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THE SEARCH FOR MUON-ELECTRON DIFFERENCES*†

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I. INTRODUCTION

This paper is a review of some recent experimental studies on the fundamental nature of the muon and the electron, and on the relationship between these particles. The paper begins with a summary of our present knowledge as to the existence of other charged leptons. This is followed by a brief discussion of some of the static and atomic properties of the muon and electron. The major portion of the paper is then concerned with the high energy behavior of the charged leptons in electromagnetic and strong interaction processes. Only a few references will be made to the behavior of the charged leptons in weak interaction processes.

For this audience there is no need to present an extended description of what we know about the fundamental nature of the muon and the electron. Therefore I simply present in Table I a summary of our present knowledge — or better a portrait — of the muon and the electron. The set of properties, (listed in Table I) that the muon and electron possess in common are collectively described by the phrase "muon-electron universality".

This portrait of the charged leptons leads to numerous questions. Are there heavier charged leptons? If there are no heavier charged leptons, why are there two charged leptons? Are the charged leptons really point particles, or do they have a structure which has not yet been detected? Are the electron and muon related in any profound way, or are they simply unrelated particles both of which just happen to obey the Dirac equations? Are there differences between the muon and the electron other then those listed in Table I?

We must admit that at present we do not possess a fundamental theory which can provide answers to these questions. We must also admit that we do not even possess a theory which can guide us as to how we might try to answer these questions experimentally. Therefore the experimentalist is on his own in searching

Table I

Property	Comparison between muon and electron	If property is different Muon	Electron
Intrinsic spin	both 1/2		
Statistics	both Fermi-Dirac		
Fundamental equation	both Dirac equation		
Structure	both point particles (within present experimental precision as discussed in this article)		
Interact through the strong interactions	both no (within present experimental precision as discussed in this article)		
Interact through the electromagnetic interaction	both yes		
Magnitude of electric charge	same for both		
Sign of electric charge	both + or −, neither 0		
Gyromagnetic ratio	both given by quantum electrodynamics and particle's mass		
Interact through the weak interactions	both yes		
Magnitude of weak interaction coupling constant	same for both		
Associated neutrino	yes but different neutrinos	ν_μ	ν_e
Mass (MeV/c^2)		106	0.51

for answers to these questions. These searches, which in their very nature must be speculative, have taken two directions. One direction consists of attempts to find heavier members of the electron-muon family. The other direction consists of comparative measurements of the properties of the muon and of the electron in the hope that hitherto unknown differences between the two particles will be discovered. Of course, for this second direction to be fruitful, one must measure known properties with greater precision or one must measure properties which have not been previously measured. The recent high precision measurements of the gyromagnetic ratio of the muon are an illustration of the first type of measurement. The deep inelastic scattering experiment, which I will describe later, is an illustration of the second type of measurement.

A number of comprehensive reviews of the properties of the muon and the electron have appeared in the last ten years[2,3,4]. I shall not repeat the material contained in those reviews, but only summarize their conclusions. Thus my emphasis will be on very recent experimental results. These new results have not altered the portrait, presented in Table 1, in an experimentally significant way. But these new results do indicate what could be the most fruitful directions for future investigation. This forms the subject of the last sections of this paper — the sections entitled "Speculations".

II. ARE THERE HEAVIER CHARGED LEPTONS?

An old and obvious speculation is that the electron and muon are the lowest mass members of a larger family of charged leptons,

$$e, \mu, \mu', \mu'' \ldots ,$$

with associated neutrinos,

$$\nu_e, \nu_\mu, \nu_{\mu'}, \nu_{\mu''} \ldots$$

If μ' and μ have the same lepton number then electromagnetic decays such as

$$\mu' \rightarrow \mu + \gamma \tag{1}$$

can occur. If the μ' has a unique lepton number there will be decay modes such as

$$\begin{aligned} \mu'^- &\rightarrow \nu_{\mu'} + \mu^- + \bar{\nu}_\mu \\ \mu'^- &\rightarrow \nu_{\mu'} + e^- + \bar{\nu}_e \end{aligned} \tag{2}$$

