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## Coherent Soft Particle Production in $Z$ Decays into Three Jets

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### Abstract

Low energy particle production perpendicular to the event plane in three jet events produced in  $Z$  decays in  $e^+e^-$ -annihilation is measured and compared to that perpendicular to the event axis in two jet events. The topology dependence of the hadron production ratio is found to agree with a leading order QCD prediction. This agreement and especially the need for the presence of a destructive interference term gives evidence for the coherent nature of gluon radiation. Hadron production in three jet events is found to be directly proportional to a single scale function of the inter jet angles. The slope of the multiplicity with respect to the scale is expected to be equal to the colour factor ratio  $C_A/C_F = 9/4$ . The measurement yields:

$$\frac{C_A}{C_F} = 2.211 \pm 0.014_{\text{stat.}} \pm 0.053_{\text{syst.}} \quad .$$

This result strongly supports the assumption of local parton hadron duality, LPHD, at low hadron momentum.

# 1 Introduction

Interference effects are basic to quantum-mechanical gauge theories of fundamental interactions like Quantum Chromodynamics, QCD. The probabilistic Parton Model of strong interactions, however, neglecting interference effects still successfully describes many processes as incoherent sum of the individual sub-processes. In fact it proved difficult to show strong interaction interference effects in high energy inelastic processes like  $e^+e^-$ -annihilation, deep inelastic scattering or  $p\bar{p}$ -interactions. Evidence for coherence effects comes from the necessity to include colour coherence in the fragmentation models in order to describe the energy dependence of hard interactions, from the so called hump backed plateau in the logarithmic scaled momentum spectrum of hadrons due to the suppression of low energy particle production, and from the string effect in three-jet events in  $e^+e^-$ -annihilation explained by destructive interference. For a comprehensive review see [1].

Arguments against the conclusiveness of these verifications of coherence effects have been raised, however. Incoherent fragmentation models involving a large number of parameters allow for a sufficient description of the data at least at fixed centre-of-mass energy [2]. The hadronic momentum spectrum and the energy dependence of the peak of this distribution may also be accounted for when assuming a non-minimal phase-space structure [3]. The string effect measurements, except for symmetric topologies [4, 1], are influenced by boost effects.

In this letter evidence for quantum-mechanical interference or colour coherence is presented from the comparison of low energy hadron production perpendicular to the event plane of three-jet events and the event axis in two-jet events, respectively. Similar to the string effect, in this case the presence of destructive interference is expected. Presuming colour coherence the measurement is also directly sensitive to the colour of the underlying partons,  $q, \bar{q}$  and  $g$ , via the dependence on the three jet topology the ratio of the colour charges or the colour factors for gluons and quarks,  $C_A/C_F$ , can directly be verified.

This letter is organised as follows. In Section 2 a basic description of the theoretical formulae and the idea underlying this measurement is given. Section 3 gives an overview on the data, cuts and corrections applied and describes the measurement. In Section 4 results on the hadron multiplicity observed in cones oriented perpendicular to the event plane of three-jet events and the event axis of two-jet events, respectively, are presented and conclusions are given.

## 2 Motivation and Theoretical Basis of the Measurement

The hadronisation process is believed to commence with the radiation of soft gluons from the initial hard partons. For gluons, due to their higher colour charge compared to quarks, soft gluon radiation and consequently hadron production is expected to be increased in the high energy limit by the gluon to quark colour factor ratio  $C_A/C_F = 9/4$ . This simple expectation is perturbed for hadrons sharing a large energy fraction of the underlying parton. Leading particle effects become evident and, as such particles are created last in time [5], effects due to energy conservation are vital. The latter influence is stronger for gluon jets due to the stronger particle production.

In case a radiated hard gluon stays close to the radiating quark additional soft gluons may be unable to resolve the individual colour charges of the close by partons. In this limit only coherent radiation off the parton ensemble, that is from the colour charge of the initial quark occurs. This principle effect also leads to the angular ordering property of soft gluon emission.

Coherence effects should be best visible for low energy hadrons emitted perpendicular to the radiating parton as these particles cannot be assigned to a specific jet and have poor resolution.

