

# IMPACTS OF AN ATS LATTICE ON EIC DYNAMIC APERTURE

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## Abstract

The Electron-Ion Collider (EIC) project at Brookhaven National Laboratory has explored strategies for increasing the energy aperture of the Electron Storage Ring (ESR) to meet the goal of 1% for the 90 degree lattice at 18 GeV. Current strategies use a four sextupole family per arc correction scheme to increase the energy aperture and to keep the transverse aperture sufficiently large as well. A scheme called Achromatic Telescopic Squeezing (ATS), first introduced for the Large Hadron Collider, introduces a beta-beat into select arcs, allowing dynamic aperture optimizations with different sextupole strengths. The ATS scheme's mix of some higher beta-function and some lower sextupole strengths in the arcs has the potential to increase the energy aperture. Basic chromatic corrections and numeric optimizations were used to compare the ATS optics to a non-ATS scheme. In all cases, the ATS scheme performed similarly or better than the more common schemes. However, this increase in energy aperture from the ATS optics also has negative effects, such as an increase in emittance which poses complications for the current ESR design.

## INTRODUCTION

The Conceptual Design Report (CDR) [1] for the Electron-Ion Collider (EIC) sets goals for the Dynamic Aperture (DA) of the ESR. For all configurations of the Electron Storage Ring (ESR), the on energy aperture requirements have been met. The energy aperture requirement of  $\delta = 1\%$  has recently been met for the two Interaction Point (IP) configuration at 18GeV for the version 5.5 lattice, which operates with 90° cells in the arcs [2]. The previous version, 5.3, never achieved the energy aperture goal, which led to the exploration of alternative strategies. Under the current strategy, with phase trombones being used to set phases, a maximum of  $\delta = 0.07\%$  was achieved. The optimization schemes being used largely consisted of four sextupole families per arc and adjustable phases going into each arc with a fixed overall tune. In simulations without synchrotron oscillations, the energy aperture could achieve the goal with little difficulty. The mechanism creating the difficulty in energy aperture optimization with synchrotron scillations have not yet been fully analyzed, however changes in the 5.5 lattice, including somewhat relaxed beta functions in the IP, improved this issue.

On the path from version 5.3 to 5.5 of the ESR, several strategies to optimize the DA were pursued. One of these was the Achromatic Telescopic Squeezing (ATS) scheme [3] presented here. The ATS scheme has promising features for optimizing the energy aperture of the ESR, however this was not the main objective of this scheme when it was conceived

for the LHC. It's potential for the ESR therefore had to be evaluated in detail.

## ATS SCHEME

The ATS scheme was first introduced at the LHC in order to fully utilize the existing large aperture in the arcs for luminosity optimization. Because the arc has to accommodate a lower energy beam at injection, it's aperture at high energy is unnecessarily large. The ATS scheme induces a  $\beta$ -beat in the arcs neighboring the IR which uses this available aperture, and this beta beat can reduce the cross section at the IP without strengthening the final focus quadrupoles. The ratio of the new peak  $\beta$  in the arc to the original one is called the telescopic ratio, which can be optimized for the lattice. A result of this, in a 90° lattice with four sextupole families, is that two of the families will be strengthened by an increase in  $\beta$ -function and two will be weakened by a decrease in  $\beta$ -function (which will be called the Strong and Weak families) [3]. This gives a different sextupole solution for the correction of chromaticities and chromatic beta beats. The different chromatic correction, although not the primary reason for the development of the ATS scheme in LHC, has possible advantages in DA optimization that can be beneficial to the EIC.

The possible advantages of the ATS scheme can be explained by looking at resonance driving terms that sextupoles excite. All terms that are first order in the sextupole strengths  $K$  scale as  $K\beta^{3/2}$ . In a simple estimate of the ATS scheme, these terms tend to decrease. As  $\beta_S$ , the  $\beta$ -function at the strong sextupole increases, their strength  $K_S$  proportionally decreases so that the chromaticities remains corrected, which go with  $K\beta$ .

If  $\Delta\beta$  is the beta beat, then the increase in the driving term at the strong sextupole is proportional to  $\sqrt{\beta + \Delta\beta}$  while it is proportional to  $\sqrt{\beta - \Delta\beta}$  at the weak sextupoles. The average of these driving terms decreases with the beta beat, as it evaluates to  $\sqrt{\beta}(1 - \frac{1}{4}(\Delta\beta/\beta)^2)$  in a second order expansion.

