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Energy Conditions of the Five Dimensional with NMDC and Acelerating Universe

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Abstract. The Energy condition is studied for five dimensional cosmological model with nonminimal derivative coupling (NMDC) between scalar field and curvature tensor. We assume that the scale factors of three dimensional space ($a(t)$) and the extra dimension ($b(t)$) is related by $b(t) = \left(a(t)\right)^{\gamma}$, where γ is a constant. We apply the Null Energy Condition (NEC), Weak Energy Condition (WEC), Strong Energy Condition (SEC) and Dominant Energy Condition (DEC) to our model and investigate some constraint in order the energy condition violated. The constraint that we found is appropriate with cosmological model in which the four-dimensional universe expands with positive acceleration and the extra dimension decays.

1. Introduction

The general theory of relativity developed by Albert Einstein can be used to get the dynamics of our universe. This is because the general theory of relativity makes a relation between the curvature of the spacetime and the matter distribution, via the Einstein field equation,

$$G_{AB} = \kappa^2 T_{AB}, \tag{1}$$

with $\kappa^2 = 8\pi G$ is a constant. The Einstein tensor, G_{AB}, gives a geometrical quantity which is complicated unless we choose a specific symmetry of the spacetime. On the other hand, the energy-momentum tensor, T_{AB}, depends on the type and properties of the matter which we can consider in the model. Therefore, to get the "realistic" matter in our consideration, there must exist a set of conditions or constraints that restricts the matter considered [1]. This constraints are called energy conditions. For a matter with energy density ρ and pressure P, the energy condition can be written as [2],

- Null Energy Condition (NEC) : $\rho + P \geq 0$,
- Weak Energy Condition (WEC) : $\rho \geq 0$ and $\rho + P \geq 0$,
- Strong Energy Condition (SEC) : $\rho + 3P \geq 0$ and $\rho + P \geq 0$,
- Dominant Energy Condition (DEC) : $\rho \geq 0$ and $-\rho \geq P \geq \rho$.

From the previous works [3], SEC was first violated billions years ago while the NEC, WEC, and DEC seems to have violated only recently. In the context of cosmology, SEC is related to the expansion of the universe, so if SEC is violated, the universe expands with positive acceleration.

The models of extra dimensions had proposed to solve some theoretical problem in unification of all physical interactions. The models of one dimensional extra dimension was proposed by Kaluza and Klein when they developed unification theory of gravity and electromagnetism [4]. Similar to Kaluza-Klein model, the universal extra dimension (UED), proposed that all standard model fields can propagate in the extra dimensions and become dark matter candidate when one extra dimension are compactified as low as 300 GeV [5].

In this work, we study the energy conditions for the nonminimal derivative coupling (NMDC) of scalar field in five dimensions. Previously, we study the model and its stability in [6] and analyze the energy conditions for four dimensional model in [7]. This paper is written as follow. After this brief introduction, we explain the setup of the model that we used in this work. Then we derive the density energy and the pressure terms to get the energy conditions for this model. From the energy conditions, we consider the case when all energy condition is violated to get the constraint for the necessary parameter and solve it with considering some special cases. Finally, in the last section we choose the solution that appropriate with cosmological model in which the four-dimensional universe expands with positive acceleration and the extra dimension decays.

2. Model Setup

We start from the action :

$$S = \int d^5x\sqrt{-g}\left\{\frac{R}{16\pi G} - \frac{1}{2}\left[\varepsilon g_{AB} - \xi G_{AB}\right]\partial^A\phi\,\partial^B\phi\right\} \quad (2)$$

where the nonminimal derivative coupling is given by third term of the action, and ξ is the single derivative coupling parameter with dimensions of inverse mass-squared. The ε is a parameter to describe the field and takes the value of +1 for the canonical field and -1 for the phantom field. For this work, we consider canonical field case, so we take $\varepsilon = +1$.

Variation of the action with respect to the metric leads to field equations

$$G_{AB} = 8\pi G\left(T_{AB}^{(\phi)} + \xi\,\Theta_{AB}^{(\phi)}\right), \quad (3)$$

with

$$T_{AB}^{(\phi)} = \nabla_A\phi\,\nabla_B\phi - \frac{1}{2}g_{AB}\left(\nabla_C\phi\,\nabla^C\phi\right), \quad (4)$$

$$\begin{aligned}\Theta_{AB}^{(\phi)} = &-\frac{1}{2}\nabla_A\phi\,\nabla_B\phi R + \nabla_C\phi\left(\nabla_A\phi R_B^C + \nabla_B\phi R_A^C\right) + \nabla^C\phi\nabla^D\phi R_{ACBD} + \nabla_A\nabla^C\phi\,\nabla_B\nabla_C\phi \\ &-\nabla_B\nabla_A\phi\,\Box\phi - \frac{1}{2}\left(\nabla_C\phi\,\nabla^C\phi\right)G_{AB} + g_{AB}\left[-\frac{1}{2}\nabla^C\nabla^D\phi\,\nabla_C\nabla_D\phi + \frac{1}{2}(\Box\phi)^2 - \nabla_C\phi\nabla_D\phi R^{CD}\right].\end{aligned} \quad (5)$$

