Session 3

CURRENT COMMUTATORS

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The subject I discuss began, as we all know, with Gell-Mann's suggestion[1, 2] that the vector and axial vector current of the hadrons might satisfy exact equal-time commutation relations. During the last two years we have seen this suggestion develop into one of the most exciting branches of particle physics. In this talk I attempt to summarize the present status of the field, at least as I understand it. It seems to me that there are four main topics to be covered; they are:

(i) What is known (and not known) about the commutation relations satisfied by current components.
(ii) The low-energy theorems for processes involving soft pions.
(iii) Dispersion-theoretic sum rules.
(iv) Saturation of sum rules and the connection between current algebras and symmetries.

These topics are discussed in order.

I. The Commutation Relations

In any Lagrangian field theory which could produce SU(3) as an approximate symmetry, there will be eight vector currents $\mathcal{J}^0_i(x)$, with $i = 1 \cdots 8$. Using these currents, one can define eight charges $F_i(t)$ by integrating the time components over space:

$$F_i(t) = \int d^3x\, \mathcal{J}^0_i(\underset{\sim}{x}, t)\ . \qquad (1)$$

Furthermore, as Gell-Mann pointed out, these charges satisfy the equal-time commutation relations

$$[\,F_i(t),\, F_j(t)] = \sum_k i\, f_{ijk}\, F_k(t)\ , \qquad (2)$$

where the f_{ijk} are the structure constants of SU(3). I would like to stress here that the above statements are true in <u>any</u> reasonable field theory which could possibly lead to SU(3) and are in no way peculiar to the quark model. (In the present context the only thing special about the quark model is that it is the simplest model leading to the commutation relations, Eq. 2.)

Again in any field-theoretic model, there will be an octet of axial vector currents $\mathcal{J}^{50}_i(x)$, with $i = 1 \cdots 8$. In analogy with Eq. 1, one can define axial charges according to

$$F^5_i(t) = \int d^3\underset{\sim}{x}\ \mathcal{J}^{50}_i(\underset{\sim}{x}, t)\ , \qquad (3)$$

and since the axial currents belong to an octet, we have

$$[\,F_i(t),\, F^5_j(t)] = \sum_k i\ f_{ijk}\, F^5_k(t)\ . \qquad (4)$$

Thus far, I have simply pointed out some very general, formal properties of Lagrangian field theory. The physics enters when we cease to think of the $\mathcal{J}$'s and $\mathcal{J}^5$'s as purely abstract objects and identify them with the various components of the physical weak and electromagnetic currents. The well-known correspondence is[2]

$$J^\nu_{em} = \mathcal{J}_3 + (\sqrt{3})^{-1}\mathcal{J}_8 \qquad (5)$$

and

$$J^\nu_{weak} = \cos\theta\,(\mathcal{J}_{1+i2} + \mathcal{J}^5_{1+i2}) + \sin\theta\,(\mathcal{J}_{4+i5} + \mathcal{J}^5_{4+i5}), \qquad (6)$$

where, for example, $\mathcal{J}_{1+i2} = \mathcal{J}_1 + i\,\mathcal{J}_2$, and θ is the Cabibbo angle. It is worth emphasizing, I think, that the identification of the <u>physical</u> weak and electromagnetic currents with objects which satisfy the equal-time commutators 2 and 4 is a meaningful and nontrivial statement about the physics of hadrons. In principle, all the matrix elements of J^ν_{em} and J^ν_{weak} can be measured in electron and neutrino scattering experiments. Given these matrix elements, one could calculate the commutators and test Eqs. 2 and 4 directly. Such a program is, of course, out of the question at present, but as we shall see there is considerable evidence in support of Eqs. 5 and 6.

Besides 2 and 4, Gell-Mann further suggested that the commutator of the axial charges should be a vector charge, according to

$$\left[F^5_i(t),\ F^5_j(t)\right] = i\sum_k f_{ijk}\ F_k(t)\ . \qquad (7)$$

While, as I pointed out, Eqs. 2 and 4 are true in any reasonable field theory, Eq. 7 is somewhat more restrictive. Examples of theories in which 7 would be true are (i) the quark model, (ii) a model built on eight elementary baryons, and (iii) an SU(3) generalization of the σ model[2, 3] with eight elementary baryons, nine pseudoscalar mesons, and nine scalar mesons. An example of a theory for which Eq. 7 does not hold would be a model built on eight baryons and eight pseudoscalar mesons with no scalar mesons. We know, of course, from the work of Adler[4] and Weisberger[5] that the physical vector and axial vector currents do satisfy Eq. 7. My reason for discussing the above models was to bring out the following points. Since 7 is not true in all models it is a nontrivial statement about the physical currents. Nevertheless, it does not necessarily have anything to say about quarks.

In addition to the commutation relations among charges one can ask about the commutators of the densities themselves. If the theory is to be local, we must have, for two charge densities,

$$\left[\mathcal{J}^0_i(\underset{\sim}{x}, 0),\ \mathcal{J}^0_i(\underset{\sim}{y}, 0)\right] = i\sum_k \delta^3(\underset{\sim}{x}-\underset{\sim}{y})\ f_{ijk}\mathcal{J}^0_k(\underset{\sim}{x}) + \text{(more singular terms)}. \qquad (8)$$

In this particular case of two charge densities, there is no particular reason to believe that the more singular terms exist, since there is no known simple model that produces them. On the other hand, we do not have any real evidence that they are not there, either. The situation is rather different when we consider the local commutator of a charge density $\mathcal{J}^0_i$ with one of the space components of the current $\underset{\sim}{\mathcal{J}}_j$. Again, on grounds of locality we must have

$$\left[\mathcal{J}^0_i(\underset{\sim}{x}, 0), \underset{\sim}{\mathcal{J}}_j(\underset{\sim}{y}, 0)\right] = i\,\delta^3(\underset{\sim}{x}-\underset{\sim}{y})\ \sum_k f_{ijk}\underset{\sim}{\mathcal{J}}_k(\underset{\sim}{x}) + \underset{\sim}{\nabla}_x\,(\delta^3(\underset{\sim}{x}-\underset{\sim}{y})\ S_{ij}) + \cdots\ , \qquad (9)$$

where I have explicitly written out a possible gradient term. In this case one can prove[6] that

$$\langle 0 \mid S_{ij} \mid 0 \rangle \neq 0. \quad (10)$$

There are two possibilities consistent with Eq. 10; S_{ij} can be either an operator or a C number. Unfortunately, it is not possible to prove from the basic postulates of relativistic quantum mechanics such as locality, Lorentz invariance, unitarity, and so on that S_{ij} either is or is not an operator. The reason is simply that there are free-field theories consistent with all these postulates in which S_{ij} is a C number (quark model) and in which S_{ij} is an operator (any theory with elementary charged mesons). The actual character of this object S_{ij} probably depends in a detailed way upon the dynamics of hadrons.

It turns out that the singular terms like S_{ij}, which may appear in the local commutators, never seem to show up in any practical applications of the commutators. One point of view would be, then, that these complicated singular objects are not really relevant to physics and may as well be forgotten. An alternative point of view is that the very complexity and model dependence of an object like S_{ij} make it interesting. If we could (i) "measure" the matrix elements of S_{ij} by, say, a sum rule, and (ii) learn how to interpret these matrix elements theoretically, we might learn a lot about the actual dynamics of hadrons. Unfortunately, at present, it is not clear how one could do either (i) or (ii).

Finally, I would like to discuss, very briefly, commutators of the space components of the currents. I have stressed that the commutators 2, 4, and 7 of the charges F and F^5 are rather model-independent. This is not the case for the commutators of the space components. The quark model makes a prediction for $[\mathcal{F}_i, \mathcal{F}_j]$ which is not shared by any other known theory. Evidently, it would be of great interest to test these predictions. However, it is not at all clear how one could obtain such a test, and we have, in fact, essentially no knowledge as to the commutation relations among the space components of the physical currents.

