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Effects of the quark anomalous magnetic moment in the chiral symmetry restoration: magnetic catalysis and inverse magnetic catalysis

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Abstract In this work, we consider the effect of a constant anomalous magnetic moment (AMM) of quarks in the SU(2) Nambu–Jona-Lasinio model in the mean field approximation. To this end, we use the Schwinger *ansatz*, which represents a linear magnetic field term in the Lagrangian. A regularization method inspired in the vacuum magnetic regularization (VMR) is adopted to avoid ultraviolet divergences. Our results indicate a smooth decrease of the pseudocritical temperature and quark condensates for magnetic fields $B \leq 0.1$ GeV^2 when a sizable AMM is considered. We found only a small window for Inverse Magnetic Catalysis (IMC), in contradiction with NJL predictions made in the literature. For a low value of AMM, we observe for all ranges of magnetic fields considered that the pseudocritical temperature increases with the magnetic field, indicating only Magnetic Catalysis (MC). In our approach, for nonvanishing quark AMM, the chiral symmetry restoration happens always as a smooth crossover and never turns into a first order phase transition.

1 Introduction

Strong magnetic fields can be generated by heavy ion collisions, with a magnitude of $eB \sim 10^{19}$ G, and this possibility has revealed an impressive amount of effects to explore. It can be useful to understand some fundamental aspects of quantum chromodynamics (QCD) that has general interesting, e.g, the Chiral Magnetic Effect [1–3], and its role in the violation of $\mathcal{P}$ and $\mathcal{CP}$ invariance. Experimental observables,

as the azimuthal correlator γ [4], are expected to be affected by such magnetic fields. Besides, magnetars with $eB \sim 10^{16}$ G [5] are natural laboratories in the universe to study such scenarios at high densities.

The role of strong magnetic fields is widely explored in effective models due its simplicity. Several quantities are then predicted, as the magnetic catalysis effect (MC) [6–8] characterized by the increasing of the chiral condensate with the magnetic fields. This effect can alter many important quantities, as the pressure, sound velocity, heat capacity of the system [9–12] or even alter the pole-masses of light mesons [13–22]. There is also a possibility of such fields to alter significantly the measure of the elliptic flow v_2 as a direct result of the paramagnetic nature of QCD vacuum [12,23].

The evaluations taken by lattice quantum chromodynamics (LQCD) confirms the MC effect at low temperatures. But a totally different result is observed close to the pseudocritical temperature, where the chiral condensate decreases non monotonically its value as the strong magnetic fields grow, i.e., for $eB \gtrsim 0.2$ GeV^2. This is the well-know inverse magnetic catalysis effect (IMC) [24–26], more details can be found in recent reviews [27,28]. Such evaluations are considered with physical pion masses [24], where it is also observed the decreasing of the pseudocritical temperature. For heavy pions, the chiral condensate suffers a MC, but the decreasing of the pseudocritical temperature still persists [29,30] .

There are several attempts trying to explain such a contradiction between LQCD results and the lack of theoretical predictions for IMC using effective models of QCD. See Refs [6,7] for reviews. Some proposals explore the evaluations of the Nambu–Jona-Lasinio (NJL) model beyond mean field approximation [31], others study the implementation of a magnetic or thermo-magnetic dependence of the coupling constant of the model fitted by LQCD data [9,32–36] and

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with the implementation of the anomalous magnetic moment (AMM) of quarks [37–40].

The study of AMM of quarks has started more than twenty years ago, and several approaches were used [41]. But, the relation of the AMM with IMC is calling the attention very recently by some authors [37–40,42]. It is possible to draw a QCD phase diagram with the influence of the AMM phenomenologically estimated by quark models [37] or, even, to evaluate the electromagnetic vertex function to predict the value of AMM as a function dependent on the temperature and magnetic fields [43]. The mesonic excitations, as well as thermodynamic properties, have been explored with the AMM influence [39,40,44]. Also, some results at finite density with light mesons or thermodynamic properties are reported by [45,46].

In the context of the NJL model [47,48] and its extensions, as de PNJL [49], the four-fermion interaction channel provides a non-renormalizable theory at $3 + 1$ D, and one needs to adopt a regularization prescription to avoid ultraviolet divergences. In this way, based on well known results of [50–52], and recently explored in Ref. [53], the magnetic field independent regularizations (MFIR) present to be satisfactory when treating the implementation of a constant external magnetic field. This regularization prescription completely separates the magnetic field contributions from the vacuum or thermal quantities. On the other hand, most of the non-MFIR based regularizations showed unphysical results [54,55], as oscillations in the chiral quark condensate [53] or tachyonic neutral pion masses [13]. There are also other regularizations that can avoid these oscillations, as the Pauli-Villars scheme [22,31,56] or the Vacuum magnetic regularization scheme [12,36].

To include the AMM of quarks, most of the cited works made use of the linear magnetic field *ansatz* proposed by Schwinger in the context of quantum electrodynamics (QED) [57]. However, in the context of finite temperatures and zero density, none of these works apply a proper separation of the pure magnetic contributions from the vacuum, like MFIR or VMR regularization schemes. In a QED framework, it is remarkable the results of Ref. [58], where the one-loop effective potential with the AMM of the electron added through the Pauli term is investigated. Moreover, an exact expression up to the order α^2 is achieved. The range of applicability of this approach was discussed for the electron-positron pair-production [59–61], as well as when applied to quarks [62]. Other possibilities are also explored, as in Ref. [63], where the quarks' masses and the AMM appear as a result of a dynamical generation associated with a scalar and an appropriated tensor channel in the Lowest Landau Level approximation.

