TN-62-21
April 1962

COOLING THE ACCELERATOR AND WAVEGUIDE BY A SERIES FLOW ARRANGEMENT

by

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SUMMARY

The problem of maintaining a constant spacial average temperature of the waveguide at all accelerator power levels has been investigated. Particularly, the possibility of cooling the waveguide and accelerator by a series flow arrangement was examined. The conclusions which can be drawn from this investigation are that a single-pass series arrangement (water from the water temperature control system cools the waveguide, then cools the accelerator) cannot maintain a constant accelerator temperature and a constant waveguide temperature at all power levels. A two-pass series arrangement (the water flows along the waveguide, along the accelerator, and then back along the waveguide) is mathematically feasible, but appears difficult to execute. It, therefore, appears to the writer that the same water cannot be used for cooling both the waveguide and accelerator connected in series and that a parallel flow arrangement must be used.

CALCULATIONS

Let us first determine the conditions which must be satisfied in order to maintain a constant accelerator wall temperature.

Assuming there is no heat gain from or loss to the surroundings and if we demand that the accelerator wall temperature does not vary axially (with x), any heat generated at a given axial position must be transferred to the water at that axial position. Therefore, the water temperature must rise linearly as follows for constant gradient operation:

$$t(x) - t(o) = \frac{q}{wc_p} \frac{x}{\ell} \tag{1}$$

and

$$\frac{dt}{dx} = \frac{q}{wc_p \ell} \tag{2}$$

If we now consider an element of the accelerator pipe of length dx, its heat transfer area is $\left(A/\ell\right)dx$, the enthalpy rise (heat pickup) of the water while it traveled the length dx is

$$dq = wc_p \frac{dt}{dx} dx \tag{3}$$

and the heat transferred from the accelerator wall to the water must be

$$dq = h(x) \left(t_a - t(x)\right) \frac{A}{\ell} dx \tag{4}$$

Combining Eqs. (1) through (4), we get

$$t_a - t(o) = q \left[\frac{x}{wc_p \ell} + \frac{1}{h(x)A}\right] \tag{5}$$

From Eq. (5) we can determine how the heat transfer coefficient h must vary with x in order to maintain a constant accelerator wall temperature. This, however, is not our problem here. We can also see that if we do in fact vary the heat transfer coefficient in this manner and retain a constant flow rate at all power levels, $\left(\frac{x}{wc_p\ell} + \frac{1}{h(x)A}\right)$ is a constant, K_a, and that the accelerator pipe to water inlet temperature difference is a function only of the power level. We, therefore, know how the inlet water temperature must be varied with power level. At any two power levels then

$$\frac{t_a - t(o)_{I}}{t_a - t(o)_{II}} = \frac{q_I}{q_{II}} \tag{6}$$

Let us look now at the waveguide cooling problem It is not necessary to maintain the entire length of waveguide at a constant temperature, but only to maintain a constant spacial average temperature (to maintain a constant length) at all power levels (within limits). It is, therefore, not necessary to vary h with axial position. Thus, with a cooling-channel of uniform cross section, h will remain essentially constant and, since the heat generation is axially uniform, the waveguide temperature will vary linearly along with the water temperature. That is, the wall to water temperature difference will not vary with axial position. The average waveguide temperature is then obtained from

$$q = hA \left(t_{wg} - \bar{t}\right)$$

The average water temperature $\bar{t}$ is simply an average of the inlet and outlet water temperatures. Therefore

$$q = hA \left(t_{wg} - \frac{t(o) + t(\ell)}{2}\right) \tag{7}$$

Now, from the water enthalpy rise,

$$t(\ell) - t(o) = \frac{q}{wc_p} \tag{8}$$

Combining with Eq. (7),

$$t_{wg} - t(o) = q \left(\frac{1}{2wc_p} + \frac{1}{hA}\right) \tag{9}$$

we find that the average waveguide temperature minus the inlet water temperature must be proportional to the heat generation rate. And as before

$$\frac{t_{wg} - t(o)_{I}}{t_{wg} - t(o)_{II}} = \frac{q_I}{q_{II}} \tag{10}$$

For a single-pass arrangement, let us call the waveguide water inlet temperature t_1, and the accelerator water inlet temperature t_2.

