Cosmological intersecting brane solutions in string theory

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Abstract. The recent discovery of an explicit dynamical description of p-branes makes it possible to investigate the existence of intersection of such objects. We generalize the solutions depending on the overall transverse space coordinates and time to those which depend also on the relative transverse space and satisfy new intersection rules. We give classification of these dynamical intersecting brane solutions involving two branes, and discuss the application of these solutions to cosmology and show that these give Friedmann-Lemaître-Robertson-Walker cosmological solutions.

1. Introduction

The dynamics of the brane world model in five or six dimensions have been much explored (see [1, 2] and references therein) because of the possible cosmological and phenomenological interests. Although some results have recently emerged on the applications of the solutions in higher-dimensional supergravity to the brane world cosmology, e.g., in [1], the construction of the cosmological model in string theory is much less extensive. One motivation for the present work is to improve this situation. For this purpose, it is first necessary to construct dynamical brane solutions depending on the time as well as space coordinates.

It has already been known [3, 4] that dynamical brane solutions arise when the gravity is coupled not only to a single gauge field but also to several combinations of scalars and forms, as generalization of the static intersecting brane solutions in the supergravity. Here we construct dynamical brane solutions by generalizing these static solutions to a dynamical one. The first class of dynamical solutions we study has the dependence on the time as well as overall transverse space coordinates in the metric and obeys the well-known intersection rules. However, it has also been known for some time that some static intersecting brane solutions may not follow these intersection rules. These intersecting brane solutions are derived for the case when the branes depend on the relative transverse directions of the intersecting branes [5].

Our goal is to exhaust and classify all two intersecting brane solutions which depend on the time and (relative) transverse dimensions and to study their applications to the cosmological evolutions, in particular in the ten-dimensional string theory and eleven-dimensional supergravity theory. We first find cosmological solutions for possible intersections including the above exceptional cases for two intersecting branes by extending the similar solutions obeying the usual intersection rules [3, 4]. Our results on the dynamical branes are given for general cases of arbitrary dimensions and forms, but in their applications to cosmology,

we mainly focus on the dynamical branes in ten- and eleven-dimensional supergravities because these are the most important low-energy effective theories of superstrings.

2. Intersecting brane solutions in D-dimensional theory

We consider a D-dimensional theory composed of the metric g_{MN}, dilaton ϕ, and the antisymmetric tensor fields of rank $(p_r + 2)$ and $(p_s + 2)$:

$$
\begin{aligned}
S = \quad & \frac{1}{2\kappa^2} \int \left[R * \mathbf{1} - \frac{1}{2} d\phi \wedge *d\phi - \frac{1}{2} \frac{1}{(p_r + 2)!} e^{\epsilon_r c_r \phi} F_{(p_r+2)} \wedge *F_{(p_r+2)} \right. \\
& \left. - \frac{1}{2} \frac{1}{(p_s + 2)!} e^{\epsilon_s c_s \phi} F_{(p_s+2)} \wedge *F_{(p_s+2)} \right],
\end{aligned}
\tag{1}
$$

where κ^2 is the D-dimensional gravitational constant, $*$ is the Hodge operator in the D-dimensional spacetime, $F_{(n)}$ is an n-form field strength, and c_I, ϵ_I ($I = r,\ s$) are constants given by

$$
c_I^2 = 4 - \frac{2(p_I + 1)(D - p_I - 3)}{D - 2}, \quad \epsilon_I = \begin{cases} + & \text{if } p_I - \text{brane is electric} \\ - & \text{if } p_I - \text{brane is magnetic} \end{cases}.
\tag{2}
$$

To solve these field equations, we assume that the D-dimensional metric takes the form

$$
ds^2 = h_r^\alpha h_s^\beta \left[h_r^{-1} h_s^{-1} q_{\mu\nu}(\mathrm{X}) dx^\mu dx^\nu + h_s^{-1} \gamma_{ij}(\mathrm{Y_1}) dy^i dy^j + h_r^{-1} w_{mn}(\mathrm{Y_2}) dv^m dv^n + u_{ab}(\mathrm{Z}) dz^a dz^b \right],
\tag{3}
$$

where $q_{\mu\nu}$, γ_{ij}, w_{mn}, and u_{ab} are the metrics depending only on x^μ, y^i, v^m, and z^a coordinates of dimensions $(p+1)$, $(p_s - p)$, $(p_r - p)$, and $(D + p - p_r - p_s - 1)$, respectively. The parameters α and β in the metric (3) are given as

$$
\alpha = \frac{p_r + 1}{D - 2}, \quad \beta = \frac{p_s + 1}{D - 2}.
\tag{4}
$$

Here we suppose that $p_s(p_r)$-brane extends along X and Y_1 (Y_2) spaces.