In this work, we consider in an effective way the effects of the AMM of the quarks in a magnetized medium through the addition of the phenomenological Pauli term to the SU(2)

NJL model. In order to treat the vacuum divergent term, we make use of an ingenious formalism developed in the context of the QED theory in Ref. [58] to incorporate the electron AMM effectively where the vacuum term is given as an analytic expression similar to the one obtained by Euler-Heisenberg [64], Weisskop [65] and Schwinger [66], where the usual summation on the Landau levels is performed in an exact way. We adapt the latter formalism originally proposed for the renormalizable QED theory for the non-renormalizable SU(2) NJL model. This is possible, once the SU(2) NJL model with AMM is implemented with the Pauli term in the same manner as the Dirac equation in [58] . If the correct replacements for QED to QCD coefficients, i.e, number of colors, electric charges and so on, are correclty done, the solutions of the Dirac equation, as well as the one-loop approximation for the effective potential are rigorously the same of our case. Notice that all the expressions used in this work for the magnetic effects are derived based on the Schwinger proper-time formalism. In particular, when the AMM is absent it is possible to show that the proper-time integral associated with the vacuum one-loop magnetic contribution can be performed analytically and usual expressions often obtained using dimensional regularization are recovered [12,52,53]. The divergent effective potential is treated by using an appropriate subtraction scheme where we separate clearly and analytically the convergent term from the divergent ones. In contrast to renormalizable theories, the way the divergences are treated is to be considered as a part of the definition of the model. We choose to use the 3D cut-off technique in the VMR which has been proven to be reliable [12,53] for the NJL model. In fact, this formalism has been shown to guarantee that the Gell-Mann-Oakes-Renner relation is respected [12]. One of the main objectives of the present work is to show the importance of a proper regularization procedure. In the literature several ways have been used to deal with the infinities inherent to the non-renormalizable NJL model, such as Form Factors [37], B-dependent cutoff [39], Pauli-Villars [56]. Besides the infinite vacuum contribution, often in previous works the finite medium term has also being affected by the regularization procedure, which has been shown to be an inadequate procedure [53].

The inclusion of AMM effects depends on the choice of the anomalous magnetic moments of the quarks. We use two sets of AMM values given in the literature [37]. Here, our focus is to analyze the behavior of the effective quark mass as a function of temperature, magnetic field strength and its dependence on the AMM of the quarks. In addition, the importance of the regularization procedure will be emphasized. Our final aim is to study the behavior of the pseudo-critical temperature as a function of the magnetic field and the influence of the AMM in this case. This will allow us to see if the mechanism of the inverse magnetic catalysis can be associated to the inclusion of AMM of quarks, as has been reported in

the recent literature. As we have already mentioned before, the formalism adopted here has its limit of validity that, of course, has been taken into account in our calculations.

This work is organized as follows. In Sect. 2 we present the SU(2) NJL model with the AMM term included. In Sect. 3, we develop the formalism, in Sect. 4 we obtain the GAP equation. Numerical results are presented in Sect. 5 and our the conclusions in Sect. 6. In the Appendices A and B specific technical details are discussed.

2 Lagrangian of the SU(2) NJL model with AMM

The lagrangian of the SU(2) NJL model with anomalous magnetic moment in an external electromagnetic field is given by the following expression [37]

$$
\mathcal{L} = \overline{\psi} \left(i \slashed{D} - \hat{m} + \frac{1}{2} \hat{a} \sigma^{\mu\nu} F^{\mu\nu} \right) \psi
$$
$$
+ G \left[(\overline{\psi}\psi)^2 + (\overline{\psi} i \gamma_5 \boldsymbol{\tau} \psi)^2 \right], \tag{1}
$$

where A^μ, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ are respectively the electromagnetic gauge and tensor fields, G represents the coupling constant, $\boldsymbol{\tau}$ are isospin Pauli matrices, Q is the diagonal quark charge [1] matrix, Q=diag(q_u= 2/3, q_d=-1/3), $D^\mu = (\partial^\mu + i e Q A^\mu)$ is the covariant derivative, ψ is the quark fermion field, and $\hat{m}$ represents the bare quark mass matrix,

$$
\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}, \quad \hat{m} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}. \tag{2}
$$

We consider here m_u=$m_d = m$ and choose the Landau gauge, $A_\mu = \delta_{\mu 2} x_1 B$, which satisfies $\nabla \cdot \vec{A} = 0$ and $\nabla \times \vec{A} = \vec{B} = B \hat{e}_3$, i. e., resulting in a constant magnetic field in the z-direction.

