Ground-State Deformation and Radii of Hg Isotopes from HFB Calculations

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Introduction

Atomic nuclei are complex quantum many-body systems that can undergo structural changes as the number of neutrons or protons varies. One clear manifestation of this evolution is a quantum phase transition (QPT), where the ground-state properties change abruptly with nucleon number. Such shape transitions—between spherical, oblate, and prolate forms—arise from the interplay of shell effects, residual interactions, and collective correlations. To study these changes, key observables include the root-mean-square (rms) radius and the quadrupole deformation parameter. The rms radius reveals the spatial extent of nucleons, while the deformation parameter measures the deviation from spherical symmetry. Together, they provide valuable insight into how nuclear matter reorganizes with changing nucleon configurations.

Experimentally, the rms charge radii can be extracted through high-precision techniques such as electron scattering, muonic atom spectroscopy, and isotope shift measurements, all of which have significantly advanced our knowledge of nuclear size and shape systematics. These measurements, when combined with theoretical models, allow for detailed investigations of the microscopic mechanisms behind shape transitions and their connection with underlying shell structure. In this work, we focus on the Hg isotopic chain, which has long served as a key testing ground for nuclear shape evolution due to its rich variety of structural phenomena, including shape coexistence and sudden shape transitions [1]. By systematically analyzing the behavior of rms radii and quadrupole deformation, we aim to provide deeper insights into the interplay between single-particle and collective degrees of freedom that governs the emergence of shape variation in mercury isotopes.

Theoretical framework

Hartree-Fock-Bogoliubov theory (HFB) [2] is the generalised single-particle model in which both Hartree-Fock and BCS theories are given equal status. In the HFB approximation, quasiparticle states are used instead of the single-particle states. The nuclear wave function is expressed in terms of the vacuum of suitable quasiparticles. The HFB equation is given by

$$\begin{pmatrix} h-\lambda & \Delta \\ -\Delta^* & -h^*+\lambda \end{pmatrix} \begin{pmatrix} U_n \\ V_n \end{pmatrix} = E_n \begin{pmatrix} U_n \\ V_n \end{pmatrix} \tag{1}$$

where $h = t + \Gamma$. t, Γ, E_n and λ represents kinetic energy, self-consistent field, quasiparticle energy, and chemical potential. In the mean-field part, we employed the Universal energy density functional, UNEDF2 parametrisation [3], and in the pairing part, we utilised a density-dependent delta interaction with the mixed variant.

Results and Discussion

The present study explores the structural evolution of Hg isotopes extending from the neutron to the proton drip line. Fig. 1 illustrates the potential energy curves (PECs) of selected isotopes, obtained by constraining the quadrupole moment Q_{20}. The minima of these curves correspond to the ground-state energies, and the associated values of Q_{20} reveal whether a given isotope favours a prolate, oblate, or spherical configuration. Since the quadrupole moment is directly related to

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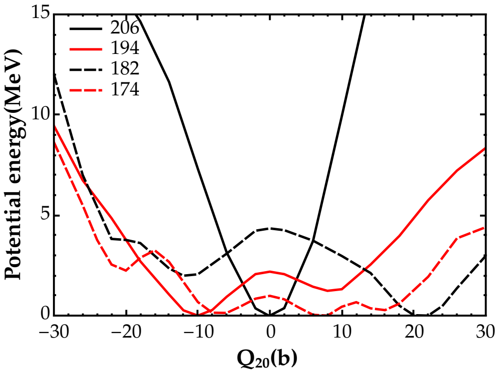

FIG. 1: Potential energy curve of ^{174,182,194,206}Hg isotopes.

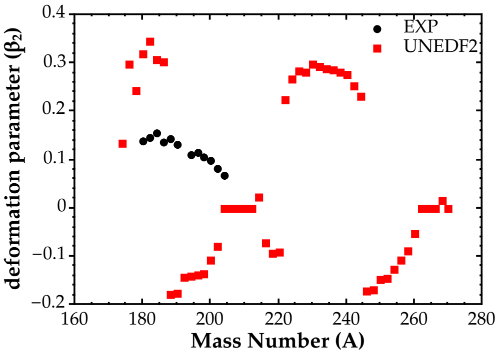

FIG. 2: Deformation parameter of $^{174-268}$Hg isotopes.

the degree of nuclear deformation, the deformation parameter β_2 was extracted using the relation, $Q_{20} = \sqrt{\frac{16\pi}{5}}(\frac{3}{4\pi}AR^2\beta_2)$, where A is the mass number and $R = 1.2 \times A^{1/3}$ fm denotes the nuclear radius. The calculated values of β_2 are shown in Fig. 2 together with available experimental data [4], providing a quantitative measure of shape evolution across the isotopic chain. The HFB results matches with the relativistic calculations and we observe similar discrepency which have to be considered in detail.

The root-mean-square (rms) radius is another important observable that shows variations in nuclear shape. Kinks or discontinuities in the rms radii systematics are the expected manifestation of sudden changes in nuclear form. Significant slope shifts are specifically displayed at $A = 222$ and $A = 246$, which, when compared with the PEC analysis in Fig. 1, show that shape transitions occur at these locations. This demonstrates that rms radii are essential for signaling structural rearrangements and have a strong correlation with nuclear deformation. The computed values for isotopes in the $A = 222$–246 range show increased deformation in comparison to nearby nuclei. Nuclei elongate over the surface when deformation increases, which causes the rms charge radius to rise as well. This tendency is obvious in the findings (not shown here).

In summary, the present analysis demonstrates that nuclear radii, alongside quadrupole deformation parameters, serve as reliable indicators of shape transitions in Hg isotopes. Also, there is a lack of experimental evidence for astrophysically significant nuclei (r-process) close to the drip-line due to their short half-lives, low production cross-sections, etc. Hence, the only way to comprehend the structure of these nuclei is through theoretical models. Moreover, we can understand the effect of the single particle energy levels on the nuclear shape still persist near drip-line.

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