqrt{m_1 / m_2} \ . \quad (18)$$

Such a mixing scheme has been suggested [45] sometime ago in connection with the Kobayashi–Maskawa (KM) matrix.

We note that in the MSW solution, $\theta_{\nu_e\nu_\mu} \approx 0.1$ is about a factor 2 smaller than the Cabbibo angle and a factor 2 larger than $\sqrt{m_\mu/m_c}$. If a similar pattern holds also for $\nu_\mu\nu_\tau$ mixing then, assuming that $\theta_{13} \leq \theta_{12}\theta_{13} \approx 0.01$ in order to consider separately the (1, 2) and (2, 3) blocks of the 3 × 3 matrix, we expect a 2–3 mixing angle, $\theta_{\nu_\mu\nu_\tau}$ between $\sqrt{m_c/m_t} \approx 0.11$ and $V_{bc}/2 \approx 0.025$.

6. Implications of the MSW solution with seesaw

In this section we point out some additional implications for particle physics, astrophysics and cosmology of the above suggested neutrino masses and mixings.

The neutrino lifetimes are extremely long. Simple dimensional arguments based on the small neutrino masses (small phase space for decay products), the small neutrino mixings and the strength of the weak interaction show that neutrino lifetimes are extremely long compared with the age of the Universe and have no observational consequences for astrophysics and cosmology.

The Majorana mass of ν_e is far too small to manifest in neutrinoless double β-decay experiments [46–47]. Also the small $\nu_e\nu_\tau$ mixing cannot give, even with $m_{\nu_\tau} \sim 10$ eV, any observable signal. The contribution of right-handed currents is also likely to be small when M_R is so big.

Appearance $\nu_\mu \rightarrow \nu_\tau$ experiments are feasible. The relatively large values, $\Delta m^2 \geq 30$ eV^2 and $2 \times 10^{-3} < \sin^2 2\theta < 4 \times 10^{-2}$, which the MSW solution and the seesaw mechanism suggest for $\nu_\mu\nu_\tau$ mixing, imply that the proposed $\nu_\mu \rightarrow \nu_\tau$ appearance experiment [48] at Fermi National Accelerator Laboratory (FNAL) has an excellent chance [49] of seeing the ν_τ. The proposed path length of ~ 1 km exceeds the oscillation length 2.47 $\bar{E}$[MeV]/Δm^2[eV^2] m ~ 200 m. The total number of ~ 10^6 interactions will lead for the above parameters to $10^3 - 10^4$ ν_τ events which should unambiguously be picked up in the massive emulsion detector. In fact, the upper portion of the mixing angle is already excluded [50].

Supernova neutrinos can be used to determine m_{μ_τ}. Future neutrino telescopes such as the SNO and Super Kamiokande will be sensitive enough to detect ν_μ and ν_τ from galactic Type II supernova explosions. A ~ 10 eV mass of the tau-neutrino may yield a noticeable time delay between the arrival times of ν_τ and of ν_e and ν_μ from the thermal neutrino burst of a nearby Type II supernova explosion, which may be used to measure [51] m_{ν_τ}.

Neutrinos can account for most of the dark matter in the universe. A stable ν_τ with a mass in the 10–50 eV range contributes a cosmological energy density [52] $\rho_\nu \approx n_{\nu_\tau} m_{\nu_\tau} \approx (3/11) n_\gamma m_{\nu_\tau} \approx 110\ m_{\nu_\tau}$ cm^{-3} $\approx 1 - 5$ keV cm^{-3}, similar to the closure density, $\rho_c \approx 10.6\ h^2$ keV cm^{-3}, where $0.5 < h < 1$, h being the Hubble constant in units of 100 km Mpc^{-1}s^{-1}. If the present Universe consists only of baryonic matter, neutrinos and photons, and if we use our "favoured" value $m_{\nu_\tau} \approx 10$ eV, suggested by the MSW solution and the seesaw estimate, and the recent estimate [53] $h \sim 0.87 \pm 0.12$, we obtain $\Omega \equiv \rho/\rho_c \approx 0.15$, in good agreement with observations and in particular with the value deduced from the dynamics of clusters of galaxies [54].

The dark matter in galactic hallos is more likely to be hadronic. Light neutrinos ($m_\nu \sim 10$ eV) are unlikely to form galactic halos [55] of dwarf and spiral galaxies. Within the standard particle physics model the remaining candidates for dark matter particles are baryons. Thus, in this particular scenario, the dark matter in galactic halos is likely to be hadronic (for example, brown dwarfs, white dwarfs, neutron stars and black holes). Interestingly, the cosmological baryonic mass density that was derived from big bang nucleosynthesis [56], $\rho_b \approx n_b m_p \approx 0.12$ keV cm^{-3}, i.e. $\Omega_b \sim 0.012\ h^{-2}$, although ~ 10 times smaller than the neutrino mass density, is consistent with the amount of dark matter in galactic halos. The measured luminosity density in the Universe [57] is $L \approx 2.4 \times 10^8\ hL_\odot/Mpc^3$. The masses of spiral and elliptical galaxies deduced from rotational curves, velocity dispersions and escape velocities, and recently also from gravitational lensing [58] yield average mass to light ratio of $\langle M/L \rangle \approx 16\ hM_\odot/L_\odot$. Consequently, $\Omega_{halo} \approx \langle M/L \rangle_G\ L/\rho_c \approx 1.4 \times 10^{-2}$, in excellent agreement with $\Omega_b \approx 1 \times 10^{-2}\ h^{-2}$ from the Standard Big Bang Nucleosynthesis (SBBN) [39] (especially if one uses the recent estimate $h = 0.87 \pm 0.12$).

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Addresses:

A. Dar
Dep. of Physics and Space Research Inst.
Technion-Israel Inst. of Technology,
IL–32 000 Haifa (Israel)

S. Nussinov
Dep. of Physics and Astronomy
Tel Aviv University
IL–69 978 Tel Aviv (Israel)

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