In the mean field approximation, the lagrangian $\mathcal{L}$ is denoted by

$$
\mathcal{L} = \overline{\psi} \left(i \slashed{D} - M + \frac{1}{2} \hat{a} \sigma_{\mu\nu} F^{\mu\nu} \right) \psi - \frac{(M-m)^2}{4G}, \tag{3}
$$

where the constituent quark mass is defined by

$$
M = m - 2G \langle \overline{\psi} \psi \rangle. \tag{4}
$$

where $\langle \overline{\psi} \psi \rangle$ is the chiral quark condensate. The AMM factor $\hat{a}$ is given by $\hat{a}$=diag(a_u, a_d) with $a_f = q_f \alpha_f \mu_B$, where $f = u, d$ represents the quark flavor. In the one-loop level approximation, the previous quantities are given by

$$
\alpha_f = \frac{\alpha_e q_f^2}{2\pi}, \quad \alpha_e = \frac{1}{137}, \quad \mu_B = \frac{e}{2M}. \tag{5}
$$

3 Effective potential with AMM

Using the approach adopted in Ref. [58], we adapt the non-regularized QED effective lagrangian in a constant external magnetic field with the AMM of the electron, in the one-loop approximation, to the SU(2) NJL model. For this purpose, we can write

$$
\mathcal{L}(B) = \sum_{f=u,d} \frac{N_c}{8\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-i\mathcal{K}_{0f}^2 s} \frac{q_f e B s}{\sin(q_f e B s)}
$$
$$
\times \cos(2M \eta_f B s), \tag{6}
$$

where we have adopted the following definitions

$$
\mathcal{K}_{0f}^2 = M^2 + a_f^2 B^2, \tag{7}
$$
$$
\eta_f = -q_f (\alpha_f + 1) \mu_B. \tag{8}
$$

Using the transformation $s \to -is$ in Eq. (6) and defining $q_f e B = B_f$, we achieve

$$
\mathcal{L}(B) = \sum_{f=u,d} \frac{N_c}{8\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-\mathcal{K}_{0f}^2 s} \frac{B_f s}{\sinh(B_f s)}
$$
$$
\times \cosh[(\alpha_f + 1) B_f s]. \tag{9}
$$

The expression for $\mathcal{L}(B)$ is clearly divergent and we will apply the vacuum magnetic regularization (VMR) scheme to regularize this quantity. After some simple steps, that are described in the Appendix A, we rewrite $\mathcal{L}(B)$ as

$$
\mathcal{L}(B) = \Omega^{mag} + \Omega^{vac} + \Omega^{field}, \tag{10}
$$

where:

$$
\Omega^{mag} = \sum_{f=u,d} \frac{N_c}{8\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-s\mathcal{K}_{0f}^2}
$$
$$
\times \left\{ \frac{B_f s \cosh[(\alpha_f + 1) B_f s]}{\sinh(B_f s)} \right.
$$
$$
\left. - 1 - \frac{1}{6}[3(\alpha_f + 1)^2 - 1](B_f s)^2 \right\}, \tag{11}
$$

The contributions Ω^{vac} and Ω^{field} must be regularized. We choose the 3D sharp cutoff scheme and the following expressions are given by

$$
\Omega^{vac} = -\frac{N_c}{8\pi^2} \sum_{f=u,d} \left\{ \Lambda[\Lambda^2 + \epsilon_f^2(\Lambda)] \epsilon_f(\Lambda) \right.
$$
$$
\left. - \mathcal{K}_{0f}^4 \ln \left[\frac{\Lambda + \epsilon_f(\Lambda)}{\mathcal{K}_{0f}} \right] \right\}, \tag{12}
$$

and for Ω^{field}

$$
\Omega^{field} = -\sum_{f=u,d} \frac{N_c B_f^2}{48\pi^2} \left[3(\alpha_f + 1)^2 - 1 \right] \ln \frac{\mathcal{K}_{0f}^2}{\Lambda^2}. \tag{13}
$$

where $\epsilon_f^2(\Lambda) = \mathcal{K}_{0f}^2 + \Lambda^2$. We will find, at the end of the derivations, the following thermodynamic potential of the SU(2) NJL model

$$\Omega = \frac{(M-m)^2}{4G} + \Omega^{vac} + \Omega^{field}$$
$$+ \Omega^{mag} + \Omega^{Tmag} + \frac{1}{2}B^2, \tag{14}$$

where Ω^{Tmag} is the finite thermo-magnetic contribution of the medium, given by the following expression:

$$\Omega^{Tmag} = -\frac{1}{\beta} \sum_{f=u,d} \frac{N_c|B_f|}{4\pi^2} \sum_{n=0}^{\infty} \sum_{s=\pm 1} \int_{-\infty}^{\infty} dp_3$$
$$\times \left[\ln\left(f^+(E_{n,s}^f) \right) + \ln\left(f^-(E_{n,s}^f) \right) \right], \tag{15}$$

We have defined the following function $f^{\pm}(E_{n,s}^f) = 1 + e^{-\beta(E_{n,s}^f \mp \mu_f)}$, where the inverse of the temperature is defined as $\beta = \frac{1}{T}$, $s = \pm 1$, is the spin index and μ_f is the chemical potential for a quark of flavor f. Differently from the former quantities, the thermo-magnetic contribution is written in terms of a summation of the Landau Levels, n, and the energy levels of the quarks, $E_{n,s}^f$, given by.

$$E_{n,s}^f = \sqrt{p_3^2 + (M_{n,s}^f - sa_fB)^2}, \tag{16}$$

with $M_{n,s}^f = \sqrt{|B_f|(2n+1-s_fs) + M^2}$, where $s_f = \text{sign}(q_f)$ being sign(x) the signal function of x.