If the μ' has a sufficiently large mass, very interesting decay modes with hadrons in the final state will occur. Examples of such decay modes are

$$\begin{aligned} \mu' &\rightarrow \nu_{\mu'} + \pi^- \\ \mu'^- &\rightarrow \nu_{\mu'} + \pi^- + \pi^0 \\ \mu'^- &\rightarrow \nu_{\mu'} + K^- \end{aligned} \tag{3}$$

Reactions (2) and (3) cause the lifetime of the μ' to rapidly decrease as the mass $m_{\mu'}$ increases. This is shown in Fig. 1 taken from the work of Beier[5]. Similar calculations have been carried out by Mann[6] and by Sakurai[7]. Assuming the μ' has the same weak interaction coupling constant as the μ, the lifetime will be about 10^{-11} sec at a mass of 1 GeV/c^2 and will approach 10^{-16} sec as the mass approaches 10 GeV/c^2. If reaction (1) can also occur, the lifetimes will be even shorter.

To the question, "Are the muon and the electron part of a larger family of charged leptons?" we must give the unsatisfactory answer which follows. As far as we know there are no other charged leptons. But this knowledge does not go very far. This partial knowledge can be summarized easily.[8]

1. Numerous experiments, many having to do with the decay of the K meson, have shown no additional leptons with masses below 0.5 GeV.[8] The only exception to this statement are some surprising effects found by Ramm.[9] These effects may be explained by postulating a neutral (and perhaps a charged) muon-pion resonance with a mass near 400 MeV/c^2. But such an unexpected and startling new phenomenon obviously requires much more investigation before its existence can be accepted. It is also not clear how such a resonance would correspond to heavy leptons of the type we are considering here.

2. No leptons with masses above 0.5 GeV/c^2 have been found. Some searches[10] have required the formation of a beam of leptons and hence lepton lifetimes of greater than 10^{-8} or 10^{-9} sec. But as shown in Fig. 1, such experiments cannot detect charged leptons with masses greater than 0.5 GeV/c^2 unless some additional and unknown conservation law closed their normal channels of decay. Other searches have been carried out to detect leptons with lifetimes shorter than 10^{-8} sec. Some of these searches[11] have assumed that the reaction

$$e^- + p \rightarrow \mu'^- + p$$

can occur. This in turn assumes that the e and the μ' have the same lepton number. No heavy leptons[11] with these properties have been found in the mass range of 0.2 to 1.0 GeV/c^2. To summarize, all searches for heavy leptons with masses greater than 0.5 GeV/c^2 have had low overall sensitivity because either the rate of production of the hypothetical leptons was unknown or the hypothetical leptons could only be detected if they had special properties.

3. Thus there is a clear need to search directly for heavy leptons with lifetimes shorter than 10^{-9} sec. Such a search should be sensitive to lifetimes as short as 10^{-16} to 10^{-20} sec, should be able to detect the decay products from reactions (1), (2) or (3), and should use a method of producing the hypothetical leptons which has a known production cross section. Fortunately such a search method now exists; <u>electron-positron colliding beam machines</u> provide a just about perfect way to carry out such searches.[12] Assuming pure quantum electrodynamics and that the charged lepton is a point Dirac particle of unit charge, the total cross section for the process

$$e^- + e^+ \rightarrow \mu'^- + \mu'^+ \tag{4}$$

is given by

$$\sigma_{\text{total}} = \frac{\pi\alpha^2(\hbar c)^2}{6E^2}\,\beta\,(3-\beta^2) \tag{5a}$$

Here E is the laboratory energy of the electron or positron. $\beta = v/c$ where v is the velocity of the μ' and c is the velocity of light. $\hbar$ is Plank's constant and α is the fine structure constant. As β approaches 1

$$\sigma_{\text{total}} \underset{\beta\rightarrow 1}{\approx} \frac{2\times 10^{-32}}{E^2}\ \text{cm}^2 \tag{5b}$$

where E is in GeV.