We take the metric for five dimensional flat universal extra-dimension (5D UED) model [8] as,

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j + b^2(t)dy^2, \quad (6)$$

where $a(t)$ and $b(t)$ is the scale factors for three dimensional space and for the extra dimension. We take an assumption that those two scale factors related as $b = a^\gamma$ for γ constant as taken in [9]. From this relation, we can get the Hubble parameter for each scale factor,

$$H_a = \frac{\dot{a}}{a} = H \quad \text{and} \quad H_b = \frac{\dot{b}}{b} = \gamma H \quad (7)$$

3. Energy Conditions for Nonminimal Derivative Coupling of Scalar Field in Five Dimensions

From equation (1) and (3), we can conclude that $T_{AB} = T_{AB}^{(\phi)} + \xi\, \Theta_{AB}^{(\phi)}$. Writing the energy-momentum tensor as $T_{AB} = diag\left(-\rho, P_r, P_r, P_r, P_t\right)$, $a = a(t)$ and $b = b(t)$ we get,

$$\rho = \frac{\dot{\phi}^2}{2}\left\{1 - \xi H^2\left(9 + 9\gamma\right)\right\}, \tag{8}$$

$$P_r = \frac{\dot{\phi}^2}{2}\left\{1 + \xi\left[H\frac{\ddot{\phi}}{\dot{\phi}}\left(4 + 2\gamma\right) + H^2\left(\gamma^2 + 2\gamma + 3\right) + \dot{H}\left(2 + \gamma\right)\right]\right\}, \tag{9}$$

$$P_t = \frac{\dot{\phi}^2}{2}\left[1 + 3\xi\left(\dot{H} + 2H^2 + 2H\frac{\ddot{\phi}}{\dot{\phi}}\right)\right]. \tag{10}$$

We define the total pressure,

$$P = P_r + \lambda P_t = \frac{\dot{\phi}^2}{2}\left\{\lambda + 1 + \xi\left[H^2\left(6\lambda + \gamma^2 + 2\gamma + 3\right) + \dot{H}\left(3\lambda + \gamma + 2\right) + H\frac{\ddot{\phi}}{\dot{\phi}}\left(6\lambda + 2\gamma + 4\right)\right]\right\}, \tag{11}$$

where λ is the contribution parameter of extra-dimension pressure (P_t). Therefore, the energy conditions can be written as,

$$NEC: \lambda + 2 + \xi\left[H^2\left(6\lambda + \gamma^2 - 7\gamma - 3\right) + \dot{H}\left(3\lambda + \gamma + 2\right) + H\frac{\ddot{\phi}}{\dot{\phi}}\left(6\lambda + 2\gamma + 4\right)\right] \geq 0 \ , \tag{12}$$

$$WEC: \xi H^2\left(9\gamma + 9\right) \leq 1 \text{ and equation (12)} \ , \tag{13}$$

$$SEC: 3\lambda + 4 + \xi\left[H^2\left(18\lambda + 6\gamma + 12\right) + \dot{H}\left(9\lambda + 3\gamma + 6\right) + H\frac{\ddot{\phi}}{\dot{\phi}}\left(18\lambda + 6\gamma + 12\right)\right] \geq 0 \text{ and equation (13)} \ , \tag{14}$$

$$DEC: \lambda + \xi\left[H^2\left(6\lambda + \gamma^2 + 11\gamma + 12\right) + \dot{H}\left(3\lambda + \gamma + 2\right) + H\frac{\ddot{\phi}}{\dot{\phi}}\left(6\lambda + 2\gamma + 4\right)\right] \geq 0 \text{ and equation (13)} \ , \tag{15}$$

Now we consider the case that all energy conditions are violated. So the requirement condition is,

$$\lambda + 2 + \xi\left[H^2\left(6\lambda + \gamma^2 - 7\gamma - 6\right) + \dot{H}\left(3\lambda + \gamma + 2\right) + H\frac{\ddot{\phi}}{\dot{\phi}}\left(6\lambda + 2\gamma + 4\right)\right] < 0 \ . \tag{16}$$

We can simplify the equation by substituting the expression of $\ddot{\phi}/\dot{\phi}$ from the field equation (3-6) for 00 index as,

$$\frac{\ddot{\phi}}{\dot{\phi}} = \frac{\dot{H}}{H\left[1 - \xi H^2\left(9\gamma + 9\right)\right]}. \tag{17}$$