4 The gap equation

To evaluate the effective quark masses under external constant magnetic fields, we evaluate $\frac{\partial \Omega}{\partial M} = 0$. The gap equation can be written as:

$$\frac{M-m}{2G} - \sum_{f=u,d} [h_f^{vac}(M) + h_f^{field}(M, eB)$$
$$+ h_f^{mag}(M, eB) + h_f^{Tmag}(M, eB, T)] = 0. \tag{17}$$

Each piece of the above equation is defined as follows. The vacuum-magnetic $h_f^{vac}(M)$ contribution is given by

$$h_f^{vac}(M) = \frac{MN_c}{2\pi^2} \left\{ \Lambda \epsilon_f(\Lambda) - \mathcal{K}_{0f}^2 \ln\left[\frac{\Lambda + \epsilon_f(\Lambda)}{\mathcal{K}_{0f}} \right] \right\}, \tag{18}$$

The remaining magnetic field dependent contributions at $T = 0$, $h_f^{field}(M, eB)$, and $h_f^{mag}(M, eB)$ are given by

$$h_f^{field}(M, eB) = \frac{MN_cB_f^2}{24\pi^2} \frac{[3(\alpha_f + 1)^2 - 1]}{\mathcal{K}_{0f}^2}, \tag{19}$$

$$h_f^{mag}(M, eB) = \frac{MN_c}{4\pi^2} \int_0^{\infty} \frac{ds}{s^2} e^{-s\mathcal{K}_{0f}^2}$$
$$\times \left\{ \frac{B_fs \coth[(\alpha_f + 1)B_fs]}{\sinh(B_fs)} \right.$$
$$\left. - 1 - \frac{1}{6} \left[3(\alpha_f + 1)^2 - 1 \right] (B_fs)^2 \right\}. \tag{20}$$

At last, the thermal $h_f^{Tmag}(M, eB, T)$ contribution is, therefore, defined as

$$h_f^{Tmag}(M, eB, T)$$
$$= -\frac{MN_c|B_f|}{4\pi^2} \sum_{n=0}^{\infty} \sum_{s=\pm 1} \int_{-\infty}^{\infty} \frac{dp_3}{E_{n,s}^f}$$
$$\times \left(\frac{1}{1 + e^{\beta(E_{n,s}^f - \mu_f)}} + \frac{1}{1 + e^{\beta(E_{n,s}^f + \mu_f)}} \right)$$
$$\times \left(1 - \frac{sa_fB}{M_{n,s}^f} \right). \tag{21}$$

Solving the Eq. (17) we obtain the effective quark masses with the influence of AMM. To this end, we adopt the criteria of Ref. [37], where the anomalous magnetic moment is considered as a constant value through the relation $k_f = \alpha_f/2M$. Therefore, $\mathcal{K}_{0f}$ in Eq. (7) is redefined as

$$\mathcal{K}_{0f}^2 = M^2 + k_f^2 B_f^2. \tag{22}$$

The last equation results from the definition $a_f = q_f \alpha_f \mu_B \rightarrow a_f = q_f e k_f$.

5 Results

In this work we adopt the following set of parameters: $\Lambda = 591.6$ MeV, $m = 5.7233$ MeV and $G = 2.404/\Lambda^2$. These parameters are chosen such that we have the following values in the vacuum: pion mass $m_\pi = 138$ MeV, pion decay constant $f_\pi = 92.4$ MeV, and the chiral condensate $\langle \overline{u}u \rangle = (-241\text{MeV})^3$ [53]. We adopt two sets of parameters for the constant values of k_f that are obtained in [37]. In the first set $\kappa^{[1]}$, we consider the sizable quark AMM values

$$k_u^{[1]} = 0.29016 \text{ GeV}^{-1}, \quad k_d^{[1]} = 0.35986 \text{ GeV}^{-1},$$
$$\alpha_u^{[1]} = 0.242, \qquad\qquad \alpha_d^{[1]} = 0.304, \tag{23}$$

and the second set $\kappa^{[2]}$ is given by

$$k_u^{[2]} = 0.00995 \text{ GeV}^{-1}, \quad k_d^{[2]} = 0.07975 \text{ GeV}^{-1},$$
$$\alpha_u^{[2]} = 0.006, \qquad\qquad \alpha_d^{[2]} = 0.056, \tag{24}$$

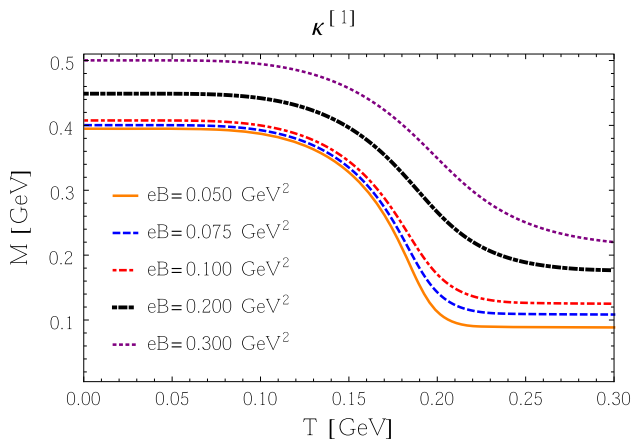The details of this parametrization can be seen in Ref. [37, 41].