The D-dimensional metric (3) implies that the p-brane solutions are characterized by a function which depends on the coordinates transverse to the brane as well as the worldvolume coordinate. For the configurations of two branes, we should sort the coordinates in three sets and the powers of harmonic functions are different for each set of coordinates according to the intersection rules. One set of the coordinates is the overall worldvolume coordinates, which are common to the two branes. The others are overall transverse coordinates and the last are the relative transverse coordinates, which are transverse to only one of the two branes.

The field equations of intersecting branes allow for the following three kinds of possibilities on p_r- and p_s-branes in D dimensions [5]:

(I) Both h_r and h_s depend on the overall transverse coordinates: $h_r = h_r(x, z)$, $h_s = h_s(x, z)$.

(II) Only h_s depends on the overall transverse coordinates, but the other h_r does on the corresponding relative coordinates: $h_r = h_r(x, y)$, $h_s = h_s(x, z)$.

(III) Each of h_r and h_s depends on the corresponding relative coordinates: $h_r = h_r(x, y)$, $h_s = h_s(x, v)$.

For completeness, let us first consider case (I) though this has been already discussed in [4]. We assume that the the scalar field ϕ and the gauge field strengths are given as

$$
e^\phi = h_r^{\epsilon_r c_r/2} h_s^{\epsilon_s c_s/2},
\tag{5}
$$

$$
F_{(p_r+2)} = d\left[h_r^{-1}(x, z) \right] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y_2}), \quad F_{(p_s+2)} = d\left[h_s^{-1}(x, z) \right] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y_1}),
\tag{6}
$$

where $\Omega(X)$, $\Omega(Y_1)$, and $\Omega(Y_2)$ denote the volume forms of dimensions $(p+1)$, $(p_s - p)$, and $(p_r - p)$, respectively:

$$\Omega(X) = \sqrt{-q}\, dx^0 \wedge \cdots \wedge dx^p, \quad \Omega(Y_1) = \sqrt{\gamma}\, dy^1 \wedge \cdots \wedge dy^{p_s - p}, \quad \Omega(Y_2) = \sqrt{w}\, dv^1 \wedge \cdots \wedge dv^{p_r - p}. \quad (7)$$

Here, q, γ, and w are the determinant of the metric $q_{\mu\nu}$, γ_{ij}, and w_{mn}, respectively. With the assumptions of fields, it is easy to show that the field equations reduce to

$$R_{\mu\nu}(X) = 0, \quad R_{ij}(Y_1) = 0, \quad R_{mn}(Y_2) = 0, \quad R_{ab}(Z) = 0, \quad (8)$$

$$h_r = h_0(x) + h_1(z), \quad h_s = k_0(x) + k_1(z), \quad (9)$$

$$D_\mu D_\nu h_0 = 0, \quad \triangle_Z h_1 = 0, \quad \triangle_Z h_s = 0, \quad \text{for } \partial_\mu h_s = 0, \quad (10)$$

$$D_\mu D_\nu k_0 = 0, \quad \triangle_Z k_1 = 0, \quad \triangle_Z h_r = 0, \quad \text{for } \partial_\mu h_r = 0, \quad (11)$$

where D_μ is the covariant derivative with respect to the metric $q_{\mu\nu}$, and $\triangle_Z$ is the Laplace operators on the space of Z, and $R_{\mu\nu}(X)$, $R_{ij}(Y_1)$, $R_{mn}(Y_2)$, and $R_{ab}(Z)$ are the Ricci tensors of the metrics $q_{\mu\nu}(X)$, $\gamma_{ij}(Y_1)$, $w_{mn}(Y_2)$, and $u_{ab}(Z)$, respectively, and we used the intersection rule $\chi = 0$:

$$\chi \equiv p + 1 - \frac{(p_r + 1)(p_s + 1)}{D - 2} + \frac{1}{2}\epsilon_r \epsilon_s c_r c_s. \quad (12)$$

If $F_{(p_r+2)} \neq 0$ and $F_{(p_s+2)} \neq 0$, the functions h_1 and k_1 are nontrivial.