Equations (6) and (10) now become

$$\frac{t_a - t_{2I}}{t_a - t_{2II}} = \frac{q_{aI}}{q_{aII}} \tag{6'}$$

$$\frac{t_{wg} - t_{1I}}{t_{wg} - t_{1II}} = \frac{q_{wgI}}{q_{wgII}} \tag{10'}$$

The heat generation rate in the waveguide, q_{wg}, will be a constant fraction of the heat generated in the accelerator, q_a. Knowing this and with the aid of Eq. (8), we can combine (6)' and (10)' to yield

$$\frac{t_a - t_{wg}\big)_I}{t_a - t_{wg}\big)_{II}} = \frac{q_{aI}}{q_{aII}} \tag{11}$$

Since we demand that the values of t_a and t_{wg} are fixed regardless of power level, we see from Eq. (11) that this is true only if $q_{aI} = q_{aII}$. Therefore, the single-pass series arrangement cannot be made to work at more than one power level.

Let us now examine a two-pass arrangement where the waveguide is cooled by the water flowing to the accelerator pipe (pass m) and also by the water returning from the accelerator pipe (pass n). We can then write equations similar to Eq. (9) for each of the passes. Calling the pass m water inlet temperature t_1, pass m outlet and accelerator inlet t_2, and accelerator outlet and pass n inlet t_3,

$$t_{wg}\big)_m - t_1 = q_m K_{wg} \tag{12}$$

and

$$t_{wg}\big)_n - t_3 = q_n K_{wg} \tag{13}$$

In maintaining a constant waveguide length, it is not important that $t_{wg})_m$ and $t_{wg})_n$ remain constant for all power levels (and we have proven in the single-pass case that they cannot be), but that their average, t_{wg}, remain constant. Therefore,

$$t_{wg} = \text{const.} = t_1 + t_3 + (q_m + q_n)\, K_{wg} \tag{14}$$

Now, for the accelerator pipe, we obtain from Eq. (5) and from $wc_p \left(t_3 - t_2\right) = q_a$ that

$$t_a = t_3 - q_a \left(\frac{1}{wc_p} + K_a\right) \tag{15}$$

If we note again that the total heat generated in the waveguide, $q_m + q_n$, is a constant fraction of the accelerator pipe heat generation, combination of Eqs. (14) and (15) yields

$$\frac{t_{wg} - t_a - t_{1I}}{t_{wg} - t_a - t_{1II}} = \frac{q_I}{q_{II}} \tag{16}$$

It should, therefore, be possible to maintain the accelerator pipe at a constant temperature and the waveguide at a constant average temperature for all power levels by properly adjusting t_1.

A practical problem arises now, that of maintaining a part of the waveguide at an average temperature $t_{wg})_m$ and another part at the higher temperature $t_{wg})_n$. It is evident that at the low heat generation rate of the waveguide, it would not be possible to run an inlet cooling tube and an outlet cooling tube in close proximity along the waveguide wall without transferring heat from the outgoing cooling water to the incoming cooling water (a counterflow heat exchanger).

Your comments on other possible water flow arrangements would be appreciated.

Symbols

A	total heat transfer area
c_p	cooling water specific heat
h	convective heat transfer coefficient
K_a	accelerator constant $= \frac{x}{wc_p \ell} + \frac{1}{h(x)A}$
K_{wg}	waveguide constant $= \frac{1}{wc_p} + \frac{1}{hA}$
ℓ	accelerator section length
q	heat transfer rate
t	temperature
$t(x)$	water temperature at position x
t_a	accelerator section temperature
t_{wg}	average waveguide temperature
w	cooling water flow rate
x	axial position

Subscripts

1	waveguide inlet, first pass
2	waveguide outlet, first pass, and accelerator inlet
3	accelerator outlet and waveguide inlet, second pass
m	first waveguide, cooling pass
n	second waveguide, cooling pass