As I will discuss in Section VIIC, the reaction

$$e^+ + e^- \rightarrow \mu^+ + \mu^- \tag{6}$$

is copious, has been studied and the measurements agree with the predictions of Eq. (5). When the colliding beams have sufficient energy, the reaction

$$e^+ + e^- \rightarrow \mu'^+ + \mu'^-$$

is almost as copious and the μ' can be detected through decay modes like reactions (2) and (3). The search for higher mass leptons produced by electron-positron colliding beams has just begun. A preliminary search[12] for heavy charged leptons with masses up to 0.8 GeV/c^2 has not produced any evidence for such leptons; but the experimental conditions were such that the required sensitivity was not achieved.[6,7] In particular, the luminosity of colliding beam apparatus was not quite large enough. As indicated by Eq. (5b), a luminosity of at least 10^{30} events/(cm^2 sec.) is required for the search to be definitive. Such luminosities will soon be available.

III. COMPARISON OF SOME STATIC PROPERTIES OF THE MUON AND THE ELECTRON

In this section I compare some of the static properties of the muon and the electron, properties which are particularly relevent to later discussions. I adopt the point of view that the property of the electron has been well established and that the corresponding property of the muon requires comment.

A. Electric Charge

Four properties of the muon — the electric charge e_μ, the mass m_μ, the magnetic moment μ_μ and the gyromagnetic ratio g_μ — are connected by the relation

$$\mu_\mu = \left(\frac{g_\mu}{2}\right)\left(\frac{e_\mu h}{2m_\mu c}\right) \tag{7}$$

g_μ and μ_μ have been determined with great precision;[1,4] .3 parts per million and 12 parts per million respectively. Therefore e_μ can be determined if m_μ is known from an independent measurement. Such a measurement is provided by the study of the mu-mesic atom, an atom in which a negative muon is captured in an atomic orbit.[13] Ignoring relativistic corrections, fine structure and hyperfine structure, the n^{th} energy level of such an atom is given by the Bohr formula

$$E_n = \frac{-m_\mu e_\mu^2 (Ze_p)^2}{2n^2 \hbar^2} \tag{8}$$

I have distinguished the muon charge e_μ from the charge on the nucleus Ze_p. By measuring the energy difference between levels, m_μ or more precisely the combination $m_\mu e_\mu^2$ can be determined. This measured value of $m_\mu e_\mu^2$ combined with Eq. (7) and the known values of μ_μ, g_μ, $\hbar$, c and e_e (the charge on the electron) yields[4]

$$e_\mu / e_e = 1 \pm (4 \times 10^{-5})$$

But a much lower limit can be obtained[14] by observing that if charge is conserved in the muon decay process

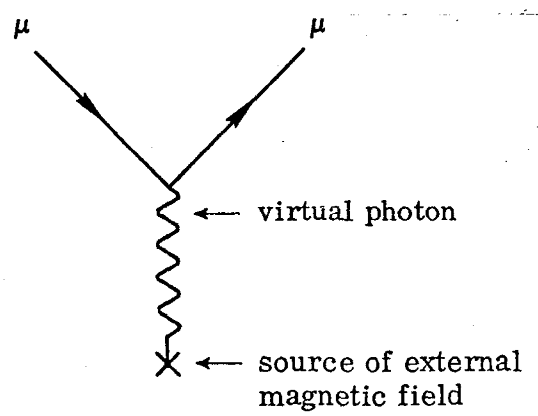
$$\mu \rightarrow e + \nu_\mu + \bar{\nu}_e$$

then one or both neutrinos will have a nonzero charge if $e_\mu \neq e_e$. However neutrinos could then be pair produced by low energy photons, leading to an additional mechanism for energy loss in stars! Astrophysical considerations then set an upper limit on the charge that could be possessed by a neutrino. This limit leads to the conclusion that