At second order, there are favourable and unfavourable terms, with some second order terms that could be greatly reduced if the weak sextupoles were turned off (This is true whether or not the ATS optics are used) [4]. This brings up the topic of interleaved versus non-interleaved sextupole schemes. The arcs in the ATS scheme are not fully non-interleaved if the weak sextupoles turned off, as the strong families will still be interleaved, this setup will be referred to as a partially interleaved sextupole scheme. The ATS scheme also has the benefit of reducing the effects of the remaining interleaved families due to the  $\beta$ -function being increased at only one family per plane. The usefulness of a partially interleaved system without the added benefit from the ATS optics was also tested, but yielded poor results.

## APPLICATION TO THE ESR

The ATS scheme can be applied simply to the 1-IP configuration of the ESR, with the six arcs being split into three groups. The two on either side of the IP (group 1 and 3) will correct the W-function [5], and the two opposite the IP (group 2) will correct the remaining chromaticity. Groups 1 and 3 each have strong and weak sextupole families for each plane, SF, SD, WF, and WD. This arrangement will be used to test the 4-Family scheme against the ATS scheme. The group 2 arcs have one sextupole family per plane, SX1 and SX2.

For the purpose of this test, two chromatic effects will be corrected, the chromaticity and the W-function [3, 5]

$$Q' = -\frac{1}{4\pi} \int_0^C ds [K_1(s) - K_2(s)D_x(s)]\beta_{x,y}(s) \equiv 1 \quad (1)$$

$$\int_{S_0}^{S_0+C} ds [K_1(s) - K_2(s)D_x(s)] \beta_{x,y}(s) e^{2j[\mu_{x,y}(s) - \mu_{x,y}(s_0)]} \equiv 0 \quad (2)$$

The chromaticity will be set to one and the W-function will be set to zero at the IP and brought down over the first two arcs.

As described previously, the partially interleaved 2-family scheme (shortened to 2-family scheme) uses the sextupoles designated as strong families, leading to the chromatic conditions for correcting the IR being

$$N_{Sx}\beta_{x,Sx}D_{x,Sx}K_{Sx} + N_{Sy}\beta_{x,Sy}D_{x,Sy}K_{Sy} = I_x - 1 \quad (3a)$$

$$N_{Sx}\beta_{y,Sx}D_{x,Sx}K_{Sx} + N_{Sy}\beta_{y,Sy}D_{x,Sy}K_{Sy} = I_x - 1 \quad (3b)$$

$$e^{2j(\Delta\mu_x - \pi/2)} [N_{Sx}\beta_{x,Sx}D_{x,Sx}K_{Sx} + e^{2j\pi/4} N_{Sy}\beta_{x,Sy}D_{x,Sy}K_{Sy}] = I_x \quad (4a)$$

$$e^{2j(\Delta\mu_y - \pi/2)} [e^{2j\pi/4} N_{Sx}\beta_{y,Sx}D_{x,Sx}K_{Sx} + N_{Sy}\beta_{y,Sy}D_{x,Sy}K_{Sy}] = I_y \quad (4b)$$

Where  $I_{x,y}$  is the contribution from the IR in each plane,  $N_{Sx,Sy}$  are the number of sextupoles,  $\Delta\mu_{x,y}$  are the phase advances from the IP. These equations represent six conditions and cannot all be corrected by the two sextupole families and phase advances available. This issue is solved by having the W-function corrected by the arcs neighboring the IR and correcting the remaining chromaticity in the group 2 arcs using the families SX1 and SX2, as seen in Fig. 1.

### ATS Scheme

The ATS scheme introduces the  $\beta$ -beat in the arcs from group 1 and 3. The sextupoles for the ATS scheme were

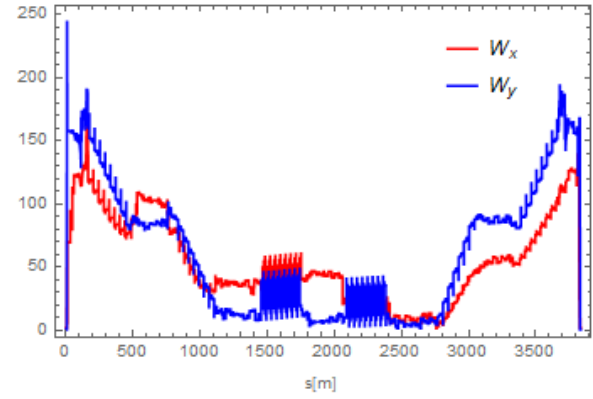


Figure 1: The W-function is plotted for the ATS scheme. Group 1 arcs bring down the W-function and group 2 arcs correct the additional chromaticity.

chosen using the same conditions as the 2-family scheme, resulting in different sextupole strengths due to the now present  $\beta$ -beat. For this test, a telescopic ratio of 2 was used, and kept for the remaining tests. This roughly doubles the  $\beta$ -function at the strong sextupole families.