The cross-section for soft gluon radiation in  $(q\bar{q}g)$  three-jet events has been calculated in [6]. It varies strongly as function of the three-jet topology. For radiation out of the  $q\bar{q}g$  plane this leading order prediction takes a particularly simple form [7]:

$$d\sigma_3 = d\sigma_2 \cdot \frac{C_A}{C_F} \cdot r_t \quad r_t = \frac{1}{4} \left\{ \widehat{qg} + \widehat{\bar{q}g} - \frac{1}{N_c^2} \widehat{q\bar{q}} \right\} \quad (1)$$

with the so called radiator function

$$\widehat{ij} = 1 - \cos \theta_{ij} = 2 \sin^2 \frac{\theta_{ij}}{2} \quad .$$

Here  $d\sigma_3$  is the cross-section for soft gluon emission perpendicular to the  $q\bar{q}g$  plane and  $d\sigma_2$  the corresponding cross-section for emission perpendicular to the two-jet event  $(q\bar{q})$  axis.  $\theta_{ij}$  denotes the angle between the partons  $i$  and  $j$ . The complete topology dependence is included in the kinematical factor  $r_t$  which obviously plays the rôle of the evolution (“energy”) scale of the low energy hadron multiplicity. The term inversely proportional to the square of the number of colours,  $N_c = 3$ , in  $r_t$  is due to destructive interference and, in the more general case, causes the string effect.

The interest of the measurement presented in this paper is:

- the experimental test of the predicted topology dependence,
- the verification of the  $1/N_c^2$ -term directly manifesting the coherent nature of hadron production and
- the measurement of the slope of the homogenous straight line given by Equation 1 and thereby of the colour factor ratio  $C_A/C_F$ .

In order to experimentally verify the above prediction charged particle production has been studied in cones with  $30^\circ$  opening angle situated perpendicular to both sides of the three-jet plane or the two-jet event axis. For two-jet events the azimuthal orientation of the cones is taken randomly. In order to distinguish events with two, three and four or more jets the angular ordered Durham (aoD) algorithm is used with a jet resolution parameter  $y_{cut} = 0.015$ . The aoD algorithm corresponds to the Cambridge algorithm without soft freezing [8]. It has been chosen as it is expected to lead to optimal reconstruction of jet directions but avoiding complications due to soft freezing. The choice of a fixed jet resolution is necessary as the leading order prediction Equation 1 applies to two- and three-jet events. Consequently events with less than two or more than three jets are explicitly excluded from the analysis.

These experimental choices made in order to approximate the theoretical prediction imply systematic uncertainties of the data to theory comparison. The possible systematic

variations of the data were assessed by varying the cone opening angle in the range  $20^\circ \leq \vartheta_{cone} \leq 40^\circ$  and the jet-resolution in the range  $0.01 \leq y_{cut} \leq 0.02$ . Moreover the Cambridge [8] and Durham [9] algorithms were used alternatively.

In order to directly compare the prediction, Equation 1, to the data, the different possibilities within one event of assigning the gluon to a jet were weighted in the prediction according to the three-jet matrix element. The kinematic term  $r_t$  as given in Equation 1 in principle varies between  $4/9 \sim 0.44$  for two-jet like configurations and 1 when the gluon recoils with respect to the quark anti-quark pair. In this analysis the lower limit is  $r_t^{min} \sim 0.5$  as the cut in the jet resolution implies a minimum angle cut between the gluon and a quark. As gluon-jets stay unidentified and gluon radiation at low angles dominates, the maximum value  $r_t^{max} \sim 0.71$  is obtained for fully symmetric three-jet events.

### 3 Data and Data Analysis

This analysis is based on hadronic  $Z$  decays collected in the years 1992 to 1995 with the DELPHI detector at the LEP  $e^+e^-$ -collider at CERN. The DELPHI detector was a hermetic collider detector with a solenoidal magnetic field, extensive tracking capabilities including a micro-vertex detector, electromagnetic and hadronic calorimetry as well as strong particle identification capabilities. The detector and its performance is described in detail elsewhere [10, 11].