Fig. 1 Effective quark mass as a function of the temperature for different values of fixed magnetic field strength. For these results, it was adopted the set $\kappa^{[1]}$
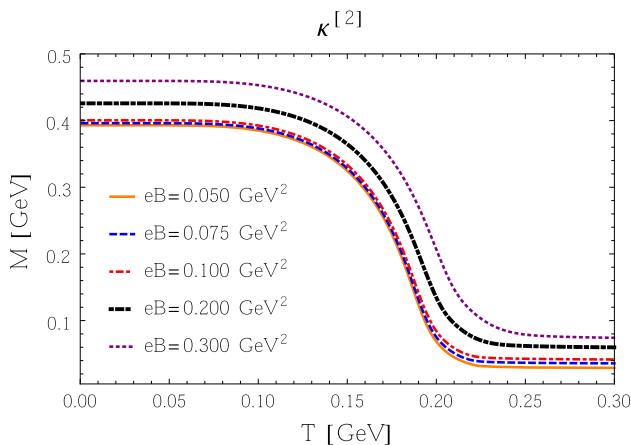

Fig. 2 Effective quark mass as function of the temperature for different values of fixed magnetic field strength. For these results, was adopted the set $\kappa^{[2]}$

Fig. 3 Effective quark masses as a function of the temperature for $eB = 0.05$ GeV^2 (top panel) and $eB = 0.2$ GeV^2 (low panel) with fixed values of κ

In Fig. 1, we show the effective quark mass as a function of the temperature for fixed values of the magnetic field using the set $\kappa^{[1]}$. As observed, the partial chiral symmetry restoration is not fully obtained for all eB values considered. The limit $M \approx m$ is not reached at temperatures higher than the pseudocritical temperature, $T >> T_c$. This is a pure effect of the lower bound in the effective quark masses generated by the constrain obtained in Appendix B. For instance, for $eB = 0.1$ GeV^2, $M \gtrsim 120$ MeV. The lower bound on the effective quark masses increases as we grow the magnetic field strength.

We show in Fig. 2 as in Fig. 1 the behavior of the effective quark masses as a function of the temperature for fixed eB, fixing in this case $\kappa = \kappa^{[2]}$. The inferior limit of the effective quark masses are lower when compared with the results of the $\kappa^{[1]}$ set, therefore, the chiral symmetry is almost restored in the high temperature regime through a smooth crossover. It is clear from Figs. 1 and 2 that for both sets used, the magnetic
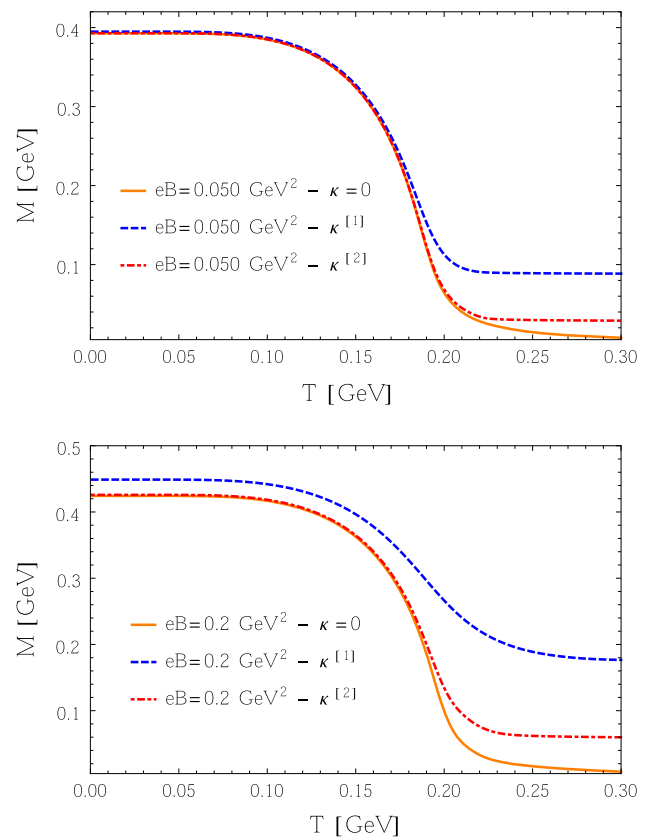
catalysis effect for $T = 0$, in contradiction with the results presented in Ref. [39,42]. Also, the type of transition that we observe is a smooth crossover, differently from the first order observed in Ref. [37]. In Ref. [38], the authors also observe a first order phase transition, but they consider confinement effects through the Polyakov loop in their calculations.

We present in Fig. 3 the effective quark mass varying with the temperature for $\kappa = 0$, $\kappa = \kappa^{[1]}$ and $\kappa = \kappa^{[2]}$. In the top panel of Fig. 3, we fix $eB = 0.05$ GeV^2. All the three curves shows similar results for $T < 200$ MeV. On the other hand, the lower bound of the $\kappa^{[1]}$ and $\kappa^{[2]}$ determines the limit of the effective quark mass, and different values are obtained in the region $T > 200$ MeV. In the low panel, we show the same quantities with the magnetic field, $eB = 0.2$ GeV^2, fixed. As we can see at low temperatures, the results show a clear difference, with $M_{\kappa^{[1]}} > M_{\kappa^{[2]}} > M_{\kappa=0}$. At high temperatures, the lower bounds strongly determines the limit of the effective quark masses in the partial chiral symmetry restoration phase, which is not fully completed for $\kappa^{[1]}$ and $\kappa^{[2]}$.

