Probing the Temporal Fine Structure of Hubble Parameter Evolution with Linear Cosmography

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Abstract. Given recent indications of various tensions in the "Cosmic Concordance" when combining complementary data sets (CMB studies, Baryon Acoustic Oscillations, Type Ia Supernovae, etc.) – such as the "Hubble Tension", and the possible departure from LambdaCDM time evolution – the potential of such discrepancies to have a real cosmological origin persists. From a model-independent standpoint, this impels one to precisely locate the source of the problem with concordance cosmology. The extraordinarily good fit to the CMB data shows that LambdaCDM was once truly an excellent match to the real universe; yet, at some point between the last scattering surface and the more local region with standard-candle-measured expansion, the universe has "gone off the track" of simple LambdaCDM, if the signs of a mismatch between the high- and low-redshift universe are to be believed. Much as a doctor probes the patient and asks, "Where does it hurt?", linear cosmography can be used to map out the temporal evolution of the Hubble parameter to ask the question: "When (and how) did the Universe begin to go wrong?" Our studies of the DES Supernova data shows hints of interesting temporal behaviors, such as possible oscillations in the Hubble Parameter; and further study is ongoing to examine how statistically significant these hints can be found to be, and how sensitive they are to choices of SNIa redshift binning and dataset selection.

1 Introduction, Motivation and Methodology

For several years, complementary data sets were presumed to be narrowing in on a conclusive set of parameters for the ΛCDM "Concordance Model" – based upon vacuum energy as the Dark Energy – until a number of tensions and discrepancies recently arose, creating a level of confusion for this program.

Perhaps most notable is the "Hubble Tension" [1], a discrepancy in estimates for the Hubble Constant H_0 from the Cosmic Microwave Background (CMB) versus late-time Standard Candles such as Type Ia Supernovae (SNIa). Arguments exist [2] attributing it to systematics (particularly for Cepheids) in the use of distance indicators; but if this tension is truly cosmological, it may indicate a misinterpretation in how we extrapolate our estimates of H_0 from high redshifts down to the low-redshift universe.

Beyond SNIa, DESI Baryon Acoustic Oscillations (BAO) results [3] indicate potential departures from a Cosmological Constant Λ in the late universe; yet, CMB studies by ACT [4] show early agreement with ΛCDM. Aside from (mostly ISW-related) CMB effects, and barring "Early Dark Energy" models (seemingly disfavored [5]), the precise form of the Dark Energy (DE) has virtually no effect on the physics at CMB times (serving mainly to set the distance to the surface of last scattering). Therefore, our hypothesis is that a basic misunderstanding of the true nature of "Dark Energy" (DE) may be manifesting itself as an early-universe agreement with the "stand-in" model of ΛCDM, but then breaking down at low-redshift, just as the explicit physical nature of the DE first becomes cosmologically relevant.

To determine when (and if) there is a specific time in cosmic history when the evolution began departing from ΛCDM, we probe the expansion history with "tomography", via linear cosmography.

Here we do not fit the data with cosmological models, or with multiple-term cosmographic expansions, both of which smear the results across a wide redshift range. We instead proceed by slicing the Dark Energy Survey (DES) SNIa data [6] into redshift bins small enough so that we may accurately fit each bin with a straight-line segment, and then use those line slopes to derive $H(z)$ for each bin, thus giving us a detailed trail of the evolution of the Hubble Parameter over time.

We use y-redshift, $y \equiv z/(1+z)$, for its better convergence properties (since $y \to 1$ as $z \to \infty$). From the Luminosity Distance (d_{Lum}) integrand being $\propto 1/H(y)$ [7], it can be shown that:

$$H(y) = c \, \{(1-y)^2 \, \frac{d}{dy}[(1-y) * d_{Lum}] \, \}^{-1} \, , \qquad (1)$$

Thus we can determine the "instantaneous" value of the Hubble Parameter $H(y)$ at any epoch, by using SNIa data to evaluate the slope of the fitting function, $F_{fit} = [(1-y) * d_{Lum}]$, for each y-redshift bin.

Previous cosmographic studies (including ours [8]) have been useful, though are usually expanded around $z = 0$ (not likely a physically interesting pivot point); and without any clear knowledge of the epoch or form of a potential deviation from ΛCDM, it is very difficult to know how many or what kind of cosmographic expansion parameters are best to use (a problem avoided by binning). Binning studies with various versions of the data have been done, such as directly binning the distance modulus μ, or the inferred equation of state $w(z)$ [3]; but given that the Hubble Tension is usually quoted in terms of estimated H_0 values – both from high- and low-redshift data sets – and since $H(y)$ has an evident interpretation in terms of the universe "speeding up" or "slowing down" (relative to a baseline model), we find it illustrative to turn the information in the bins into a piecewise estimate of the $H(y)$ curve.