In order to select well measured particles originating from the interaction point the cuts shown in Tables 1 and 2 were applied to the measured tracks and electromagnetic or hadronic calorimeter clusters. Here  $p$  and  $E$  denote the particles momentum and energy,  $\vartheta_{polar}$  denotes the polar angle with respect to the beam,  $\epsilon_k$  is the distance of closest approach perpendicular to  $(xy)$  or along  $(z)$  the beam, respectively,  $L_{track}$  is the measured track length and  $N_{charged}$  is the observed charged multiplicity.

variable	cut
$p$	$\geq 0.4\text{GeV}$
$\vartheta_{polar}$	$20^\circ - 160^\circ$
$\epsilon_{xy}$	$\leq 5.0\text{cm}$
$\epsilon_z$	$\leq 10.0\text{cm}$
$L_{track}$	$\geq 30\text{cm}$
$\Delta p/p$	$\leq 100\%$

Table 1: Cuts applied to charged tracks.

calorimeter	$E_{min} [\text{GeV}]$	$E_{max} [\text{GeV}]$
HPC	0.5	50
EMF	0.5	30
HAC	1.0	50

Table 2: Cuts applied to calorimeter clusters. HPC (EMF) denotes the barrel (forward) electromagnetic calorimeter, respectively, HAC the hadronic calorimeter.

The cuts shown in Table 3 select hadronic decays of the  $Z$  and suppress background from leptonic  $Z$  decays,  $\gamma\gamma$  interaction or beam gas interactions to a negligible level. Further reduction of background is due to the jet selection cuts given in Table 4. The cut variables are the visible charged energy,  $E_{charged}^{total}$  ( $E_{charged}^{hemisph.}$ ), observed in the event or in one event hemisphere, respectively. Event hemispheres are defined by the plane perpendicular to the sphericity axis. The polar angle of this axis with respect to the beam is  $\vartheta_{sphericity}$ . Events are discarded, if they contain charged particles with momenta above the kinematic limit.

Events are then clustered into jets using the aoD (or alternatively the Durham or Cambridge) algorithm. The quality requirements shown in Table 4 assure well measured jets in three-jet events. Here  $E_{\text{visible}}/\text{jet}$  denotes the total visible energy per jet and  $\theta_i$  the angle between the two jets opposite to jet  $i$ . The sequence number  $i$  of a jet is given by ordering the jets with increasing jet energy. These exact jet energies were calculated from the inter-jet angles assuming massless kinematics (see eg. [12]). The quality of the selected two-jet events is warranted already by the event cuts (see Table 3). Arbitrary three-jet<sup>1</sup> and mirror-symmetric three-jet topologies (defined by  $\theta_3 - \theta_2 \leq 5^\circ$ ) are considered in later cross-checks. Note that the symmetric event sample is almost completely included in the arbitrary sample.

In total 1797429 Z events fulfill the above event cuts, therefrom 1031080 are accepted as two-jet event, 309227 as arbitrary and 53344 as symmetric three-jet event.

variable	cut
$E_{\text{charged}}^{\text{hemisph.}}$	$\geq 0.03 \cdot \sqrt{s}$
$E_{\text{charged}}^{\text{total}}$	$\geq 0.12 \cdot \sqrt{s}$
$N_{\text{charged}}$	$\geq 5$
$\vartheta_{\text{sphericity}}$	$30^\circ - 150^\circ$
$p_{\text{max}}$	45 GeV

Table 3: Event selection cuts.

variable	cut
$\sum_{i=1}^3 \theta_i$	$> 355^\circ$
$E_{\text{visible}}/\text{jet}$	$\geq 5 \text{ GeV}$
$N_{\text{ch}}/\text{jet}$	$\geq 2$
$\vartheta_{\text{jet}}$	$30^\circ - 150^\circ$

Table 4: Selection cuts applied to three-jet events.

The charged particle multiplicity in cones perpendicular to the event axis of a two-jet event or the event plane of a three-jet event has then been determined. Besides the cuts mentioned above it has been demanded that the axis of the cones has a polar angle of at least  $30^\circ$  with respect to the beam.