$$e_\mu/e_e = 1 \pm 1 \times 10^{-13}$$

B. Gyromagnetic Ratio

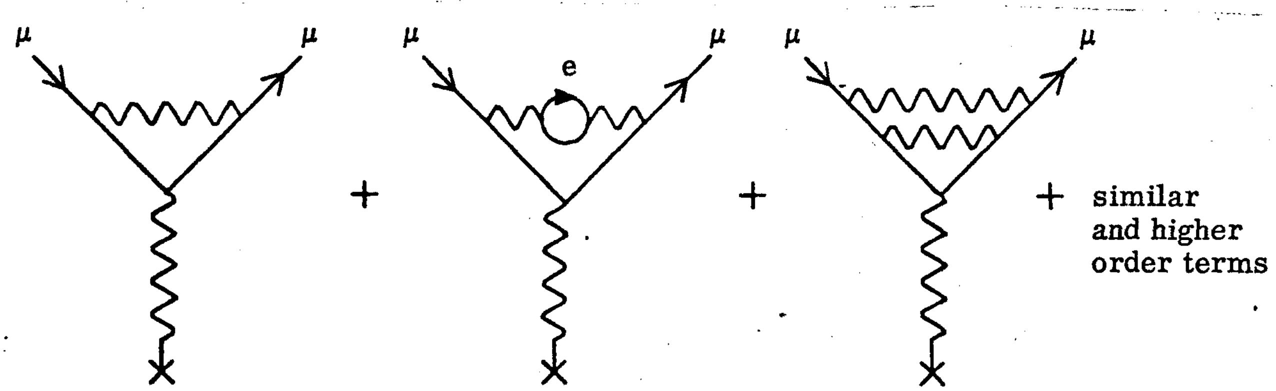
The gyromagnetic ratio, g_μ, can be calculated exactly from quantum electrodynamics, once the muon mass is known, if strong interactions are ignored. (Fortunately the influence of the strong interactions on g_μ is small; I will give the estimated size of the effect below.) The Dirac relativistic theory of the electron or the muon yields g = 2. The Feynman diagram for the interaction of a muon with an external magnetic field (which yields g = 2) is



But quantum electrodynamics shows that there is an anomalous magnetic moment so that g is not exactly 2. It is conventional to set

$$(g_\mu - 2)/2 = a_\mu = \frac{1}{2}\left(\frac{\alpha}{\pi}\right) + A_2\left(\frac{\alpha}{\pi}\right)^2 + A_3\left(\frac{\alpha}{\pi}\right)^3 + \ldots$$

The coefficients A_i are all of the order of magnitude of 10 or less so that α_μ is very small. Nevertheless it has been measured to great accuracy. The measurement of a_μ is a measurement of the combined effect of terms like



The most recent results of Farley, Picasso and their colleagues[1] at CERN yield

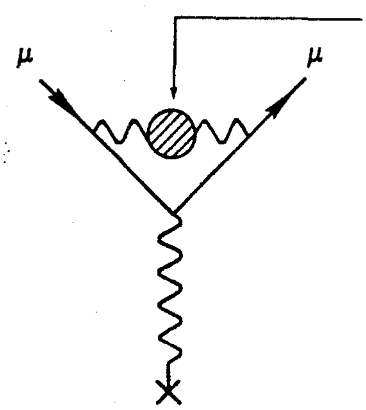
$$a_\mu^{exp} = (116616 \pm 31) \times 10^{-8}$$

and quantum electrodynamics yields[2]

$$a_\mu^{theory} = (116581) \times 10^{-8}$$

Thus experiment and theory are in agreement. Even more precise agreement is found for the electron.[15]

Therefore with respect to the measurement of g-2, once the mass of the muon is taken into account, there is no observable difference between the muon and the electron. For future use I note that the strong interactions enter the g-2 calculation through the diagram



From electron-positron colliding beam measurements it is estimated[1,16] that the effect of this diagram should be

$$a_{\mu}^{hadronic} \approx (5 \text{ to } 25) \times 10^{-8}$$

Therefore the precision of the existing g-2 measurement for the muon is not sufficient to detect strong interaction effects. But an even more precise experiment is now being constructed[16], and this new experiment will be sensitive to $a_{\mu}^{hadronic}$.

To summarize, the static properties of the muon (only some of which have been discussed here) compared to the static properties of the electron show no differences other than those explained by the mass difference.