II. Low-Energy Theorems for Soft Pions

One of the most fruitful applications of the current commutators has been their use in the derivation of low-energy theorems for processes involving soft pions. What I mean by a low-energy theorem for pions is, perhaps, best explained by an example. Let us consider a scattering process in which the initial state contains a heavy particle i and a pion of isotopic spin a and momentum q_a, and the final state contains a heavy particle f and a pion of isotopic spin b and momentum q_b. We wish to find an approximate expression for the scattering amplitude that will be valid for small q_a and q_b.

The derivation of the desired result would proceed as follows. One defines a function A as

$$A(q_b b, q_a a) = \frac{i f_\pi^2}{4 m_\pi^4} (q_a^2 - m_\pi^2)(q_b^2 - m_\pi^2) \int d^4x\, e^{-i(q_a - q_b)\cdot x}$$

$$\times \langle f \mid T(\partial_\nu \mathcal{F}_b^{5\nu}(x)\, \partial_\mu \mathcal{F}_a^{5\mu}(-x)) \mid i \rangle \;, \quad (11)$$

where $(2f_\pi)^{-2} = m_\pi^{-2} \langle \pi \mid \partial_\nu \mathcal{F}^{\nu 5}(0) \mid 0\rangle$. It is simply a mathematical fact that (i) when $q_a^2 = q_b^2 = m_\pi^2$, A is equal to the scattering amplitude for $\pi_a + i \rightarrow \pi_b + f$, and (ii) for small q_a and q_b, A can be determined up to order $q_a q_b$ in terms of known pole diagrams and the equal-time commutators $[F_a^5, F_b^5]$ and $[\dot{F}_a^5, F_b^5]$. The physics enters when we assume that the pion mass is sufficiently small and A is a sufficiently slowly varying function of q_a and q_b that the values of A as calculated from the known low q limit can give useful information about actual low-energy pion scattering. I would like to emphasize that this physical assumption is by no means trivial. In an ordinary Lagrangian field theory with elementary pions and nucleons interacting through a pseudoscalar coupling, both (i) and (ii) above are true, order by order in perturbation theory. But in this theory, at least to low orders in the perturbation expansion, A is an extremely rapidly varying function and its low q limit is not useful for discussing pion scattering. In the real world, however, it does seem to be true that A is slowly varying over the relevant range of q.

Here I have to mention one technical point. The expression one obtains in the low q limit contains an unknown operator $\Sigma_{ab} \equiv [\dot{F}_a^5, F_b^5]$. This object has the dimensions of mass and is something like the term in the Hamiltonian which violates conservation of the axial vector current. It turns out that if A is to be slowly varying, as appears to be the case, then Σ_{ab} must be very small and can be neglected.

Neglecting, then, a presumably small term proportional to Σ_{ab}, one finds, for small q, the remarkably simple formula

$$A(q_b b, q_a a) = \sum_c i f_\pi^2 (q_a + q_b)\cdot(P_i + P_f)\epsilon_{abc}(T^c)_{fi} + \text{poles} + O(q_a a_b) \;, \quad (12)$$

where $(T)_{fi}$ is the isospin matrix between the initial and final heavy particles.

Equation 12 can be used to predict the πN scattering lengths. The predictions[7] are $a_{1/2} = 0.20\, m_\pi^{-1}$ and $a_{3/2} = -0.10\, m_\pi^{-1}$, which are in good agreement with the experimental values of $0.17\, m_\pi^{-1}$ and $-0.09\, m_\pi^{-1}$.

The low-energy theorem for pion scattering which I have just described is typical of a number of theorems for processes involving soft pions. I would like to summarize, here, the main results of the theorems. When they are put together, I feel that they constitute an impressive bit of physics. These results can be listed as follows:

(a) $\pi + N \rightarrow N + 2\pi$

L. Chang[8] has calculated the amplitude for production of two soft pions in a pion-nucleon collision. A preliminary comparison with experiment indicates good agreement up to incident pion energies of about 300 MeV.

(b) $\gamma + N \rightarrow \pi + N$

The low-energy theorems for photoproduction[9] are

$$A^+(q_\pi = 0) = (G/4M^2)(\mu_p' - \mu_n), \quad (13)$$

$$A^0(q_\pi = 0) = (G/4M^2)(\mu_p' + \mu_n) \;, \quad (14)$$

where the A^+ and A^0 are the standard CGLN photoproduction amplitudes. A dispersion integral evaluation[10] of $A^+(q_\pi = 0)$ indicates that Eq. 13 is satisfied to within 15%, and it is known that $A^0(q_\pi = 0)$ is small in agreement with Eq. 14.

(c) Leptonic K Decays

All amplitudes for K_{e2}, K_{e3}, and K_{e4} decays[11-13] can be expressed in terms of two matrix elements of the weak current. One prediction is $f_+ (q_\pi = 0) + f_- (q_\pi = 0) = f_\pi/f_K$, where f_+ and f_- are the K_{e3} form factors. As pointed out by Cabibbo, this relation is compatible with experiment. The ratio $R = \tau(K^+ \to \pi^0 + e + \nu)/\tau(K_2^0 \to \pi^+ + \pi^- + e + \nu)$ can also be predicted; comparison of theory and experiment gives R theory/R exp = 1.0 ± 0.2.

(d) Nonleptonic K Decays

Here[11, 14, 15] we have to make some assumption about the weak Hamiltonian. The simplest hypothesis is that H_W is a function of the weak current only, which will be the case if there is an intermediate boson or a local current-current interaction. With this assumption one finds that all K decays should respect the $|\Delta I| = 1/2$ rule even though H_W may contain a $|\Delta I| = 3/2$ piece. The ratio $R = \tau(K_1^0 \to \pi^+\pi^-)/\tau(K^+ \to \pi^+\pi^+\pi^-)$ is predicted with the result R' theory/R' exp = 0.80 ± 0.05; all other ratios of rates follow from the $|\Delta I| = 1/2$ rule. Finally, the energy dependence of the matrix elements for $K \to 3\pi$ can be calculated, and again, there is good agreement between theory and experiment.

(e) Nonleptonic Decay of the Hyperons

Making the same assumption as above about H_W, one can obtain a low-energy theorem for the S-wave amplitudes in the decays $B \to B' + \pi$.[16, 17] The conclusion is that "octet dominance," which implies the $|\Delta I| = \frac{1}{2}$ rule and the Lee-Sugawara triangle, will hold if, and only if, the S-wave amplitude $S(\Sigma_+^+) = 0$. Experimentally, we seem to have both octet dominance and $S(\Sigma_+^+) = 0$. There are no predictions for the P-wave amplitudes, unless one makes additional dynamical assumptions such as the validity of the pole model; the pole model does not seem to work very well.

I should make one final remark about the low-energy theorems. It is very hard to understand how they could work so well if there is a strong $\pi\pi$ interaction at low energies. Weinberg[7] has applied the current algebra techniques to $\pi\pi$ scattering and finds scattering lengths

$$a_0 = 0.20\ m_\pi^{-1}, \qquad a_2 = -0.06\ m_\pi^{-1}, \tag{15}$$

which are, in fact, small. It would be very helpful to have more accurate experimental data on the $\pi\pi$ interaction.

III. Dispersion-Theoretical Sum Rules

The basic idea of a sum rule is, of course, very simple: one simply sandwiches a commutation relation between two states, inserts a complete set of intermediate states, and thus obtains a sum rule. Unfortunately, the sum rules obtained in the simple way are not very useful: first, they are hard to compare with experiment and, secondly, they are not explicitly covariant. For this reason, there has been considerable work on the problem of transforming sum rules into a covariant, dispersion-theoretic form. I would like to sketch now how this transformation is accomplished and what the limitations of the method are. Then I will give some of the experimentally testable rules which have emerged.