The multiplicity has been corrected for the applied cuts and the limited detector acceptance and resolution by a multiplicative correction factor. This has been calculated using simulated events as the ratio of the multiplicities of generated events to accepted events on detector level. The events were generated with the JETSET 7.3 parton shower model [13] as tuned to DELPHI data [14]. The influence of the magnetic field, detector material, signal generation and digitisation was simulated. The simulated data were then treated like the measured events.

It is well known that the Monte Carlo model underestimates the production of particles at large angles [14]. Therefore the hadron multiplicity in cones perpendicular to the two-jet event axis or the three-jet event plane has been reweighted with a small overall multiplicative correction. This correction was deduced from a comparison of data and simulation for two-jet events. After correction the model describes the data and its dependencies on the three-jet topology well.

In particular, due to the 400 MeV momentum cut on the charged particles the obtained correction factors for the charged multiplicity in the cones are comparably big ( $\sim 2$ ), but vary only slightly with event topology. Moreover, in the important ratio of the multiplicity of three- and two-jet events the correction factors cancel to large extend. The ratio of the corrections for three-jet to two-jet events is only  $\sim 0.9$ . Due to the smallness of this correction and the good description of the data by the simulation, the uncertainty for the multiplicity ratio due to imperfections of the model turned out to be small with respect

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<sup>1</sup>i.e. any selected three-jet topology

to the statistical errors and has been neglected for the quoted charged multiplicities. This uncertainty is, however, considered in global fits of the data.

## 4 Results

Experimental results on the average charged hadron multiplicities in cones are presented for cones perpendicular to the event axis in two-jet events and cones perpendicular to the event plane in three-jet events. Moreover the ratio of the cone multiplicities in three- and two-jet events is given as well as the momentum spectra in cones perpendicular to three-jet events.

The charged hadron multiplicity in cones perpendicular to the event axis in two-jet events is given in Table 5 for three different cone opening angles and three different values of the jet resolution parameter  $y_{cut}^{aoD}$ . The multiplicity shows the expected slight increase with  $y_{cut}$  and, due to the increase of phase-space an approximately quadratic increase with cone opening angle.

$\theta_{cone}$	$y_{cut}$		
	0.01	0.015	0.02
20°	0.231 $\pm 0.001$	0.245 $\pm 0.001$	0.255 $\pm 0.001$
30°	0.537 $\pm 0.001$	0.570 $\pm 0.001$	0.593 $\pm 0.001$
40°	1.007 $\pm 0.002$	1.067 $\pm 0.002$	1.111 $\pm 0.002$

Table 5: Hadron multiplicity in cones perpendicular to the event axis in two-jet events in dependence of the cone opening angle and the jet resolution for the aoD algorithm.

The corresponding three-jet multiplicity is shown in Figure 1 as function of the inter-jet angles  $\theta_2$  and  $\theta_3$  for arbitrary and as function of  $\theta_3$  for symmetric three-jet topologies. The cone opening angle is  $\theta_{cone} = 30^\circ$  and  $y_{cut}^{aoD} = 0.015$ . Numerical values of the results are given in [15]. The inner error bars represent the statistical error, the outer bars also include the systematic uncertainty relevant for the comparison to the theoretical expectation, Equation 1, added in quadrature. As only the ratio of multiplicities in three and two-jet events enters in the comparison to the theoretical expectation (see Equation 1), the relative systematic uncertainty of the multiplicity has been taken to be identical to the relative error of the three- to two-jet multiplicity ratio. It has been determined as the R.M.S. of the results obtained when varying the cone size, jet resolution and jet algorithm as described in Section 2.

The theoretical expectation shown as full line in Figure 1 describes the data well for all topologies except for large  $\theta_3$  for symmetric three-jet topologies. This region corresponds to small  $\theta_1$ , i.e. close by low energy jets. In this case the event plane is ill determined and data and theory cannot be compared reliably. The prediction increases (by  $\sim 7\%$ , see dashed line in Figure 1) if the interference term of the theoretical prediction  $\propto 1/N_c^2$  is omitted. In this case the prediction is significantly above the data.