After substitution, equation (16) can be written as,

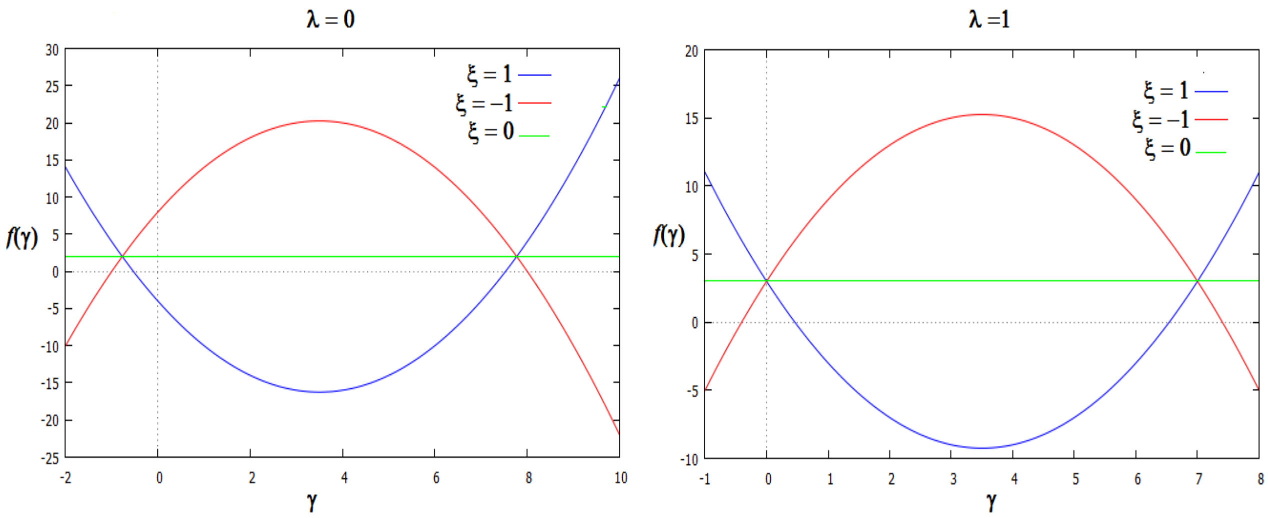
$$\frac{\lambda + 2 + \xi\left\{H^2\left[\gamma^2 - 25\gamma - 24 - \lambda\left(9\gamma + 3\right)\right] + \dot{H}\left(9\lambda + 3\gamma + 6\right)\right\} - \xi^2 H^2\left\{H^2\left[54\lambda\left(\gamma + 1\right) + 9\gamma^3 - 54\gamma^2 - 117\gamma - 54\right] + \dot{H}\left[27\lambda\left(\gamma + 1\right) + 9\gamma^2 + 27\gamma + 18\right]\right\}}{\left[1 - \xi H^2\left(9\gamma + 9\right)\right]} < 0. \tag{18}$$

This condition applies to all solutions for scale factor *a*. Here we restrict our work to consider de Sitter solution

($a \propto \exp\left(H_0 t\right)$, H_0 constant, $\dot{H} = 0$). For this case, equation (18) gives us,

$$f\left(\gamma\right) = \frac{\lambda + 2 + \xi H_0^2\left\{\left[\gamma^2 - 25\gamma - 24 - \lambda\left(9\gamma + 3\right) - \xi H_0^2\right] - \xi H_0^2\left[54\lambda\left(\gamma + 1\right) + 9\gamma^3 - 54\gamma^2 - 117\gamma - 54\right]\right\}}{1 - \xi H_0^2\left(9\gamma + 9\right)} < 0. \tag{19}$$

Because the $\lambda-$parameter does not have particular constraint, here we analyze two special cases that extra-dimension pressure has no contribution to total pressure ($\lambda = 0$) and has same contribution with common space pressure ($\lambda = 1$). For each case we consider $\xi = -1, 0, 1$ and $H_0 = 1$ to get the characteristic of the energy conditions depends on the type of ξ (negative, zero, positive). The plot of $f\left(\gamma\right)$ versus γ is given in figure 1.



(a) (b)

Figure 1. Curve Plot of Two Cases Considered, (a) case $\lambda = 0$ and (b) case $\lambda = 1$.

4. Conclusion

We have studied the energy conditions for five dimensional nonminimal derivative coupling model of scalar field, and found some constraint for ξ and γ so that the energy conditions is violated. The violation of the energy conditions leads to condition that four dimensional universe is expands with positive acceleration. From the relation between two scale factors ($b = a^{\gamma}$), the $\gamma < 0$ condition indicates that the extra dimension decays if the four dimensional universe expands. For de Sitter solution, two cases concerning the contribution of extra-dimensional pressure to total pressure is considered when all energy conditions are violated. For the case that extra-dimensional pressure have no contribution on total pressure (or $\lambda = 0$), a negative ξ gives us $\gamma < -1$ or $\gamma > 8$, while positive ξ gives us $-0,53113 < \gamma < 7,5311$. On the other hand, for the case that extra-dimensional pressure have same contribution with common space pressure on total pressure ($\lambda = 1$), a negative ξ gives us $\gamma < -0,40512$ or $\gamma > 7,4051$, while positive ξ gives us $0,4582 < \gamma < 6,5414$. We also see that for $\xi = 0$ have no solution, so we can conclude that in five dimensions. It means that the NMDC term is required to have all energy conditions violated.

However, from previous study [7], the ξ parameter must be negative, so both $\lambda = 0$ and $\lambda = 1$ require negative γ to have all energy conditions violated.

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