It is convenient to start, not with the equal-time commutator itself, but with the Fourier transform of a retarded product. Let us define

$$M(k_0, \underset{\sim}{k} \cdots) = i \int d^4x\, e^{-ik\cdot x}\, \theta(x_0) \langle f | [A(\underset{\sim}{x}), B(0)] | i \rangle , \tag{16}$$

where A and B are local operators which we will later take to be various components of the currents $\mathcal{J}_i^\nu$ and $\mathcal{J}_i^{\nu 5}$. As is well known, M is an analytic function of k_0 and satisfies the Low equation

$$M(k_0, \underset{\sim}{k} \cdots) = \int \frac{dk_0'\ \mathrm{Im}\ M(k_0', \underset{\sim}{k} \cdots)}{k_0' - k_0} \tag{17}$$

Furthermore, one can convince himself that the equal-time commutator is given by

$$\langle f | \left[\int d^3x\, e^{i\underset{\sim}{k}\cdot \underset{\sim}{x}} A(\underset{\sim}{x}, 0),\ B(0) \right] | i \rangle = - \lim_{k_0 \to \infty} k_0 M(k_0, \underset{\sim}{k} \cdots) = \int dk_0'\ \mathrm{Im}\ M(k_0', \underset{\sim}{k} \cdots) . \tag{18}$$

So far, what has been accomplished is to relate the equal-time commutator to something which looks like the integral over the absorptive part of a scattering amplitude: the sum rule is still not covariant because of the noninvariant integration contour, dk_0 with $\underset{\sim}{k}$ held fixed. Evidently, the next step will be to change to an invariant contour of integration, just as one does in going from the Low equation to a dispersion relation.

From the proofs of dispersion relations, we know that Eq. 17 can be rewritten as

$$M(S \cdots) = \int \frac{\mathrm{Im}\ M(S' \cdots)}{S' - S}\, dS' + (\text{subtractions}), \tag{19}$$

where $S = (k + P_i)^2$, the variables being held fixed in the integration are invariant scalar products such as $t = (P_i - P_f)^2$, k^2, and $q^2 = (P_i + k - P_f)^2$, and M is an invariant function which may differ from M by kinematical factors. In the same manner, Eq. 18 can be transformed into

$$\langle f | [\ \ ,\ \ ] | i \rangle = G(t) = \int dS'\ \mathrm{Im}\ M(S', k^2, q^2, t) + (\text{subtractions}), \tag{20}$$

where the "form factor" G(t) represents the matrix element of the equal-time commutator. The sum rule has now been written in an invariant form, but before discussing applications we must face the problem of subtractions, the possible presence of which is indicated in Eqs. 17 and 20.

It is well known that in going from Eq. 17 to 19 one may pick up subtraction terms, i.e., polynomials in S with coefficients which are functions of t, k^2, and q^2; the same thing can happen when one goes from Eq. 18 to 20. Evidently, the sum rule will be useless if subtractions are present, since the right-hand side will contain unknown functions of t. In commutators of current densities $\mathcal{J}^\nu$ and $\mathcal{J}^{5\nu}$, the situation with regard to subtractions is as follows: (i) For the commutator of two time components $\mathcal{J}^0$ or $\mathcal{J}^{05}$, subtractions are not present and Eq. 20 gives a useful sum rule; (ii) for one time component and one space component $\underset{\sim}{\mathcal{J}}$ or $\underset{\sim}{\mathcal{J}}^5$, the question of whether subtractions are present depends on dynamical details and cannot be answered in general; (iii) for two space components subtractions are present and Eq. 20 is not a useful sum rule.

Earlier, I stated that we know quite a bit about the commutators of time components, but hardly anything about commutators of space components of the currents. This is, in large part, because one can write covariant dispersion-theoretic sum rules in the former case but not in the latter.

Even in its dispersion-theoretic form, the general sum rule

$$G(t) = \int \mathrm{Im}\ M(S', k^2, q^2, t)\ dS' \qquad (21)$$

is still not easy to test experimentally: for practical reasons one has to resort to special cases. Setting $k^2 = q^2 = 0$ and taking the derivative with respect to t at $t = 0$ in Eq. 21 yields, for the commutator of $\mathcal{J}_i^0$ and $\mathcal{J}_j^0$, the relation[18]

$$\frac{1}{3}\left(\langle r^2\rangle_p - \langle r^2\rangle_n\right) - (\mu_p - \mu_n)^2$$

$$= \frac{1}{2\pi^2 a}\int \frac{\sigma^V_{3/2} - 2\sigma^V_{1/2}}{\omega}\ d\omega, \qquad (22)$$

where $\sigma^V_{3/2}$ is the cross section for (isovector γ) + proton $\to I = 3/2$ states, $\sigma^V_{1/2}$ is the cross section to make isospin 1/2 states, ω is the photon energy, and the $\langle r^2\rangle$'s and μ's are charge radii and magnetic moments. A similar sum rule which makes use of the commutator $[\mathcal{J}^0, \mathcal{J}]$, and is therefore plagued by subtraction questions, is[19]

$$\mu_p - \mu_n = (1/4\pi^2 a)\int\Big[(\sigma_{3/2} - 2\sigma_{1/2})^V_A$$

$$- (\sigma_{3/2} - 2\sigma_{1/2})^V_P\Big]\ d\omega, \qquad (23)$$

where the subscripts A and P refer to proton and photon spins antiparallel and parallel.

An interesting fact about the sum rule 22 is that the left-hand side is negative, while the contribution of the (3,3) resonance to the right-hand side is positive. The resulting failure of a simple resonance approximation in Eq. 22 caused some worry when the sum rule was first derived, but a more careful evaluation of the dispersion integral,[20] including S waves and higher resonances, indicates that Eq. 22 is, in fact, satisfied.

The sum rule 23 is interesting because, as mentioned above, it depends on the commutator $[\mathcal{J}^0, \mathcal{J}]$. Unlike 22, Eq. 23 is not satisfied in perturbation theory--a reflection of the subtraction question which shows up in these commutators. For these reasons it would be very interesting to know if this sum rule is satisfied. As far as I know, no numerical evaluation has been attempted.

In addition to the explicit evaluation of dispersion integrals, there is another way to test special cases of Eq. 21. One can show that the difference in neutrino cross sections $(d/dk^2)\ \sigma(\bar{\nu}P) - \sigma(\nu P)$, where k^2 is the momentum transfer from the leptons to the hadrons, is essentially equal to

$$\int_0^{S_{max}} \mathrm{Im}\ M(S', k^2, k^2, 0)\ dS',$$

where Im M comes from the commutator of j^0_{weak} and j^{+0}_{weak}, and S_{max} is determined by the incident neutrino energy. For large neutrino energy, $S_{max} \to \infty$, and one has[21]

$$\lim_{E_\nu \to \infty} \frac{d}{dk^2}\left(\sigma(\bar{\nu}p) - \sigma(\nu p)\right) = \int \mathrm{Im}\ M(S', k^2, k^2, 0)\, dS' \qquad (24)$$

$$= G(0) = (\text{known constant independent of } k^2).$$

A similar calculation for electron scattering produces only an inequality

$$\lim_{E_e \to \infty} \frac{d}{dk^2}\left[(\sigma(ep) + \sigma(en)\right] > \frac{2\pi a^2}{k^4}, \qquad (25)$$

where the k^{-4}, which is missing in Eq. 24, comes from the photon propagator. One hopes that this inequality can be tested in the near future.

The sum rules 22 and 23 are probably the only predictions of the local commutators which are of immediate experimental interest. This is not to say, however, that the local commutators and dispersion sum rules do not deserve further study. These commutators presumably contain a considerable amount of information about the relation between strong interactions and the weak and electromagnetic interactions.