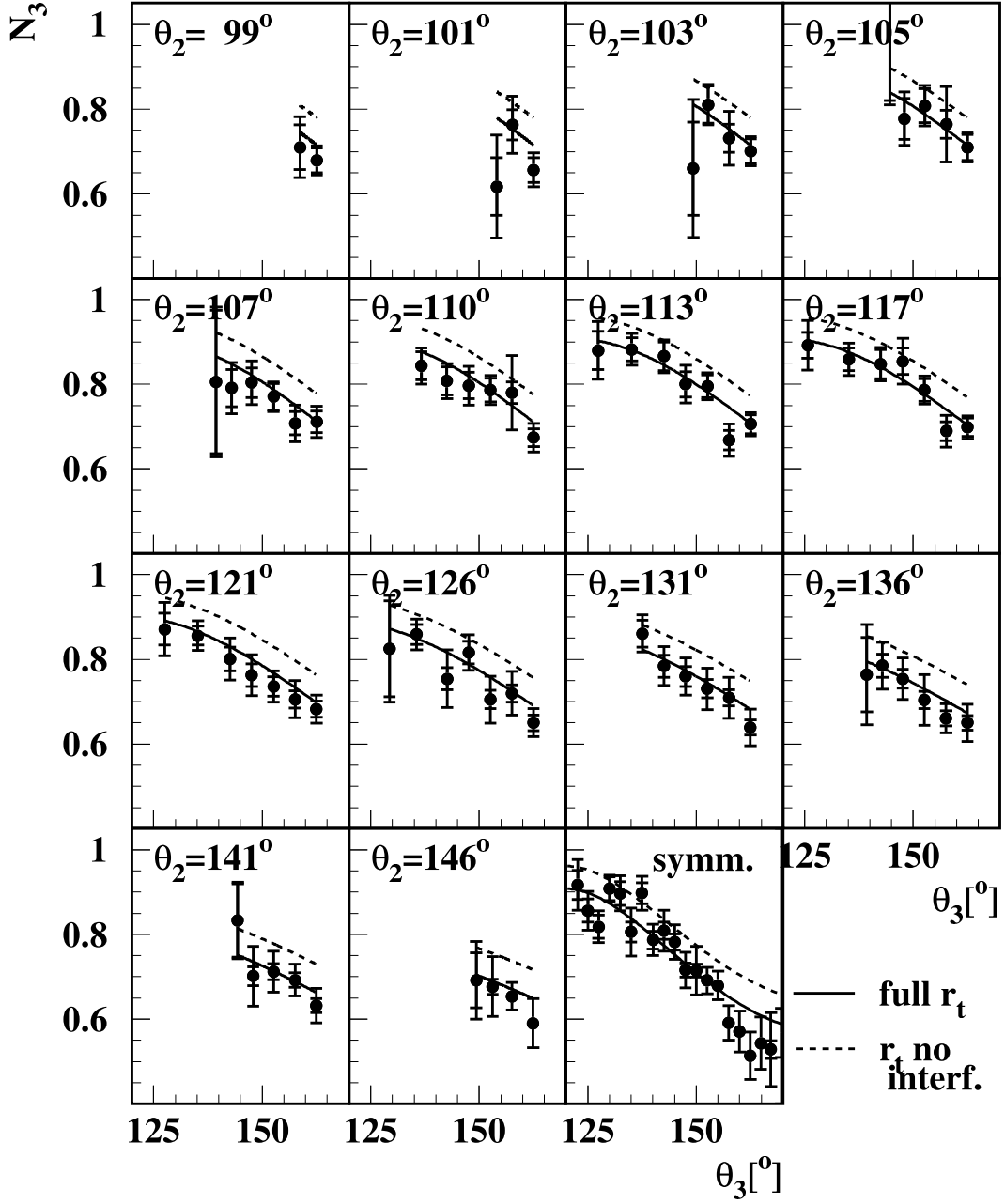


Figure 1: Multiplicity in cones of  $30^\circ$  opening angle perpendicular to the three-jet event plane as function of the opening angles  $\theta_2, \theta_3$ . The inner error bars are statistical, the outer also include systematic uncertainties (see text). The full line is the expectation deduced from Equation 1 using  $N_2$  from Table 5. For the dashed line the interference term in Equation 1 ( $\propto 1/N_c^2$ ) has been omitted.