To study how the pseudocritical temperature changes with the magnetic field when we consider fixed κ_f values, we evaluate $-\partial M/\partial T$. The peak of each curve in Fig. 4 repre-

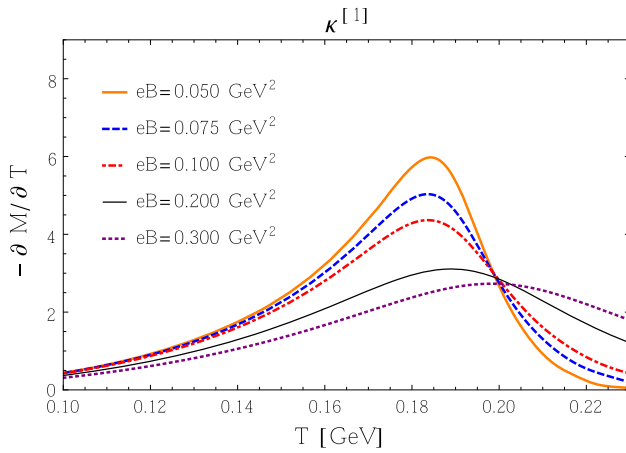**Fig. 4** $-\partial M/\partial T$ as function of the temperature, for different fixed values of magnetic field with fixed $\kappa^{[1]}$
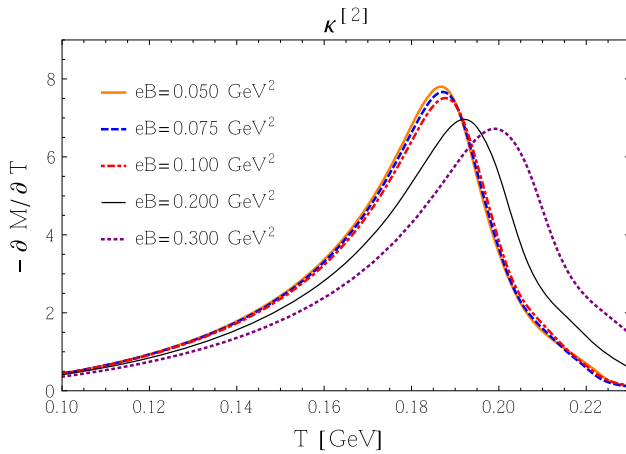

Fig. 5 $-\partial M/\partial T$ as function of the temperature, for different fixed values of magnetic field with fixed $\kappa^{[2]}$

Fig. 6 The pseudocritical temperature for the chiral transition of magnetized quark matter as a function of the magnetic field for fixed values of AMM κ

Fig. 7 Effective quark mass as function of the magnetic field for $T = 0.05$ GeV (top panel) and $T = 0.140$ GeV (low panel) with fixed values of κ

sents the transition point, T_c with $\kappa^{[1]}$. We can see a smooth decreasing of T_c at $eB \leqslant 0.1$ GeV^2. Beyond this, the pseudocritical temperature grows as we increase the magnetic fields. On the other hand, when we fix $\kappa^{[2]}$, one observes the increase of the pseudocritical temperature in Fig. 5 for all values of magnetic fields. The behavior of the pseudocritical temperature, T_c, as function of the magnetic fields is clarified in the Fig. 6. We can clearly see that the values of T_c are bigger in the $\kappa = 0$ when compared to the and $\kappa^{[2]}$ and $\kappa^{[1]}$.

We note that our results for T_c as a function of the magnetic field do not present any kind of oscillation as some NJL predictions in the literature [37,39].

We also evaluate the effective quark mass as function of the magnetic fields with both sets of fixed κ and $\kappa = 0$, for the temperatures $T = 0$ in the top panel of Fig. 7 and $T = 0.140$ GeV in the low panel. We can see that the values of the effective quark mass with $\kappa^{[1]}$ are almost the same as the one evaluated with $\kappa^{[2]}$ and $\kappa = 0$ for both temperatures considered in the weak magnetic field regime, e.g,
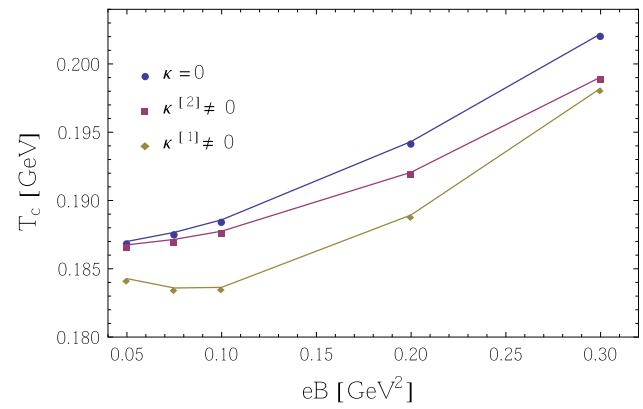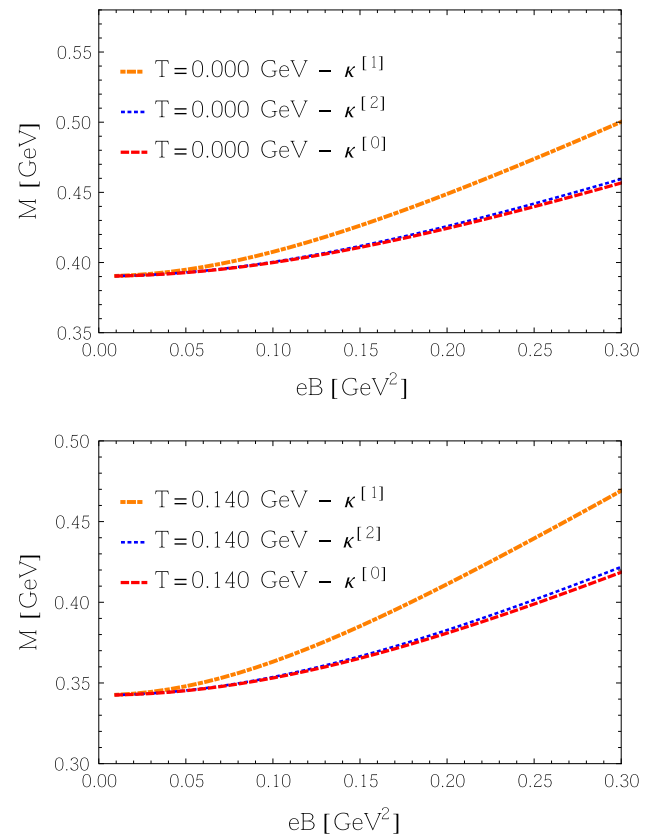
$eB \lesssim 0.05$ GeV^2, and are always bigger than the ones with $\kappa^{[2]}$ and $\kappa = 0$ in the strong magnetic field regime. In our evaluations, there are no oscillations in the effective quark masses, as observed in several evaluations in the $\kappa = 0$ [53] or with $\kappa \neq 0$ [37,39,40,43]. These nonphysical oscillations observed in the literature for the constituent quark mass, pseudocritical temperature for chiral symmetry restoration