IV. Saturation and Approximate Symmetries

The final topic I will discuss concerns the use of current commutators to predict the properties of hadron states. Some examples of this idea, in its simplest form, are:

(a) The commutator

$$\left[F_i^5, F_j^5\right] = \sum_k i\, f_{ijk}\, F_k$$

leads to a sum rule[4] for $\pi\pi$ scattering which is not satisfied if one includes only the contribution of the known resonances ρ and f^0. There have been some guesses that a strong low-energy $\pi\pi$ interaction is necessary to bring the sum rule into agreement with experiment, but it is, of course, very possible that the discrepancy is due to high- rather than low-energy effects.

(b) If, in the corresponding sum rule for $\pi\rho$ scattering, one assumes that the ω and ϕ mesons dominate, then the $\omega\rho\pi$ coupling can be predicted with results that are in good agreement with experiment.[22]

In the examples described above, one simply examines a specific sum rule and tries to extract whatever information is available. While this approach may lead to interesting results, it is not very systematic. A more powerful approach is suggested by the fact that the sum rules are equivalent to the statement that the F's and F^5's generate the algebra of $SU(3) \otimes SU(3)$: thus the analysis of sum rules is really a problem in group theory. Now, in group theoretical language, the hypothesis that the sum rules are saturated by a small number of single-particle and resonant states is equivalent to the supposition that the physical particle states are mixtures of, at most, a few irreducible representations of $SU(3) \otimes SU(3)$. One may, then, try to classify the hadrons according to representations of $SU(3) \otimes (SU(3)$ and obtain, in some rather vague sense, a sort of higher symmetry.

A considerable amount of work has been done on this problem of using current algebra to classify states. For a number of technical reasons, the problem is not easy. I will not go into any of the details here, but will simply describe the results of this program as applied to the low-lying baryon states.

In zeroth approximation, one assigns the baryon octet and decuplet of resonances to the pure representation (6, 3). To start with, at least, this assignment seems to be fairly good. Some predictions are (i) $|G_A| = 5/3$ as opposed to the experimental value of ≈ 1.20, (ii) a D/F ratio of 3/2 for the matrix element of the axial current between baryons, and

(iii) a value of G^*, the octet-decuplet matrix element of the axial current, which is in rather good agreement with experiment. The trouble comes when one looks at the baryon anomalous magnetic moments, which, it turns out, are predicted to be zero. The way out of this difficulty is, of course, to introduce some mixing; one also hopes that the mixing will improve the prediction of G_A.

Several people[23-25] have studied the possibility of representing the octet and decuplet as $(6,3)$ along with various admixtures of $(8,1)$, $(3,\bar{3})$, and $(\bar{3},3)$. It turns out that there is more than one satisfactory solution to the problem, so that which is the "right" mixture is still not known. Nevertheless, there is some reason for optimism. The mixing schemes typically have three or four free parameters but can predict six quantities (G_A, the D/F ratio for the axial current, G^*, the neutron and proton magnetic moments, and the N-N^* transition moment) to a very good accuracy.

Acknowledgment

I would like to thank Dr. S. Adler, Dr. Y. Dothan, Dr. P. Freund, Dr. F. Gilman, and Dr. B. Lee, the discussion leaders and scientific secretaries for this session, both for their direct help in gathering the material presented here and for many discussions which greatly increased my understanding of current algebras.

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Discussion

Balachandran (Syracuse): M. G. Gundzik, F. Nicodemi, and I have obtained, besides the S-wave π-N and K-N scattering lengths, the S-wave effective ranges and P-wave scattering lengths for these processes. The agreement between theory and experiment is good except for the π-N S-wave $I = 1/2$ effective range and the π-N 3-3 scattering length. In collaboration with Dr. Narayanaswami, we have also obtained preliminary results on low-energy photopion production.

We have also done the π-π calculation. The S-wave scattering lengths we get are somewhat different from Professor Weinberg's, although the signs and orders of magnitude are the same. The difference seems to be due to the choice of different extrapolations. The σ-meson term which was written down vanishes on the mass shell. Therefore, the Green's function with this term subtracted out is also an extrapolation in the pion mass of the S matrix. We do not of course know whether the gently varying extrapolation must have the σ-meson term or not. I would like to point out that the σ-meson term cannot be evaluated by comparing the predictions with experiment, as the extrapolation with this term included may not be gently varying.

Lovelace (CERN): I understand that both the Weinberg π-π scattering lengths and the Adler sum rule are true in the Gell-Mann – Levy σ-model, at least in perturbation theory. So any contradiction must depend on additional assumptions, e.g., that the π-π S wave has a weak energy dependence.

Dashen: Remember that the σ model does not fix the mass of the σ meson; it could be anything. The Weinberg π-π scattering length has effectively given the σ meson a large mass, large compared with the pion mass. Now, offhand I don't know if you can give the σ a very large mass and still satisfy the sum rule.

Weinberg (Berkeley): The Adler sum rule for π-π scattering is saturated by a σ meson appearing in the t and u channels as well as the s channel, but not by an S-wave resonance alone unless its mass is extremely low. We have to learn how to put particles into sum rules in a crossing-symmetric way, but of course this is a problem for many people.

Lovelace: Can Balachandran predict the π-N S-wave effective ranges? These were always the difficulty with vector-meson dominance. If you look at the π-N S-wave scattering lengths by themselves, it appears that they are dominated completely by the vector mesons, because the spin-0 exchange vanishes. As soon as you go to the effective ranges, this goes completely wrong.

Balachandran: Let me emphasize that the extrapolation is highly nonunique, because the S matrix is not defined off the mass shell. You have to make a choice as to what extrapolation you use, and you hope that you've got the most gently varying one. We have used one particular extrapolation, and this gives the S-wave effective range and the P-wave scattering lengths for π-π scattering to be zero, and the S-wave π-π scattering lengths are of the same order of magnitude and the same sign as Professor Weinberg's, but they are not numerically equal, they are slightly different. But I would not like to swear by these numbers because I think that the problem of extrapolation for the π-π problem is extremely difficult and ambiguous.

Lovelace: But what about π-N scattering?

Balachandran: For the I = 1/2 S-wave effective range, the number we get is 0.092 in pion masses, while the Hamilton-Woolcock number is -0.021. There is violent disagreement between the two numbers. For the I = 3/2 effective range we get -0.046, while the Hamilton-Woolcock number is -0.051. I was told by Professor Wali that there is some experimental disagreement between the S-wave fit of Hamilton and Woolcock and some low-energy data which have become available.

Adler (Inst. for Advanced Study): If Weinberg's prediction of small π-π scattering lengths is verified experimentally, this does not imply any contradiction with the π-π sum rule. In evaluating the sum rule, I inserted only the contribution from the known ρ and f^0 mesons, and assumed that the remainder needed to saturate the sum rule came from the I = 0 S wave, and not from possible π-π resonances at energies higher than the f^0 or from a high-energy tail. If the I = 0 S wave turned out to be small, it would indicate that the region above the f^0 meson must be important; I don't believe this would contradict any known experimental facts.

Weinberg: Professor Khuri has looked at the problem of the extrapolation in the π-π scattering amplitude and has found that the method of extrapolation is essentially unique, providing that the π-π interaction is not too strong.

Moffat (Toronto): You mentioned at the beginning of your talk that the current algebras are essentially model-independent. It appears that the Adler-Weisberger result for g_A is based on choosing the quark current, so that you get $2I_3$ on the right-hand side of the commutator. Has anyone succeeded in calculating the deviation (if any) from the Adler-Weisberger result for g_A, based on quarks, resulting from the use of a model based on elementary pions, e.g., the Yukawa coupling model associated with the "eightfold way"? In such a model one gets $2I_3$ plus something else on the right-hand side of the commutator in the $SU(2) \otimes SU(2)$ algebra. Is it possible to check the model dependence of the Adler-Weisberger sum rule?