In order to quantify this result the interference term in Equation 1 has been multiplied by an amplitude factor  $k$  and then fitted to the three-jet to two-jet multiplicity ratio. The fit yields:

$$k = 1.37 \pm 0.05_{stat.} \pm 0.33_{syst.} \quad \chi^2/Ndf = 0.72$$

for arbitrary three-jet topologies and

$$k = 1.30 \pm 0.06_{stat.} \pm 0.38_{syst.} \quad \chi^2/Ndf = 2.4$$

for symmetric three-jet events with  $\theta_1 > 32^\circ$ . The systematic uncertainty was taken to be the R.M.S. of the results obtained when varying the cone size, jet resolution and jet algorithm as described in Section 2. Moreover a 10% uncertainty of the acceptance correction applied for the multiplicity ratio has been assumed. Thus a significant destructive interference term is observed in the data to theory comparison. The size of the term is consistent within error with expectation. This results represents a novel verification of the coherent nature of hadron production. Note that it is based on the direct comparison of data to an absolute prediction. Especially no construction of an unphysical incoherent model is needed for the comparison.

Figure 2 a) shows the multiplicity ratio as function of  $r_t$  for different  $\theta_3$ . The data for different values of  $\theta_3$  agree well as is already expected from the good description of the data by the expectation, Equation 1, shown in Figure 1. The exclusive dependence on  $r_t$  is shown in Figure 2 b) compared to the homogeneous straight line  $N_3/N_2 = C_A/C_F \cdot r_t$  indicated by the dashed line. Multiple use of events from the arbitrary and symmetric samples has been prevented here. The deviation of the points at low values of  $r_t$  is again due to symmetric events with close by jets. The slope of the increase of the multiplicity ratio with  $r_t$  is expected to be the colour factor ratio  $C_A/C_F$ . A homogeneous straight line fit of the data for  $r_t > 0.55$  yields:

$$\frac{C_A}{C_F} = 2.211 \pm 0.014_{stat.} \pm 0.053_{syst.} \quad \chi^2/Ndf = 0.34$$

in very good agreement with QCD expectation  $C_A/C_F = 2.25$ . The systematic uncertainty was again determined as for the interference term,  $k$ . The fit result is indicated by the full line in Figure 2 b). The dashed line is the QCD expectation. This result for the first time quantitatively verifies the color factor ratio  $C_A/C_F$  from hadronic multiplicities and a leading order prediction. The verification of the simple prediction is possible here due to the particularly simple perturbative situation where higher order contributions prove unimportant in contrary to the case of radiation inside jets [7]. Moreover soft particles emitted at large angles are least affected by effects due to energy conservation and leading particle effects.

For completeness it should be mentioned that the data do not allow to simultaneously verify the colour factor ratio and the coherence term due to the obvious high correlation of both terms. If such a fit is attempted the data can as well be described with  $C_A/C_F \sim 2$  and  $N_C \rightarrow \infty$ . This result is also expected from gauge theories with large number of colours. It is clear, however, that this solution does not apply as  $N_C = 3$  is well known experimentally, e.g. from the ratio,  $R$ , of the hadronic to the leptonic cross-section at the  $Z$  pole.



The dependence of the multiplicity on the scale  $r_t$  only allows to study inclusive distributions of the produced hadrons. In Figure 3 a) the differential momentum distribution is shown for different values of  $r_t$ . The momentum distributions are observed to scale approximately with  $r_t$ . This is most clearly seen in Figure 3 b) where the ratio of the momentum spectra normalised to the average spectrum is shown. Moreover these ratios have been scaled by  $\langle r_t \rangle / r_t$  such that a unit value for the overall multiplicity ratio is expected. For small momenta  $p < 1$  GeV the data are consistent with unity, for higher momenta less (more) particles are produced for small (high) scales  $r_t$ , respectively. The constance of the ratio is expected if the production of hadrons is directly proportional to gluon radiation, thus the observed result strongly supports the conjecture of local parton hadron duality, LPHD [6], for low energy hadrons. The deviation from unity for higher momenta may be understood due to the increase of hadronic phase-space for high  $r_t$ .