and other physical quantities, can be traced to the fact that the regularization procedure depends explicitly on the magnetic field. We and other authors in the literature have shown in several works [9,12,14–16,20,32,36,45,46,52–55,67–71] that these nonphysical oscillations disappear when the divergent terms are disentangled from the pure magnetic contributions by using the MFIR/VMR schemes.

6 Conclusions

In this work we have studied the effect of a constant anomalous magnetic moment in the SU(2) Nambu–Jona-Lasinio model. To this end, we have employed the Schwinger *ansatz* representing the effect of AMM. Making use of VMR-type regularization, we obtain a set of well defined expressions for the thermodynamical potential and the gap equation. For the effective quark mass, we choose two sets of fixed values of AMM. For the set $\kappa^{[1]}$, which consider a sizable value of κ_f, we observe a smoothly decrease of T_c for the magnetic field region, i.e., $eB \lesssim 0.1$ GeV^2. This is the only observation for the decreasing of T_c as function of the magnetic field obtained in this work, which is not previously observed in the literature. For the set $\kappa^{[2]}$, we observe the increase of T_c for all magnetic field range adopted, whereas this effect also take place with set $\kappa^{[1]}$ for the strong magnetic field region. Our results also shows that the pseudocritical temperature is always bigger in the case of $\kappa = 0$ than the $\kappa^{[1]}$ and $\kappa^{[2]}$ case. Therefore, one main effect of the AMM is to decrease the values of T_c as we increase the value of κ_f. The magnetic catalysis also holds for low temperatures with no oscillations, which is expected in the MFIR or VMR inspired regularization procedures. We hope that this work can clarify that these non-physical oscillations are an artifact of some regularization prescriptions that entangled the magnetic medium with the vacuum. Furthermore, these oscillations cannot be confused with de Haas-van Alphen oscillations.

It is important to mention that the magnetic field still appear in the vacuum contributions in our equations. This happens once there is a mixed contribution with effective quark masses and quark AMM parameters. One could expect that this mixing be the reason behind the non-restoration of the observed chiral symmetry breaking. However, as properly demonstrated, this is directly associated with the pure magnetic field contribution which induces restrictions on the effective quark masses through the AMM parameters. Future analysis, in order to find a MFIR-type vacuum representation, can clarify even more these points. Nonetheless, the VMR prescription applied here represents an improvement in disentangle vacuum and magnetic field contributions in the gap equation and thermodynamical potential, which is very different from previous works, avoiding nonphysical results and related phenomena.

Our results when we consider nonvanishing quark AMM show that chiral symmetry restoration happens always as a smooth crossover and never turns into a first order phase transition. We will report about the results exploring the AMM effects in the thermodynamical properties of magnetized, dense and hot quark matter with VMR scheme in a future publication.

Appendix A: Vacuum magnetic regularization (VMR)

To obtain the thermodynamical potential Eq. (14) and the gap equation Eq. (17), we adopt a vacuum magnetic regularization scheme. First, the trigonometric part of the integrand of Eq. (9) should be expanded in Taylor series at $s \sim 0$ as

$$\coth(c_f B_f s) \sim 1 + \frac{(c_f B_f s)^2}{2} + \mathcal{O}(s^4),$$
$$\sinh(B_f s)^{-1} \sim \frac{1}{B_f s} - \frac{B_f s}{6} + \mathcal{O}(s^3), \quad \text{(A.1)}$$

where $c_f = 1 + \alpha_f$. Now we can see that, the integrand behaves like

$$\frac{\cosh(c_f B_f s)}{\sinh(B_f)} \sim \left[1 + \frac{(c_f B_f s)^2}{2}\right]\left[\frac{1}{B_f s} - \frac{B_f s}{6}\right]$$
$$= \frac{1}{B_f s} + \frac{B_f s}{6}\left(1 - 3c_f^2\right) + \mathcal{O}(s^3), \quad \text{(A.2)}$$