Dashen: As far as I know, given the eightfold-way model, if that's what you call it, with eight elementary spin-1/2 baryons and eight elementary pseudoscalar mesons, there wouldn't be any way to evaluate the sum rule because you wouldn't know what the commutator is physically. Mathematically, it would be the part of the isotopic spin current that comes from nucleons, but you cannot separate that from the part that comes from the pions. It might be zero because it amounts to something like the core of the nucleon. Because of this, I don't think that one could learn anything about the possible validity of the eightfold model by use of a sum rule.

Mandelstam (Berkeley): At the beginning of your talk you pointed out that the dispersion-relation sum rules obtained from commutation relations of space components of currents need subtractions. If I am not mistaken you need space components of currents in order to get SU(6) from current algebra taking only low-lying states. How do you handle the matter of subtraction terms in this?

Dashen: The original use of the commutation relations for the space components of the current was not done with covariant sum rules. One used a sum rule that looks like the Low equation, where there is no question about subtractions, but there are serious questions about the rate of convergence.

Goldberger (Princeton): Most discussions of the applications of current commutators relate to scattering amplitudes for massive vector mesons on baryons or, through PCAC, to pseudoscalar meson-baryon amplitudes. The absence of similar relations for baryon-baryon amplitudes has always seemed to me to indicate a possible incompleteness in the theory. The reason why the baryon-baryon problem is difficult is, of course, clear in that one has not had Gell-Mann propose a set of commutation relations for the source of the baryon fields, and I wondered if there was any wisdom on that subject that I was unaware of?

Dashen: There is one thing. Whenever all strongly interacting particles are on the mass shell, all the sum rules become trivial. They are related to the superconvergent sum rules of Fubini and his collaborators, and will be discussed by Francis Low in his section. What happens is the following. If one writes down a sum rule for currents and then converts it into a sum rule for actual strongly interacting particles, he finds that the conclusion is that a certain integral must vanish. The statement that the integral vanishes is equivalent to a statement about the asymptotic behavior of the integrand, and it always turns out that in the derivation of the sum rule one has always assumed precisely this asymptotic behavior. Another way to state it is that one might say that the ρ-meson field is like the electromagnetic current. However, from the commutation relations of the vector current it's impossible to get any nontrivial restrictions on ρ-meson scattering, that is, restrictions that are not already implied by the assumptions used in deriving the sum rules. There is somewhat of a difference in the case of pions. If you say that the pion field is the divergence of an axial-vector current, then the explicit presence of a time derivative allows you to get some information about pion scattering, but only for fictitious zero-mass pions. If one wants to do a similar thing for the baryons, what is going to be necessary is to say that a spin-1/2 field is something like the divergence of a Rarita-Schwinger field, and you would have to know something about the commutation relations of these things. I don't know whether anybody has made such an attempt; it is certainly possible, but to what extent it's useful I don't know.

Ne'eman (Tel Aviv): A partial answer to Professor Goldberger's question (amplitudes, though no direct sum rules as yet) is provided by the identification of an algebraic structure in the system of space integrals of the factorized Regge residues. In that case, which involves generators with scalar densities, one gets predictions such as the Levin-Frankfurt ratio $\sigma_{\pi p}/\sigma_{pp} = 2/3$.

Hwa (Stony Brook): In connection with Goldhaber's comment, I want to remark that Nuyts and I have looked into just that problem. We postulated PCBC--that is, we assumed the existence of a baryon "current" whose divergence is related to the baryon field. We constructed the baryon current from a suitable product of three quarks. The anticommutator relation between a baryon and an antibaryon current was calculated on the basis of the canonical commutation relations of the quarks. It turned out, however, that the right-hand side of the anticommutation relation is terribly complicated. We could get no useful result out of it. We went on to consider the commutation relation between a baryon current and a meson current and did obtain a very simple relationship. Taking matrix elements of the commutator between baryon and meson states, we obtained constraint relations among various form factors.

Khuri (Rockefeller U.): Before we can answer Professor Goldberger's question we must ask about PCAC for the K meson. In all the impressive list of successes listed by the rapporteur we find only pion processes and not one soft kaon process. I would like to know if such results exist, and how they compare with experiment? It might be that even in the case of the K meson, PCAC would turn out to be not quite so useful.

Dashen: I know that in the K-decay predictions one runs into some funny business if he tries to let the K-meson mass go to zero. However, some of the sum rules written in analogy with the Adler-Weisberger sum rule use PCAC for the K meson, and as far as one can tell they work. I'm sure it's possible to predict the K-nucleon scattering lengths, and I suppose it has been done, although I don't happen to know what the results are; that would be a possible way of telling. I think I should say that if you're going to assume something like PCBC, then you are only going to be able to make predictions for zero-mass baryons, just as you can only make predictions for zero-mass pions. I suppose one can make many predictions for the emission of soft N^*'s, but I suspect that you will have to go to very high energy before baryons become soft. Of course, it's not necessarily out of the question.

Weinberg: I would like to use my prerogative as chairman, and give another answer to Goldberger's question. I think you can't get something for nothing. Behind the successful predictions of PCAC lies a symmetry, that of $SU(3) \otimes SU(3)$ or $SU(2) \otimes SU(2)$, which could be exact if the pion had zero mass, and which would then lead to exact low-energy theorems. PCAC just says that the real world is not too different from the ideal world, so we get approximate predictions only, a situation not too different from that with which we're familiar in SU(3).

Adler: The sum rules relating the strangeness-changing axial-vector coupling constants to kaon-nucleon cross sections are not sensitive tests of PCAC for the strangeness-changing current. The reason is that they typically involve the ratio $\sigma_0(KN)/g_0(KNB)^2$, where $\sigma_0(KN)$ and $g_0(KNB)$ are respectively the kaon-nucleon cross section and the kaon-nucleon-baryon coupling constant at zero kaon mass. It is possible that $\sigma_0(KN)$ and $g_0(KNB)$ are appreciably different from their on-mass-shell values, but that the ratio involved in the sum rule is still nearly the same as its on-mass-shell value. To check PCAC for the $\Delta S \neq 0$ currents, one should look at relations connecting hyperon beta decay, kaon beta decay, and kaon-nucleon coupling constants, analogous to the usual Goldberger-Treiman relation relating nucleon beta decay, pion beta decay, and the pion-nucleon coupling constant.

Feynman (Cal Tech): I am not commenting on Mr. Goldberger's question, but on something else. It is interesting that if one writes some axial-vector meson field is proportional to the axial current $\rho^5_\mu = F^5_\mu$, then current algebra permits one to calculate some properties of the propagator for the meson (vacuum expectation value of $F^5_\mu F^5_\nu$). The result is that the propagator no longer is purely transverse, but a new term, purely longitudinal, arises with an independent denominator permitting new poles and thus representing pseudoscalar particles coupled to the divergence of F^5_μ, and satisfying the conditions of PCAC.

Logunov (Serpukhov): You presented a calculation for the cross section of the reaction $\pi + N \rightarrow \pi + \pi + N$. In what way was the direct $\pi\pi$ interaction taken into account, if at all? The same question about the K decay into three pions.

Dashen: In the production of two soft pions, the one-pion-exchange pole is explicitly taken into account, using the scattering lengths for the $\pi\pi$ interaction predicted by the low-energy theorems.

Tavkheledze (Dubna): N. Bogolubov drew attention to the fact that many results usually obtained from the algebra of currents are in fact the consequence of dispersion relations and assumptions about the subtractions in these dispersion relations. Consider the amplitudes

$$T(s,t) = \int e^{iq\cdot x}\theta(x_0)\langle p'|j(x)j(0)|p\rangle ,$$

where j is one of V^a_μ, A^a_μ, $\partial_\mu A^a_\mu = j^a$.