In summary the multiplicity in cones perpendicular to the event plane of three-jet events produced in  $e^+e^-$ -annihilation has been measured for the first time and compared to the corresponding multiplicity in two-jet events. The multiplicity ratio is well described by a leading order QCD prediction also including a destructive interference term. The presence of destructive interference and thereby of the coherent nature of soft hadron production could be verified from the comparison of the data to the absolute QCD prediction. The slope of the hadron multiplicity with evolution scale  $r_t$  directly represents the colour factor ratio  $C_A/C_F$ . This measurement for the first time is able to verify the colour factor ratio using a hadron multiplicity measurement and a leading order QCD prediction. This possibility is a consequence of the favorable perturbative situation studied and the focus on low energy large angle particles. A simultaneous study of the scale and momentum dependence of the hadron multiplicity provides evidence for the local parton hadron duality, LPHD, conjecture.

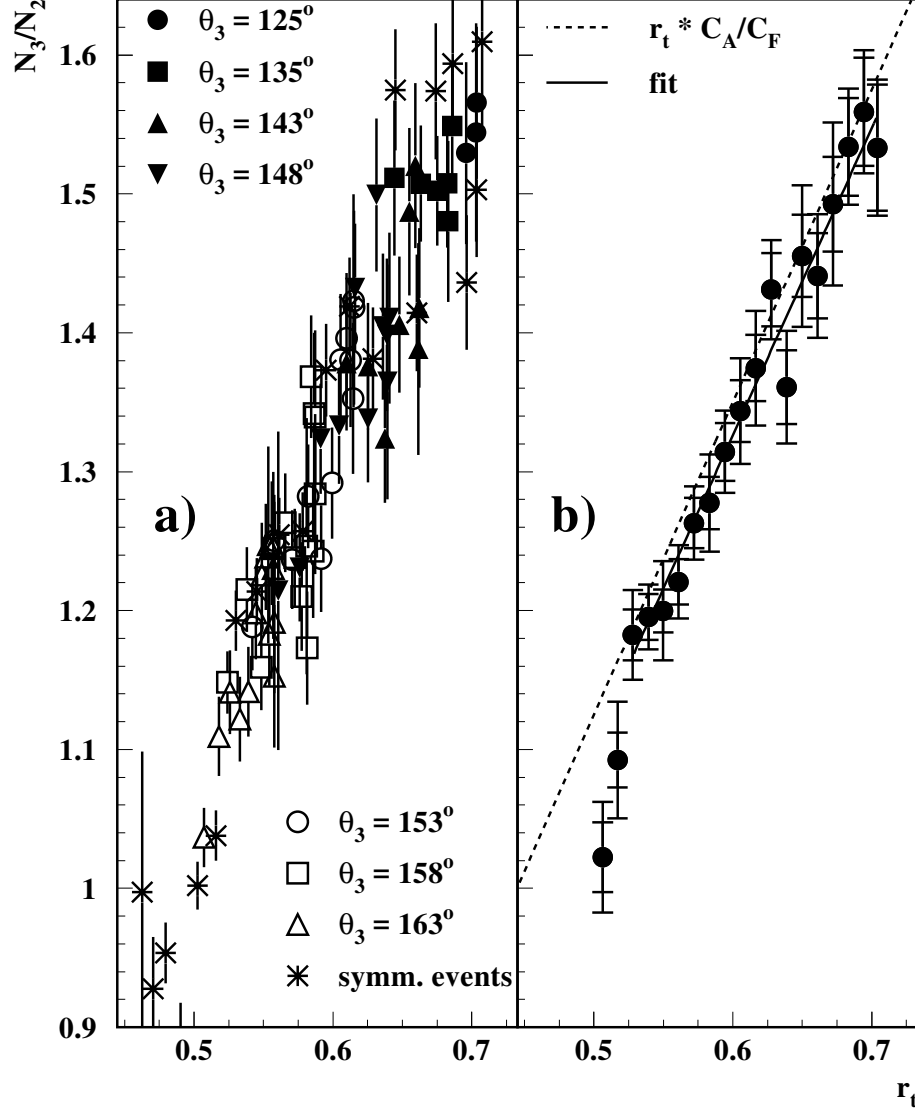


Figure 2: Multiplicity in cones of  $30^\circ$  opening angle perpendicular to the three-jet event plane as function of  $r_t$ . a) for different angles  $\theta_3$ , errors are statistical. b) averaged over  $\theta_3$ , inner errors are statistical, outer errors include systematic uncertainties (see text). The dashed line is the expectation Equation 1, the full line is a fit to the data in the indicated  $r_t$ -interval.