Bin sizes must be small enough to ensure an accurate straight-line fit; to make the quadratic term in the expansion of d_{Lum} (for a comparable ΛCDM model) less than 1% of the linear term, we require $\Delta y < 0.024$. But the bins cannot be too small, to avoid bins having too few SNIa (especially given gaps in the DES data set); and redshift uncertainties for these photometric SNIa are significant compared to the bin widths, and can cause bin assignment errors for some SNIa. We tried many binning schemes.

2 Results and Discussion

Shown here are results for our "best" (most illustrative) binning: $\Delta y = 0.012$ (but $\Delta y = 0.024$ needed for DES data gaps). Figures 1, 2 show the bin linear fit segments to F_{fit}, with and without the data visible. Figure 3 shows the binned $H(y)$ values, versus ΛCDM, and with DESI BAO (parallel distance) data [3] as purple squares; the latter two are set to the CMB best-fit values from the ACT Data Release 6 [4]: $H_0 = 68.22$, $\Omega_M = 0.3032$. Figure 4 shows the residuals (versus ΛCDM) normalized by their σ_H values, with point size scaled by $1/\sqrt{\sigma}$ to emphasize the most reliable points.

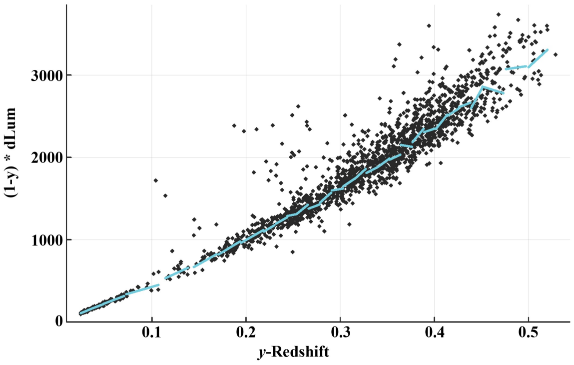

Figure 1: DES SNIa w/Piecewise Linear Fits.

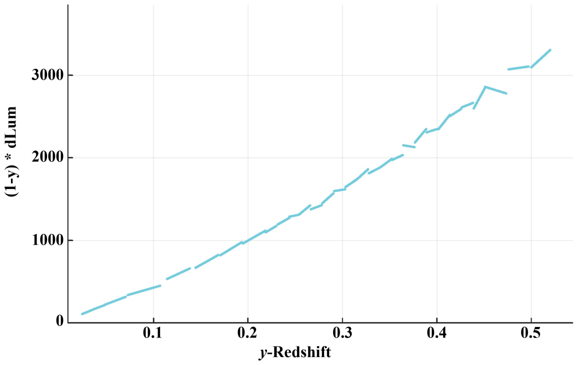

Figure 2: Same Fits as Fig. 1, DES data Removed.

In Fig. 4, note the *sudden jump* in BAO data where the acceleration begins ($y \simeq 0.4$, $z \simeq 0.7$). Our SNIa fits also show volatility there, and show hints of *possible oscillations* in the Hubble Parameter. Such oscillations are one prediction of this author's "Causal Backreaction" paradigm [9, 10] for the cosmic acceleration; but could perhaps also be predicted by a variety of theories. These results are of low statistical significance, and hampered by data gaps in the most rapidly varying regions ($y \sim z \sim 0.1$); but more robust results will be forthcoming, using combined and (eventually) updated data sets. For now, this paper is intended to serve as a trial demonstration of a new method of temporal cosmological tomography.

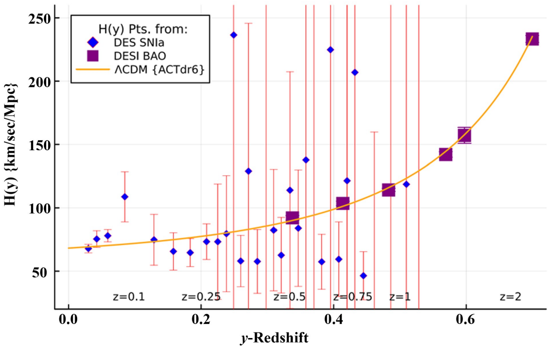

Figure 3: Best-Fit $H(y)$ Values for DES SNIa.

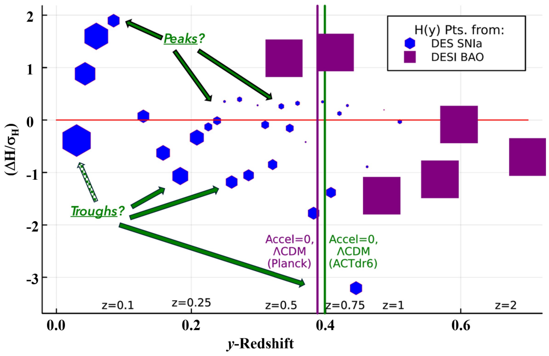

Figure 4: $H(y)$ Residuals (vs. ΛCDM)$/\sigma_H$.

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