As a special example, let us consider the case

$$q_{\mu\nu} = \eta_{\mu\nu}, \quad \gamma_{ij} = \delta_{ij}, \quad w_{mn} = \delta_{mn}, \quad u_{ab} = \delta_{ab}, \quad (13)$$

where $\eta_{\mu\nu}$ is the $(p+1)$-dimensional Minkowski metric and δ_{ij}, δ_{mn}, δ_{ab} are the $(p_s - p)$-, $(p_r - p)$-, and $(D + p - p_r - p_s - 1)$-dimensional Euclidean metrics, respectively. For $\partial_\mu h_s = 0$, the solution for h_r and h_s can be obtained explicitly as

$$h_r(x, z) = A_\mu x^\mu + B + \sum_l M_l |z^a - z_l^a|^{-D - p + p_r + p_s + 3}, \quad h_s(z) = C + \sum_c M_c |z^a - z_c^a|^{-D - p + p_r + p_s + 3},$$

$$(14)$$

where A_μ, B, C, M_l, and M_c are constant parameters, and z_l^a and z_c^a are constants representing the positions of the branes. We can also choose the solution in which the p_s-brane part depends on both t and z.

Now we discuss the intersecting brane solutions in eleven-dimensional supergravity and in ten-dimensional string theories. For the M-branes in eleven-dimensional supergravity, there is 4-form field strength without dilaton, so the intersection rule $\chi = 0$ gives

$$p = \frac{(p_r + 1)(p_s + 1)}{9} - 1, \quad (15)$$

where p denotes the number of overlapping dimensions of the p_r and p_s branes. Then we get the intersections involving the M2 and M5-branes [5]

$$\text{M2} \cap \text{M2} = 0, \quad \text{M2} \cap \text{M5} = 1, \quad \text{M5} \cap \text{M5} = 3. \quad (16)$$

For the ten-dimensional string theories, the couplings to dilaton for the RR-charged D-branes are given by

$$\epsilon_r c_r = \frac{1}{2}(3 - p_r), \quad \epsilon_s c_s = \frac{1}{2}(3 - p_s). \quad (17)$$

The condition $\chi = 0$ then gives

$$p = \frac{1}{2}(p_r + p_s - 4). \quad (18)$$

The intersections for the D-branes are thus given by [5]

$$\mathrm{D}p_r \cap \mathrm{D}p_s = \frac{1}{2}(p_r + p_s) - 2. \tag{19}$$

We finally consider the intersections for NS-branes. The parameters c_r for fundamental string (F1) and solitonic 5-brane are $\epsilon_1 c_1 = -1$ (for F1) and $\epsilon_5 c_5 = 1$ (for NS5), respectively. Then the intersections involving the F1 and NS5-branes are [5]

$$\mathrm{F1} \cap \mathrm{NS5} = 1, \quad \mathrm{NS5} \cap \mathrm{NS5} = 3, \quad \mathrm{F1} \cap \mathrm{D}\bar{p} = 0, \quad \mathrm{D}\bar{p} \cap \mathrm{NS5} = \bar{p} - 1, \quad (1 \le \bar{p} \le 6). \tag{20}$$

There is no solution for the F1-F1 and D0-NS5 intersecting brane systems because the numbers of space dimensions for each pairwise overlap are negative by the intersection rule.

We next consider case (II). We also take the following ansatz for the scalar field ϕ and the gauge field strengths:

$$\mathrm{e}^{\phi} = h_r^{\epsilon_r c_r/2} h_s^{\epsilon_s c_s/2}, \tag{21}$$

$$F_{(p_r+2)} = d\left[h_r^{-1}(x,y)\right] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y_2}), \quad F_{(p_s+2)} = d\left[h_s^{-1}(x,z)\right] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y_1}), \tag{22}$$

where $\Omega(\mathrm{X})$, $\Omega(\mathrm{Y_1})$, and $\Omega(\mathrm{Y_2})$ are defined in Eq. (7). Since we use the same procedure as in the case (I), we can derive the intersection rule $\chi = 0$ from the field equations. For $\chi = 0$ and $\partial_\mu h_s = 0$, it is easy to show that the field equations reduce to

$$R_{\mu\nu}(\mathrm{X}) = 0, \quad R_{ij}(\mathrm{Y_1}) = 0, \quad R_{mn}(\mathrm{Y_2}) = 0, \quad R_{ab}(\mathrm{Z}) = 0, \tag{23}$$

$$h_r = h_0(x) + h_1(y), \quad h_s = h_s(z), \quad D_\mu D_\nu h_0 = 0, \quad \triangle_{\mathrm{Y_1}} h_1 = 0, \quad \triangle_{\mathrm{Z}} h_s = 0, \tag{24}$$

where $\triangle_{\mathrm{Y_1}}$ is the Laplace operators on the space of $\mathrm{Y_1}$. If $F_{(p_r+2)} \ne 0$ and $F_{(p_s+2)} \ne 0$, the functions h_1 and k_1 are nontrivial.