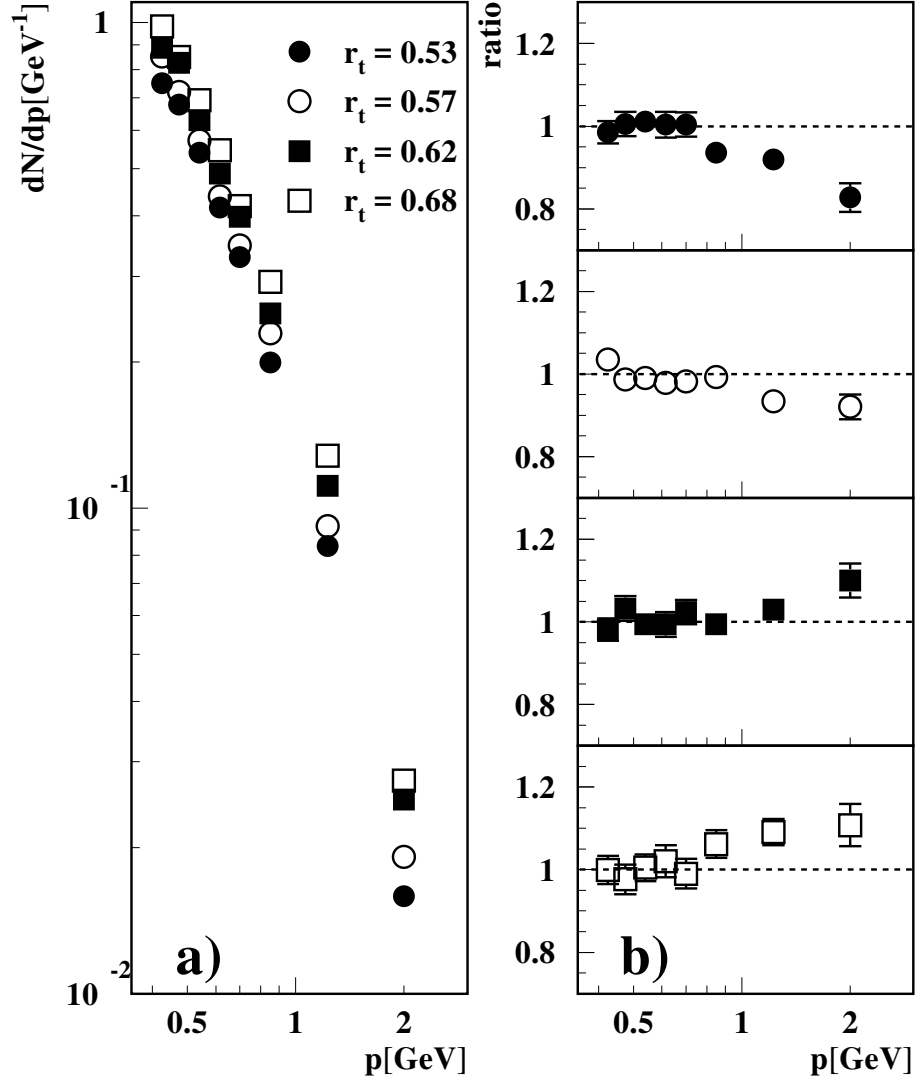


Figure 3: a) Differential momentum distribution of hadrons in cones of  $30^\circ$  opening angle perpendicular to the three-jet event plane for different values of  $r_t$  (aoD algorithm,  $y_{cut} = 0.015$ ). b) Ratio of the momentum distributions to their average reweighted using Equation 1 using the individual  $r_t$  values.

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