### 4-Family Scheme

The 4-family scheme adds two new parameters, more than are fixed by the current conditions. A choice to fix the sextupoles of group 2 to match those of the ATS scheme was made. This choice of sextupole strengths for group 2 makes the changes between the two schemes only in the group 1 and 3 arcs, making it a more direct comparison between the two schemes.

## PRE-EXCITED SEXTUPOLES

In the prior sections, the ATS scheme was discussed using solely the strong sextupole families, however, use of the weak sextupoles may improve performance if the strengths are well chosen. For this test, it was chosen to initially set both the strong and weak sextupole families to correct the linear chromaticity of the arc. This choice makes the phase advance of each cell first order energy independent, giving a clean initial setup. On top of this initial sextupole setting, the schemes in the previous section were applied. This results in lower sextupole strengths in the chromaticity correction arc.

## RESULTS

The dynamic aperture results from the ATS scheme performed favorably when compared to the 4-family scheme, increasing the energy aperture by 0.1% before the pre-excited sextupoles are used. After the implementation of the pre-excited sextupoles, the energy aperture is increased by another 0.1% over the 4-family scheme, resulting in a 0.2% advantage in the ATS scheme for this test.

As seen, the ATS scheme applied to the ESR may offer marginal benefits to the energy aperture over the competing schemes. This benefit in the energy aperture needs to be

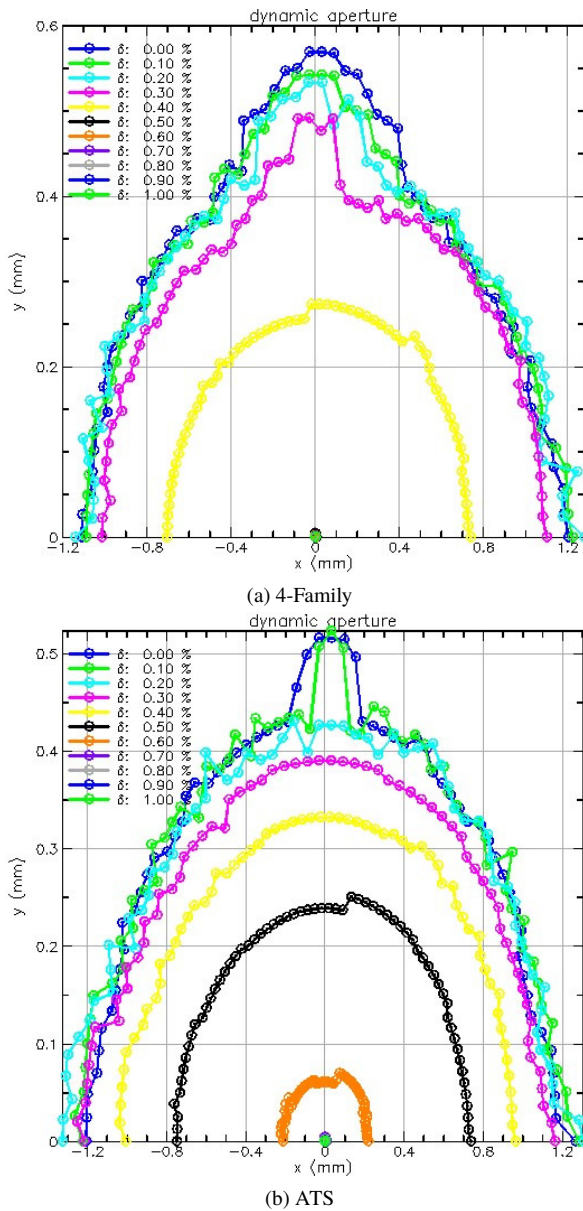


Figure 2: Dynamic Aperture in the ESR for the 4-family and ATS schemes. The ATS scheme uses pre-excited sextupoles, and performs better than the 4-family scheme in energy aperture by  $\delta=0.2\%$

balanced with potential concerns of implementing a version of the ATS scheme in the ESR.

The increase of the  $\beta$ -function used in the ATS scheme is accompanied by an increase of the emittance. The ESR lat-

tice used started with an emittance of 29.6nm. Applying the ATS scheme over one arc saw an increase of 5.3% and over two arcs saw an increase of 10%. In addition, the increased beam size in the arcs would require an increase in magnet aperture for the arcs where the ATS scheme is applied. It is questionable if the marginal benefits to energy aperture seen would justify these increases. A larger telescopic ratio could increase the ATS scheme's performance, however this would also increase the mentioned issues.

## CONCLUSION

In the ESR lattice, a small increase in energy aperture was seen when using a variation of the ATS scheme, with an increase of around 0.1% for all cases. All tests using the ATS scheme were done with a telescopic ratio of 2. This telescopic ratio comes with an emittance increase that makes the scheme impractical to add to the current ESR design. The scheme could offer advantages in designs without this constraint.

## ACKNOWLEDGEMENTS

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