Let us consider the following case Eq. (13) in more detail. For $\partial_\mu h_s = 0$, the solution for h_r and h_s can be obtained explicitly as

$$h_r(x,y) = A_\mu x^\mu + B + \sum_l M_l |y^i - y_l^i|^{-p_s+p+2}, \quad h_s(z) = C + \sum_c M_c |z^a - z_c^a|^{-D-p+p_r+p_s+3}, \tag{25}$$

where A_μ, B, C, y_l^i, z_c^a, M_l, and M_c are constant parameters.

One can easily get the solution for $\partial_\mu h_r = 0$ and $\partial_\mu h_s \ne 0$ if the roles of $\mathrm{Y_1}$ and $\mathrm{Y_2}$ are exchanged. Since the dynamical solution (25) obeys the same intersection rule $\chi = 0$, the intersections of M-branes in eleven-dimensional supergravity and D-branes in ten-dimensional string theories are given as (16), (19), and (20).

Finally, we consider case (III). We also assume that the scalar field ϕ and the gauge field strengths are given as

$$\mathrm{e}^{\phi} = h_r^{\epsilon_r c_r/2} h_s^{\epsilon_s c_s/2}, \tag{26}$$

$$F_{(p_r+2)} = h_s\, d\left[h_r^{-1}(x,y)\right] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y_2}), \quad F_{(p_s+2)} = h_r\, d\left[h_s^{-1}(x,v)\right] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y_1}), \tag{27}$$

where $\Omega(\mathrm{X})$, $\Omega(\mathrm{Y_1})$, and $\Omega(\mathrm{Y_2})$ denote the volume $(p+1)$-, $(p_s - p)$-, and $(p_r - p)$-forms, respectively.

Under the assumption, the field equations give the intersection rule $\chi = -2$. This is different from the usual rule applicable to the cases (I) and (II). Upon using the intersection rule $\chi = -2$, it is easy to show that the field equations reduce to

$$R_{\mu\nu}(\mathrm{X}) = 0, \quad R_{ij}(\mathrm{Y_1}) = 0, \quad R_{mn}(\mathrm{Y_2}) = 0, \quad R_{ab}(\mathrm{Z}) = 0, \tag{28}$$

$$h_r = h_0(x) + h_1(y), \quad h_s = k_0(x) + k_1(v), \tag{29}$$

$$D_\mu D_\nu h_0 = 0, \quad \triangle_{\mathrm{Y_1}} h_1 = 0, \quad \triangle_{\mathrm{Y_2}} h_s = 0, \quad \text{for } \partial_\mu h_s = 0, \tag{30}$$

$$D_\mu D_\nu k_0 = 0, \quad \triangle_{\mathrm{Y_1}} h_r = 0, \quad \triangle_{\mathrm{Y_2}} k_1 = 0, \quad \text{for } \partial_\mu h_r = 0, \tag{31}$$

where $\triangle_{Y_1}$ and $\triangle_{Y_2}$ are the Laplace operators on the spaces of Y_1 and Y_2, respectively. The functions h_1 and k_1 are nontrivial for $F_{(p_r+2)} \neq 0$ and $F_{(p_s+2)} \neq 0$.

Now we consider the case in Eq. (13). For $\partial_\mu h_s = 0$, the solution for h_r and h_s can be obtained explicitly as

$$h_r(x,y) = A_\mu x^\mu + B + \sum_l M_l |y^i - y^i_l|^{-p_s+p+2}, \quad h_s(v) = C + \sum_c M_c |v^m - v^m_c|^{-p_r+p+2}, \quad (32)$$

where A_μ, B, C, y^i_l, v^m_c, M_l, and M_c are constant parameters. We can also get the solution in which the function h_s depends on both t and v.

Let us consider the intersecting brane solutions in eleven-dimensional supergravity and in ten-dimensional string theories. We first discuss the intersections of M-branes in eleven-dimensional supergravity. The intersection rule $\chi = -2$ leads to

$$p = \frac{(p_r+1)(p_s+1)}{9} - 3. \quad (33)$$

Then we get the intersection involving the M5-brane

$$M5 \cap M5 = 1. \quad (34)$$

Equation (33) tells us that the numbers of intersection for M2-M2 and M2-M5 branes are negative, which means that there is no intersecting solution of these brane systems.