To eliminate the divergences, we add and subtract in the integrand the expansion Eq. (A.2) in Eq. (9)

$$
\begin{aligned}
\mathcal{L} &= \sum_{f=u,d} \frac{N_c}{8\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-s\mathcal{K}_{0f}^2} B_f s \left\{ \frac{\cosh(c_f B_f s)}{\sinh(B_f)} - \frac{1}{B_f s} \right. \\
&\left. - \frac{(B_f s)^2}{6} \left(1 - 3c_f^2 \right) \right\} + \Omega^{vac} + \Omega^{field}, \\
&= \sum_{f=u,d} \frac{N_c}{8\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-s\mathcal{K}_{0f}^2} \left\{ \frac{B_f s \cosh(c_f B_f s)}{\sinh(B_f)} - 1 \right. \\
&\left. - \frac{(B_f s)^2}{6} \left(1 - 3c_f^2 \right) \right\} + \Omega^{vac} + \Omega^{field} \\
&= \Omega^{mag} + \Omega^{vac} + \Omega^{field}.
\end{aligned}
\tag{A.3}
$$

The same procedure is adopted for the gap equation. The functions Ω^{vac} and Ω^{field} are regularized in the usual 3D-cutoff scheme. The magnetic and thermo-magnetic contributions are finite.

Appendix B: Constraints in the effective quark mass with AMM

The application of a constant AMM value through the values of Eq. (23) for $\kappa_f^{[1]}$ and Eq. (24) for $\kappa_f^{[2]}$ limit the possible values of the effective quark mass. To see this, we can work with the magnetic contribution, h_f^{mag}, of the gap equation, Eq. (17), as

$$
\begin{aligned}
h_f^{mag}(M, eB) &= \frac{M N_c}{4\pi^2} \int_0^\infty \frac{ds}{s^2} e^{-s\mathcal{K}_{0f}^2} \\
&\times \left\{ \frac{B_f s \cosh[c_f B_f s]}{\sinh(B_f s)} - 1 \right. \\
&\left. - \frac{1}{6} \left[3(\alpha_f + 1) - 1 \right] (B_f s)^2 \right\},
\end{aligned}
\tag{B.4}
$$

where the pure trigonometric part of Eq. (B.4) non-regularized can be rewritten as

$$
\begin{aligned}
h_f^{mag}(M, eB)^{NR} &= \frac{M N_c}{4\pi^2} \int_0^\infty \frac{ds}{s^2} e^{-s\mathcal{K}_{0f}^2} \\
&\times \left\{ \frac{B_f s \cosh[c_f B_f s]}{\sinh(B_f s)} \right\}
\end{aligned}
$$

where we have adopted the superscript NR, that means "Non-Regularized". Making use of the following trigonometric relation

$$
\cosh(a+b) = \cosh(a)\cosh(b) + \sinh(a)\sinh(b), \tag{B.5}
$$

we obtain for $h_f^{mag}(M, eB)^{NR}$

$$
\begin{aligned}
h_f^{mag}(M, eB)^{NR} &= \frac{M N_c}{4\pi^2} \int_0^\infty \frac{ds}{s^2} e^{-s\mathcal{K}_{0f}^2} \\
&\times B_f s \left\{ \frac{\cosh(\alpha_f B_f s)\cosh(B_f s)}{\sinh(B_f s)} \right.
\end{aligned}
$$

$$
\begin{aligned}
&\left. + \frac{\sinh(B_f s)\sinh(\alpha_f B_f s)}{\sinh(B_f s)} \right\} \\
&= \frac{M N_c}{4\pi^2} \int_0^\infty \frac{ds}{s^2} e^{-s\mathcal{K}_{0f}^2} B_f s \\
&\times \left\{ \cosh(\alpha_f B_f s) \coth(B_f s) \right. \\
&\left. + \sinh(\alpha_f B_f s) \right\}
\end{aligned}
\tag{B.6}
$$

Making use of the following relation

$$
\sinh(x) = \frac{e^x - e^{-x}}{2}, \tag{B.7}
$$

we can rewrite the second integration, i.e., the integration of the $\sinh(\alpha_f B_f s)$, as

$$
\begin{aligned}
&\int_0^\infty \frac{ds}{s^2} e^{-s\mathcal{K}_{0f}^2} \left\{ B_f s \sinh(\alpha_f B_f s) \right\} \\
&= \int_0^\infty \frac{ds}{s^2} e^{-s\mathcal{K}_{0f}^2} \left\{ B_f s \frac{e^{\alpha_f B_f s} - e^{-\alpha_f B_f s}}{2} \right\}.
\end{aligned}
\tag{B.8}
$$

It is clear from Eq. (B.8) that we will have a limit due to the α_f and κ_f on the integration to avoid a divergence. To see this, we will have the condition

$$
\begin{aligned}
-\mathcal{K}_{0,f}^2 + \alpha_f B_f &< 0 \\
-(M^2 + (\kappa_f B_f)^2) + \alpha_f B_f &< 0 \\
M^2 &> \alpha_f B_f - (\kappa_f B_f)^2
\end{aligned}
\tag{B.9}
$$

The above expression constrain the effective quark masses, M, for each of the flavors in gap equation. As an estimative, if we take $\kappa_u^{[1]}$ in the above expression for $eB = 0.1$ GeV^2, we obtain

$$
M \gtrsim 125.693 \text{ MeV}, \tag{B.10}
$$

The estimative for $\kappa_u^{[2]}$, we obtain

$$
M \gtrsim 19.763 \text{ MeV} \tag{B.11}
$$

both values are in agreement with the results in Figs. 1 and 2.

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