IV. MU-MESIC ATOMS AND MUONIUM

If the muon is a point Dirac particle the energy levels of the mu-mesic atom[13] will be given by Eq. (8) corrected for fine structure, hyperfine structure, relativistic effects, quantum electrodynamic effects, nuclear size and nuclear charge distribution. All but the nuclear corrections can be calculated from known and accepted theory. The corrections for the size and charge distribution of the nucleus must be determined by experiment. In fact the major purpose of mu-mesic atom experiments is to measure those properties of the nucleus.

By measuring many mu-mesic X-ray lines from a mu-mesic atom, a large amount of interrelated information on the spacing of the energy levels is obtained. Some of this information, particularly that coming from lower energy levels, can be used to derive the relevant nuclear properties. These derived nuclear properties can then be used to calculate the nuclear corrections in the higher energy levels, where those corrections are relatively small. In this way one can attempt a self-consistent calculation of all the mu-mesic X-ray lines. If such a self-consistent calculation cannot be made, the usual hypothesis is that the theory of atomic energy levels contains an error or that the theory of how to derive and correct for the nuclear properties contains an error. If neither of these errors could be found, then one would have to assume that the problem lay with the muon. The muon might not be a point particle or the muon might have an anomalous interaction with the nucleus. Thus high precision measurements of the X-ray lines from mu-mesic atoms provide a test of muon-electron universality.

The search for anomalous effects will be most sensitive if the distance between the nucleus and the muon is relatively small. For if the muon is not a point particle, this will be most evident for small distances. Also any anomalous muon-nucleus interaction is likely to fall off rapidly with distance as noted in Sections

VI and IX. Thus the search for anomalous effects in mu-mesic atoms is best carried out with high Z atoms such as Pb.

At present it is not clear whether measurements on mu-mesic X-rays show a discrepancy from known and accepted theory. D. Kessler et al.[18] in a paper presented to this Conference report a small discrepancy, from conventional theoretical calculations, in a large series of very high precision measurements. But these authors have not yet examined the nature of the discrepancy in detail and they do not know if the conventional calculations can be adjusted within the limits of accepted theory. On the other hand, older less precise measurements and other recent measurements[13,19] do not show this discrepancy. Therefore we must suspend judgement as to whether there is any evidence for the anomalous behavior of muons in mu-mesic atoms. To make such a judgement it is necessary that all high precision measurments be in good agreement. And it is necessary that errors in the accepted theory be completely excluded.

If it is assumed that no discrepancies exist, then mu-mesic X-rays can be used to set an upper limit on the size of the muon. This has been done by Iachello and Lande[20] using older data. I will give that limit in Section IX.

The muonium atom (μ^+e^-) has been used to test quantum electrodynamics and thus indirectly to test if the muon is a point Dirac particle. No anomalies or discrepancies have been found[2,21]. However, compared to mu-mesic atoms, muonium does not provide nearly as sensitive a search method for the kinds of effects of interest in this paper. This is because the muon-electron spacing in muonium is relatively large compared to the muon-nucleus spacing in high Z muonic atoms.

V. HIGH ENERGY REACTIONS OF MUONS AND ELECTRONS

Although the static and atomic properties of the charged leptons show no unexplained differences, one might hope that differences will appear when the dynamic properties of the charged leptons are measured at high energy. For high energies were required to reveal the richness and complexities of the strong interactions. Might not high energies also reveal unsuspected complexities in muon and electron physics? The high energy reactions of the charged leptons may be divided into three classes.

1. One class consists of those reactions in which a neutrino is absorbed or produced. Those reactions as presently measured show no violation of muon-electron universality.[3,22] But the high energy experiments in this class only have precisions of the order of 10 percent and do not involve very large four-momentum transfers. Therefore we have not yet had stringent tests of muon-electron universality in this class of reactions.

2. Another class consists of purely electromagnetic reactions in which no hadron participates or in which the hadron has only an auxiliary role acting as an almost static source of electric charge. Examples are reaction (6) or muon bremsstrahlung

$$\mu + p \rightarrow \mu + \gamma + p$$

Many of these experiments have been recently reviewed[2] with respect to tests of quantum electrodynamics. Some have also been reviewed at this conference by M.J. Tannenbaum[23] with respect to the search for muon-electron differences. In particular he discussed a recent experiment[24] confirming that muons, like electrons, obey Fermi-Dirac statistics. Except for some early experiments, all experiments in this class confirm that the charged leptons are point Dirac particles obeying quantum electrodynamics. Thus all these experiments confirm electron-muon universality in purely electromagnetic reactions. I will review in Section VII some of the electron-positron colliding beam experiments which are relevent to the major concerns of this paper.