Next we consider the intersection in the ten-dimensional string theory. The couplings to dilaton for the RR-charged D-branes are

$$\epsilon_r c_r = \frac{1}{2}(3 - p_r), \quad \epsilon_s c_s = \frac{1}{2}(3 - p_s), \quad (35)$$

and the condition $\chi = -2$ is expressed as $p = \frac{1}{2}(p_r + p_s - 8)$. The intersections for the RR-charged D-branes are thus given by

$$Dp_r \cap Dp_s = \frac{1}{2}(p_r + p_s) - 4. \quad (36)$$

We finally consider the intersections for NS-branes. The parameters c_r for fundamental string (F1) and solitonic 5-brane are $\epsilon_1 c_1 = -1$ for F1 and $\epsilon_5 c_5 = 1$ for NS5, respectively. Then the intersection with F1-brane is forbidden by the intersection rule. The intersections involving the NS5-branes are

$$NS5 \cap NS5 = 1, \quad D\bar{p} \cap NS5 = \bar{p} - 3, \quad (3 \leq \bar{p} \leq 8). \quad (37)$$

There is no brane solution involving other intersections because the numbers of space dimensions for each pairwise overlap become negative by the intersection rule.

We list the Friedmann-Lemaître-Robertson-Walker cosmological solutions with an isotropic and homogeneous three-space for the solutions (14), (25), (32) in [5] for M-branes, D-branes, F1 and NS5-branes. Since the time dependence in the metric comes from only one brane in the intersections, the obtained expansion law is simple. In order to find an expanding universe, one may have to compactify the vacuum bulk space as well as the brane worldvolume. If we have introduced the cosmic time τ, we find that the fastest expanding case in the Jordan frame has the power $a \propto \tau^{7/15}$, which is too small to give a realistic expansion law like that in the matter dominated era $(a \propto \tau^{2/3})$ or that in the radiation dominated era $(a \propto \tau^{1/2})$, where a is the scale factor of our Universe. When we compactify the extra dimensions and go to the four-dimensional Einstein frame, the power exponents are different depending on how we compactify the extra dimensions even within one solution. We give the power exponent of the fastest expansion of our four-dimensional universe in the Einstein frame in [5]. We again see that the expansion is too small. Hence, we have to conclude that in order to find a realistic expansion of the universe in this type of models, one has to include additional "matter" fields on the brane.

3. Discussions

We have derived intersecting dynamical brane solutions and discussed their dynamics in the ten- and eleven-dimensional supergravity models. These solutions are obtained by replacing a constant A in the warp factor $h = A + h_1(y)$ of a supersymmetric solution by a function $h_0(x)$ of the coordinates x^μ [3, 4, 6]. Our solutions can contain only one function depending on both time as well as overall or relative transverse space coordinates. In particular, the solutions in Sec. 2 tell us that the brane which depends on overall transverse coordinate can be extended to the time dependent case. It is possible to get the dynamical intersecting brane solutions which obey the intersection rule $\chi = -2$ different from the usual one, as we have discussed in Sec. 2.

We have used the intersection rules to find the cosmological solution because it is not so easy to find it analytically without their rules. The intersection rules have led to the functions h_r and h_s which can be written by linear combinations of terms depending on both coordinates of worldvolume and transverse space. This feature is expected to be shared by a wide class of supersymmetric solutions beyond the examples considered in the present paper, because the result has been obtained by analyzing the general structure of solutions for warped compactification with field strength of the ten- or eleven-dimensional supergravities under ansatz that is natural to include supersymmetric solutions as a special case. We have found that these solutions give a Friedmann-Lemaître-Robertson-Walker universe if we regard the homogeneous and isotropic part of the brane worldvolumes as our spacetime [5]. Unfortunately, the power of the scale factor is so small that the solutions of field equations cannot gives a realistic expansion law. This means that we have to consider additional matter on the brane in order to get a realistic expanding universe. As the number p_r or p_s increases, the power of the scale factor becomes large. We find that the intersection with D6-brane in ten-dimensional theory give the fastest expansion of our universe because the three-dimensional spatial space of our universe stays in the transverse space to the D6-brane. Though the power of the scale factor for the transverse space in solutions with the D7- or D8-branes is larger than those with the D6-brane, the number of the transverse space to these branes is less than three. Hence, these solutions cannot provide the isotropic universe if we assume that the transverse space to the brane is the part of our universe [5].

The solutions we have obtained may give some moduli instabilities because of the flat direction of the moduli potential in the lower-dimensional effective theories after compactifications [1, 4, 6, 7]. Such instability will grow unless the global or local minimum of the potential can be produced by some correction in the effective theory.

The dynamical solutions contain only one function depending on both time and transverse space coordinates. One possible reason for this is that the ansatz concerning the structure of the D-dimensional metric is too restrictive. It is interesting to examine if our solutions can be extended to more general solutions by relaxing the assumptions of the field ansatz.

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