Using local properties (microcausality),

$$[j(x), j(y)] = 0, \text{ with x and y spacelike,}$$

we get dispersion relations for $T(s,t)$. Separating the spin and unitary spin structure, we get

$$T(s,t) = \sum_i \hat{P}_i f_i (s,t).$$

If we suggest dispersion relations for $f_i(s,t)$ and $sf_i(s,t)$ (both without subtractions), we get the sum rules

$$\int_{-\infty}^{+\infty} f_i(s,t)\, ds = 0$$

applied to the amplitude

$$T^{\alpha\beta}_{\mu\nu} = \int e^{iq\cdot x}\theta(x_0)\ \langle p'\ |[\ V^\alpha_\mu(x),\ V^\beta_\nu(0)]\ |p\rangle\ .$$

This technique allows one to obtain the well-known Cabibbo-Radicatti sum rules.

When they are applied to the amplitude

$$T^{\alpha\beta} = \int e^{iq\cdot x}\theta(x_0)\langle p'|\ j^\alpha(x),\ j^\beta(0)|p\rangle ,$$

suggesting unsubtracted dispersion relations, we obtain, for $t = 0$,

$$f(0,0) = g_A^2 + \frac{f^2}{M}\int_0^\infty \frac{k d\omega}{\omega^2}(\sigma_{\pi^- p} - \sigma_{\pi^+ p}) .$$

To calculate $f(0,0)$, we suggest that the amplitude f is composed of the quark amplitudes in an additive way. Taking account of only the Born term in the quarks amplitude gives $f(0,0) = (g_A^2)_{quark} = 1$, and we have the Adler-Weisberger relation.

Dashen: As I was saying, most of the commutation relations are rather model-independent, and in almost everything one does in practice, it looks like all the peculiar gradient of δ-function terms either never enter into the calculation or else cancel out. For that reason, in the commutation relations that I wrote down in the beginning there is only really one independent assumption, and that is that the charges F^5_+ and F^5_- when commuted give back twice the third component of isotopic spin. Almost everything else that I have said can be considered as a consequence of dispersion relations and of the conservation of isotopic spin along with SU(3). Now, if one wants to introduce this assumption in some other way than saying it came from an equal-time commutator, he is perfectly free to do so, and he will probably end up getting the same answers. There is really just one independent number, which can be taken as the one on the right-hand side of the Adler-Weisberger sum rule, and the fact that it is one and not 0.7 is not a consequence of dispersion relations or locality. It is a consequence of a commutation relation, for example.

Appendix

Approximate Hadron Symmetries

Introductory Remarks at Preliminary Session 3b

B. W. Lee, Discussion Leader

I would like to divide the contributions to this session into three categories. These are:
First: Chiral and Collinear Algebra and Classification of Hadron States
Second: Local Current Commutation Relations and Their Representations
Third: Miscellaneous

In the first category, I propose to focus our attention on the following papers:
1. Relation Between D/F and G_A/G_V from Current Algebra -- Gatto et al. [Phys. Rev. Letters 16, 377 (1966)]
2. Mixing Effects in Baryon Spectroscopy -- Gatto et al. [Phys. Rev. Letters 16, 918 (1966)]
3. Current Algebras and Magnetic Moments -- Gatto et al. [Phys. Letters 21, 459 (1966)]
4. Current Algebra and Representation Mixing -- Harari [Phys. Rev. Letters 16, 964 (1966)]
5. Current Commutators, Representation Mixing, and Magnetic Moments -- Harari [Phys. Rev. Letters 17, 56 (1966)]
6. Chiral Algebra, Configuration Mixing, Magnetic Moments, and Pion Photoproduction -- Gerstein and Lee [Phys. Rev. Letters 16, 1060 (1966) and Phys. Rev. (to be published)]
7. Saturation of Sum Rules in Particle Representation of Resonances -- Cheng and Kim [to be published]
8. PCAC and Chiral Representation Mixing--Horn [to be published]

I do not think it is necessary to reiterate why we choose to saturate current commutators at infinite momentum. I assume it is also well known that the chiral U(3) ⊗ U(3) algebra generated by the space integrals of the time components of the vector and axial vector currents, and the collinear U(3) ⊗ U(3) algebra generated by

$$V^i \equiv \int d\underset{\sim}{x}^3 \; V_0^i(\underset{\sim}{x}, t) \sim \int d\underset{\sim}{x}^3 \; q^t(\underset{\sim}{x}) \, \lambda^i q(\underset{\sim}{x})$$

and

$$A_3^i \equiv \int d\underset{\sim}{x}^3 \; A_3^i(\underset{\sim}{x}, t) \sim \int d\underset{\sim}{x}^3 \; q^t(\underset{\sim}{x}) \, \sigma_3 \lambda^i q(\underset{\sim}{x}),$$

are "equivalent," in the sense that the two algebras are isomorphic, and the diagonal matrix elements of the two algebras coincide. (We assume, from now on, that the z direction is the direction of the infinite momentum.)

At this point, I would like us to agree on notations: irreducible representations of either chiral or collinear algebra will be labeled by $(n, m)_\lambda$, where n and m are the dimensions of the SU(3) representations generated by $1/2\,(V^i + A^i)$ [or $1/2\,(V^i + A_3^i)$] and $1/2\,(V^i - A^i)$; λ is the eigenvalue of the operator $A_3^0/2$, which may be called the "quark" helicity.

It was shown by Gerstein, Dashen and Gell-Mann, and others that assigning the octet of $1/2^+$ baryons and the $3/2^+$ decimet to the representation $(6, 3)_{1/2}$ leads to the well-known SU(6) results. Since some of these predictions are not satisfactory, one must allow the baryons to transform reducibly under the algebra. Furthermore, in order to obtain nonzero magnetic moments we must allow an additional degree of freedom associated with an orbital angular momentum excitation. To be more precise we must introduce a degree of freedom which we define as the orbital helicity $\Lambda_3 = J_3 - \lambda$, where J_3 is the true helicity. In a pure quark model, we have

$$J_3 = \Lambda_3 + A_3^0/2,$$

$$\Lambda_3 = -i \int d^3\underset{\sim}{x} \; q^t(\underset{\sim}{x}) \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}\right) q(\underset{\sim}{x}) ,$$

$$A_3 = \int d^3\underset{\sim}{r} \; q^t(\underset{\sim}{r}) \, \sigma_{12} q(\underset{\sim}{r}) .$$

The papers 1 through 6 and 8 cited above discuss these questions and investigate consequences of some representation-mixing schemes.

Actually, there are two different attitudes taken by the authors of the above papers. Since there seems to be some confusion in the literature, I may be allowed to make some pedantic remarks on this point. This has to do with what subalgebra of the algebra of Dirac covariants built out of quark fields, U(12), is assumed to be good at infinite momentum. By a good algebra, we mean an algebra which (a) is not plagued by Schwinger and super-Schwinger terms, and (b) gives rise to a fairly rapidly convergent sum rule. In Papers 5, 6, and 8, the chiral (or collinear) algebra is assumed good, but not the so-called $SU(6)_W$. When one takes this attitude, it makes no sense to ask, for example, whether the state $|(6,3)_{1/2}\rangle$ comes from $\underset{\sim}{56}$ of SU(6). It can be a linear combination of an infinite number of irreducible representations of SU(6) which contain $(6, 3)_{1/2}$. On the other hand, in Papers 1 through 4, the entire $SU_W(6)$ is assumed to be good, so that physical states are assumed to be linear combinations of a few irreducible representations of SU(6). In this approach, U(3) ⊗ U(3) is just a convenient subgroup from which alone G_A/G_V and (D/F) axial can be uniquely computed. At the practical level, there are two main differences, depending on which philosophy one subscribes to:

(a) In $SU(6)_W$, to construct a state of definite j and j_z out of an irreducible representation of $SU_W(6)$ and an irreducible representation of O(3) of the "orbital" angular momentum, the usual vector addition rule is to be followed. Thus, if one is to construct a state of $j = 1/2$, $j_z = 1/2$ out of the spin-1/2 part of 20 and L = 1, one has

$$|j = 1/2, \; j_z = 1/2\rangle = \sqrt{\tfrac{1}{3}} \; |(\overline{3}, 3)_{1/2}, \; L_z = 0\rangle - \sqrt{\tfrac{2}{3}} \; |(3, \overline{3})_{-1/2}, \; L_z = +1\rangle .$$

No such constraint exists in U(3) ⊗ U(3).