3. The third class of reactions, those which I shall emphasize in this article, consist of reactions in which hadrons play an intimate role. My interest in this class of reactions has two origins. First, as I shall discuss later, these reactions provide a way to search for spatial structure in the charged leptons; a way to test if the charged leptons are truly point Dirac particles. (Some Class 2 reactions also test for spatial structure.) Second, a speculation which particularly intrigues me is that the leptons may in some very reduced manner take part directly in the strong interactions. After all, the mass difference between the muon and the electron is almost a pion mass and thus could be caused by the strong interactions. To see if the charged leptons in any way directly take part in the strong interactions, it is desirable to have hadrons present — hadrons act as a source for the strong interactions.

In the interaction of muons (or electrons) with protons we can consider two kinds of processes; elastic scattering where

$$\mu + p \rightarrow \mu + p$$

and inelastic scattering where

$$\mu + p \rightarrow \mu + \text{(any set of 2 or more hadrons)}$$

Examples of inelastic scattering are:

$$\mu + p \rightarrow \mu + p + \pi^o$$

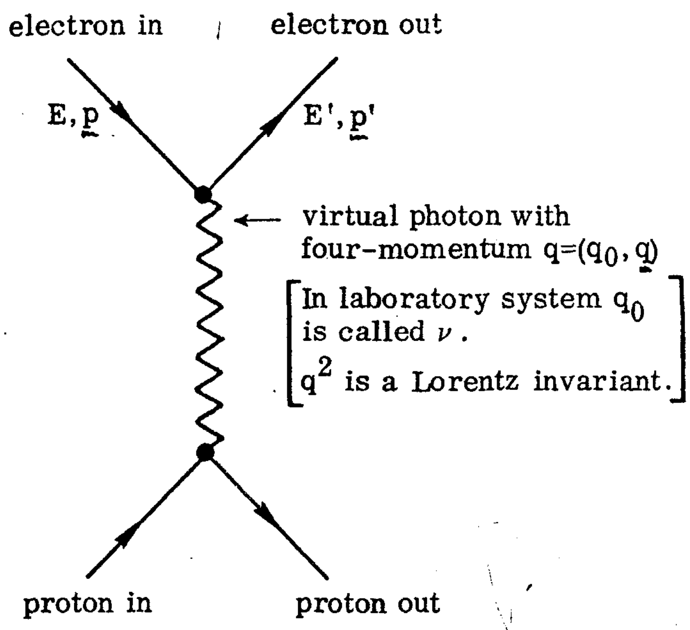
$$\mu + p \rightarrow \mu + n + \pi^+ + \pi^o$$

$$\mu + p \rightarrow \mu + \Sigma^o + K^+$$

In these elastic or inelastic scattering reactions, the charged lepton is not altered in the reaction. This distinguishes these processes from neutrino induced reactions of Class 1.

VI. MUON-PROTON ELASTIC SCATTERING AND FORM FACTORS

I will consider first elastic scattering, and to set the stage I will discuss electron-proton elastic scattering. To a precision of a few percent all data on electron-proton elastic scattering is explained by the Feynman diagram



KINEMATICS
(in laboratory system)

$\nu = q_0 = E - E'$

$\underset{\sim}{q} = \underset{\sim}{p} - \underset{\sim}{p}'$

$q^2 = q_0^2 - (\underset{\sim}{q})^2 = -2M\nu$

M is proton mass

UNITS

energy-GeV

momentum-GeV/c

mass-GeV/c^2

$\hbar = 1$

$c = 1$

$e^2/\hbar c = \alpha \approx 1/137$

in which only one photon is exchanged. All experiments agree[25] that the differential cross section for this elastic scattering process is described by the equation

$$\left(\frac{d\sigma}{dq^2}\right)_{ep,elas} = \left(\frac{d\sigma}{dq^2}\right)_{NS} \left[\frac{G_E^2(q^2) + \tau\, G_M^2(q^2)}{1+\tau} + 2\tau\, G_M^2(q^2) \tan^2\frac{\theta}{2}\right] \tag{9}$$

where $G_E(0) = 1$ and $G_M(0) = 2.79$.