(b) In $SU_W(6)$, once the SU(6) and L content of a state of definite j and j_z are given, the SU(6) and L contents of states of the same j and different j_z are uniquely determined. For instance, if one assumes $|j = 3/2, \; j_z = 1/2\rangle$ belongs to $(6,3)_{1/2}$ of $\underset{\sim}{56}$ with L = 0, the state $|j = 3/2, \; j_z = 3/2\rangle$ is given uniquely by

$$|j = 3/2, \; j_z = 3/2\rangle = |(10, 1)_{3/2}, \; L_z = 0\rangle.$$

This is not the case in U(3) ⊗ U(3) .

In all the papers I cited, discussions are based on the approximation of treating resonances as single particles. Since the same commutators lead to dispersion-relation-type sum rules such as Adler-Weisberger, it is most appropriate to compare the contribution from the idealized single-particle state J^P of mass M, isospin I [or SU(3) dimensionality] to the commutator with the Adler-Weisberger integral

$$I(M, I, J^P) = \frac{4}{3}\,\frac{m^2 G_A^2}{g_\pi{}^2_{NN}} \int \frac{d\nu}{\nu}\, \sigma_{res}(M, I, J^P),$$

where $\sigma_{res}(M, I, J^P)$ is the resonance cross section in the appropriate channel, expressed in terms, for instance, of the Breit-Wigner formula. Table 3-AI shows $I(M, I, J^P)$ for various resonances below ≈ 2 BeV in the pion nucleon channel:

Table 3-AI. Some values of $I(M, I, J^P)$.

M	I	J^P	$I(M, I, J^P)$	Source
1236	3/2	$3/2^+$	1.05 (0.65)	Cheng and Kim
			0.68	Gerstein and Lee
			0.72	Gilman and Schnitzer
1480	1/2	$1/2^\pm$	0.10	Cheng and Kim
1518	1/2	$3/2^-$	0.12	Cheng and Kim
			0.12	Gerstein and Lee
1688	1/2	$5/2^+$	0.08	Cheng and Kim
			0.09	Gerstein and Lee
1924	3/2	$7/2^+$	0.07	Cheng and Kim
2190	1/2	$7/2^-$	0.03	Cheng and Kim

Two remarks are in order here: For the $3/2^+$ decimet contribution, we find a narrow resonance approximation is a poor approximation, because of the nearness of the resonance to the threshold and the considerable variation of the width with energy. The estimate of $I(M, I, J^P)$ for the (3,3) resonance is about 2/3 of Weisberger's value, which is obtained by integrating over the total cross section over the range of the (3,3) resonance; inspection of Table 3-AI reveals that the resonance contributions listed exhaust the sum rule. In fact, as Cheng and Kim point out, with $I(1236, 3/2, 3/2^+)$ taken to be 0.65, one gets $G_A = 1.18$. Thus, I believe it is tenable to assume that the background contributions (that below and near 2 BeV and the Pomeranchuk tail) that have not been taken into account cancel in the commutator. It should perhaps be stressed, however, that the numbers exhibited in Table 3-AI are subject to error of order of, say, 15 to 20%.

Now, going back to individual contributions: in Paper 1, Gatto and collaborators proposed the scheme of assigning the $1/2^+$ octet of baryons to a mixture of 56 of $SU(6)_W$ with L = 0, and 20 with L = 1. The $3/2^+$ decimet is assigned to 56 with L = 0. Thus the $1/2^+$ octet is expressed, in the U(3)⊗U(3) language, as

$$|8, j = 1/2,\ j_z = 1/2\rangle = \cos\theta\,|(6,3)_{1/2}, 0\rangle + \sin\theta\left\{\sqrt{\tfrac{1}{3}}\,|(\bar{3},3)_{1/2}, 0\rangle - \sqrt{\tfrac{2}{3}}\,|(3,\bar{3})_{-1/2}, 1\rangle\right\},$$

and the decimet is

$$|10,\ j = 3/2,\ j_z = 1/2\rangle = |(6,3)_{1/2}, 0\rangle .$$

Gatto and collaborators note a relation that follows from these assignments, independently of the mixing angle θ,

$$-\frac{G_A}{G_V} = \frac{1}{3}\,\frac{D+F}{D-F},$$

which seems to be well satisfied experimentally. Harari came up with more or less the same model in Papers 4 and 5. In terms of the mixing angle θ, we have in this model

$$-G_A/G_V = \tfrac{5}{3}\cos^2\theta + \tfrac{1}{3}\sin^2\theta,$$

$$(D/F)_{axial} = \frac{\cos^2\theta + \frac{1}{3}\sin^2\theta}{\frac{2}{3}\cos^2\theta},$$

$$I(1236,\ 3/2,\ 3/2^+) = (\tfrac{4}{3})^2\cos^2\theta .$$

With the choice $\theta \approx 37°$, one gets $-G_A/G_V = 1.19$, $(D/F)_{axial} = 3/2 \times 1.2$, and $I(1236) = 1.1$.

In Paper 2, the authors discuss several alternative mixing schemes based on $SU(6)_W$. The first model considered is the mixing of (56, L = 0) and (56, L = 1), as the "orbital" excitation model suggests. They find that in this model the $(D/F)_{axial}$ remains the same, i.e., 3/2, but $-G_A/G_V \leq 5/3$. The second model considered is the mixing of (56, 0) and (70, 1), as Dalitz's classification of the negative-parity baryon states suggests. They find that this mixing scheme leads to no constraint between the values of $-G_A/G_V$ and $(D/F)_{axial}$. They also discuss the mixing of (56, 0) and (700, 0), based on the observation that the noncompact U(6,6) [which contains the chain $\{U(6) \otimes U(6)\}_{nonchiral} \supset U(6)_W$] has a ladder representation that contains the series 56, 56, 700, ---. This scheme turns out not to give a satisfactory relation between $-G_A/G_V$ and $(D/F)_{axial}$.

In Paper 6, $SU_W(6)$ is assumed to be not a good algebra and rejected. Thus the entire discussion is based on $[U(3) \otimes U(3)]_{chiral} \otimes U(1)$, the factor U(1) being the one-parameter algebra generated by Λ_3, the orbital helicity. The model proposed is

$$|\beta_{1/2}\rangle = \cos\beta\,|(6,3)_{1/2}, 0\rangle + \sin\beta\{\cos\alpha\,|(3,\bar{3})_{-1/2}, 1\rangle + \sin\alpha\,|(8,1)_\lambda, \tfrac{1}{2} - \lambda\rangle\},\quad \lambda \text{ arbitrary},$$

$$|\Delta_{1/2}\rangle = |(6,3)_{1/2}, 0\rangle,$$

where the subscripts refer to helicities. In this model

$$-G_A/G_V = \tfrac{5}{3}\cos^2\beta + \sin^2\beta,$$

$$(D/F)_{axial} = \frac{\cos^2\beta + \sin^2\beta\cos^2\alpha}{\frac{2}{3}\cos^2\beta + \sin^2\beta\sin^2\alpha},$$

$$I(1236) = (\tfrac{4}{3})^2\cos^2\beta .$$

The choice $\cos^2\alpha = 3/5$ yields

$$(D/F)_{axial} = \frac{3}{2}$$

independently of the value β. It is noted that $\cos\beta = (3/8)^{1/2}$ gives

$$-G_A/G_V = 1.25,\quad I(1236) = 0.67.$$

The expectation value of the electric dipole operator D_1,

$$D_1 = \int d^3\underset{\sim}{x} \times j_0(\underset{\sim}{x}, t),$$

between the spin $1/2^+$ baryons at infinite momentum is equal to the anomalous magnetic moment. The structure of D_1 with respect to $U(3)\otimes U(3)\otimes U(1)$ is such that it transforms like $(8,1)_0 + (1,8)_0$ with $L_z = \pm 1$. Thus in the expression

$$\langle B_{-1/2} \mid D_1 \mid B_{1/2} \rangle$$

there are two nonvanishing reduced matrix elements

$$\{(6,3)_{1/2}, 0\} \leftrightarrow \{(\bar{3},3)_{1/2}\ L_z = -1\} \text{ and}$$

$$\{(\bar{3},3)_{1/2},\ L_z = 0\} \leftrightarrow \{(\bar{3},3)_{1/2}\ L_z = -1\}$$

in the Gatto et al. — Harari model; and this is only one reduced matrix element $\{(6,3)_{1/2}, 0\} \leftrightarrow \{(\bar{3},3)_{1/2}, L_z = -1\}$ in the Gerstein-Lee model. The relation

$$\mu^* = -\frac{\sqrt{2}}{\cos\theta}\mu_A(n) = +\frac{\sqrt{2}}{\cos\beta}\mu_A(p)$$

follows from either model, while the latter model predicts in addition $\mu_A(p) = -\mu_A(n)$. The value for μ^* is estimated by Dalitz and Sutherland to be

$$(\mu^*)_{0-s} \cong (1.28\pm0.02)\ (\mu^*)_{SU(6)} = (1.28\pm0.02)\ \frac{2\sqrt{2}}{3}\mu_{total}(p)\ .$$

Gerstein and Lee carried out an analysis of μ^* based on the correspondence between the commutation relation $[A_0^i, D_1]$ and the Fubini-Furlan-Rossetti sum rule and the photoproduction data, and obtained

$$(\mu^*)_{GL} \cong 1.4\ (\frac{2\sqrt{2}}{3}\mu_p),\ \mu_p = \mu_{total}(p)\ .$$

The Gatto-Harari model, with $\theta = 37°$, gives

$$(\mu^*) \simeq 1.3\ (\frac{2\sqrt{2}}{3}\mu_p)\ ,$$

while the Gerstein-Lee model, with $\beta = 47°$, gives

$$\simeq 1.6\ (\frac{2\sqrt{2}}{3}\mu_p)\ .$$

In the above discussion, we have assumed that the photoexcitation of the (3,3) resonance proceeds only through the M1 multipole, as there is overwhelming evidence for this. In Paper 3, Gatto and collaborators point out that in $SU_W(6)$, the moments $\mu_A(p,n)$, μ^* are all zero. The reason is as follows: since in Gatto et al.'s scheme $|\Delta_{1/2}\rangle$ is assigned to $|(6,3)_{1/2}, 0\rangle$ and the SU(6) 56, it follows that $|\Delta_{3/2}\rangle$ must be assigned to $|(10,1)_{3/2}, 0\rangle$. However, there is no nonvanishing matrix element of D_1 between $|(10,1)_{3/2}, 0\rangle$ and $|B_{1/2}\rangle$. Thus $\mu^* = 0$ (since the E2 moment is assumed to be zero), and from the relation $\cos\theta\,\mu^* = -\sqrt{2}\,\mu_A(n)$, $\mu_A(n) = 0$. This difficulty arises from assuming $SU_W(6)$ to be a good algebra, and does not exist in $U(3)\otimes U(3)$. For, in the latter scheme, it is not necessary to assign $|\Delta_{3/2}\rangle$ to a pure $|(10,1)_{3/2}, 0\rangle$.

Gerstein and Lee consider the possibility that the stable baryons, the $3/2^+$ decimet, the 1405 singlet, and the $3/2^-$ and $5/2^+$ octet form the reducible representation

$$\{(6,3)_{1/2}, 0\}, \{(3,\bar{3})_{-1/2}, 1\} \text{ and } \{(8,1)_\lambda, \tfrac{1}{2} - \lambda\}\ .$$

The strengths of N^{**}, $N^{***} \to N + \pi$ predicted are compared with experiment and found not too satisfactory. It appears that further chiral representations should be mixed into the $3/2^-$ and $5/2^+$ resonances. This means that more states are needed to saturate the Adler-Weisberger sum rule than those already taken into account. A similar conclusion can be drawn about the Harari model from the work of D. Horn.

An interesting observation made by Gerstein and Lee is that the photoproduction amplitude of the isospin 1/2 resonances ($3/2^-$ and $5/2^+$) from nucleons transforms like pure isovector. There seems to be some support for this experimentally.

In the third category, we have the contributions:

1. Relations between the Pion-Nucleon and the Meson Coupling Constants from Pion Scattering Length - Sakurai
2. Symmetry Predictions from Sum Rule Without Saturation - Gilman and Schnitzer
3. Axial Vector Current Consisting of Pseudoscalar Octet - Kao and Sugawara
4. Universality and Quark Models - Freund
5. Calculation of Transition Probabilities from Noncompact Dynamical Groups - Barut

The first two papers rederive the Kawarabayashi and Suzuki relation

$$\left(\frac{f_\rho^2}{4\pi}\right) = \left(\frac{G_{\pi NN}^2}{4\pi}\right)\left(\frac{G_V}{G_A}\right)^2\left(\frac{m_\rho^2}{m_N^2}\right)\ ,$$

which was first derived by applying current algebra to the process $\rho \to 2\pi$. Sakurai derives it by equating the Tomozawa-Weiberg formula for πN scattering length,

$$A_T = -\left(\frac{G_{\pi NN}^2}{4\pi}\right)\left(\frac{G_V}{G_A}\right)^2\left(\frac{m_\pi m_N}{m_\pi + m_N}\right)\frac{1}{m_N^2}\,\vec{T}_\pi\cdot\frac{\vec{\tau}_N}{2}\ ,$$

with the expression one gets if one assumes A_T comes from the ρ-meson exchange:

$$A_T = -2\left(\frac{f_\rho^2}{4\pi}\right)\left(\frac{m_\pi m_N}{m_\pi + m_N}\right)^2\frac{1}{m_\rho^2}\,\vec{T}_\pi\cdot\frac{\vec{\tau}_N}{2}\ .$$

Gilman and Schnitzer, on the other hand, note that two Adler-Weisberger-type relations result for the process $\rho + \pi \to \rho + \pi$, depending on whether one chooses to contract two ρ^- meson fields or pion fields. In the former case, one obtains the commutator of vector currents through the assumption that

$$\vec{\rho}_\mu \approx \frac{f_\rho}{m_\rho^2}\vec{V}_\mu\ ,$$

while in the latter, one gets the commutator of axial vector currents through PCAC. In the two sum rules that ensue, the continuum contributions are the same (in the approximation of neglecting the off-mass-shell corrections), and one obtains the formula exhibited above. Thus the Kawarabayashi-Suzuki relation appears to be a consistency requirement on the chiral algebra, PCAC and the hypothesis about the ρ-meson dominance of the vector current form factors. It is well to recall that substantially the same formula follows from SU(6) with $(G_A/G_V)^2$ restricted to $(1/2)(25/9)$.

Another interesting point that Gilman and Schnitzer make is that if the relation

$$\sigma(\pi^- p) - \sigma(\pi^+ p) = \sigma(\rho^- P) - \sigma(\rho^+ P)$$

due to Lipkin and Scheck (this also follows from the assumption that in charge exchange and forward πN and ρN processes, the t channel is dominated by the same octet coupled to the isospin current) is used to represent the continuum contribution beyond the (3,3) resonance in the Adler-Weisberger and Cabibbo-Radicati sum rules, one obtains relations of the type

$$\mu_p - \mu_n \simeq 2g_{\pi NN}/f_\rho \simeq 2\sqrt{2}\,M\left(-\frac{G_A}{G_V}\frac{1}{m_\rho}\right),$$

which agrees with experiment to within 10%.