This equation, the Rosenbluth formula, assumes that the electron is a point Dirac particle with only electromagnetic and weak interactions. The equation is written for scattering in the laboratory system, θ is the electron scattering angle and

$\tau = |q^2|/4M^2$. $(d\sigma/dq^2)_{NS}$ is the differential cross section for the scattering of an electron by a spin-zero point proton; NS denotes no spin. $(d\sigma/dq^2)_{NS}$ is a function only of the total energy of the system and θ ; it is completely specified by quantum electrodynamics. $G_E(q^2)$ and $G_M(q^2)$ are the proton form factors. They take into account that the proton has nonzero spatial extent, and that the proton has strong interactions. If the proton were a point Dirac lepton G_E and G_M would both equal unity for all values of q^2. The crucial variable is q^2, the square of the four-momentum transferred from the lepton vertex. q^2 is always spacelike in this process and in our metric is negative. In this article energy units will always be GeV, momentum units will be GeV/c and the units of q^2 will be $(\mathrm{GeV/c})^2$. Also unless $\hbar$ and c appear explicitly in a formula, they have both been set equal to 1. I remind you that it is found experimentally[25] that

$$G_E(q^2) \approx 1/\left[1 + |q^2|/.71\right]^2 \qquad (10)$$

units are $(\mathrm{GeV/c})^2$

and

$$G_M(q^2) \approx 2.79\ G_E(q^2) \qquad (11)$$

G_E and G_M are functions of q^2 which is a Lorentz scalar. Thus they express in a relativistically correct way the effects of the hadronic and non-pointlike nature of the proton. When $|q^2|$ is small compared to M^2, we can treat the proton nonrelativistically and provide a simple physical picture of the meaning of these G's.[26,27] For $|q^2| << M^2$, $|q^2| \approx |\mathbf{q}|^2$ where $\mathbf{q}$ is the three-momentum transferred to the proton. Then $G_E(q^2) \approx G_E(|\mathbf{q}|^2) = G_E(\mathbf{q})$ where $G_E(\mathbf{q})$ is the three-dimensional Fourier transform of the electric charge distribution. Explicitly

$$G_E(q^2) \approx G_E(\underset{\sim}{q}) = \int \rho_E(\underset{\sim}{r})\, e^{i\, \underset{\sim}{q} \cdot \underset{\sim}{r}}\, d^3 r \tag{12a}$$

$\rho_E(\underset{\sim}{r})$ is the charge density distribution and is normalized by

$$\int \rho_E(\underset{\sim}{r})\, d^3 r = 1$$

$G_M(q^2)$ can be similarly interpreted. If the proton is contained within a sphere of R, then for $|\underset{\sim}{q}|\, R \ll 1$

$$G_E(\vec{q}) = 1 - (\tfrac{1}{6})\, |\underset{\sim}{q}|^2 \langle r^2 \rangle_E + (\tfrac{1}{120})\, |\underset{\sim}{q}|^4 \langle r^4 \rangle_E + \ldots \tag{12b}$$

Here $\langle r^2 \rangle_E$ and $\langle r^4 \rangle_E$ are the average values of r^2 and r^4 respectively over the charge distribution of the proton. It is often possible to expand the function $G_E(q^2)$ in the relativistically invariant form

$$G_E(q^2) = 1 + a_1\, q^2 + a_2 (q^2)^2 + \ldots\ldots \tag{12c}$$

But the coefficients in Eq. (12c) can be rigorously assigned[27] their corresponding meanings in Eq. (12b) only if $|q^2| \ll M^2$.

Now if the muon is a pure Dirac point particle we can use Eq. (9) for muon-proton elastic scattering. There are small effects due to the muon mass, which I have not exhibited explicitly; but these are known. Then

$$\left(\frac{d\sigma}{dq^2}\right)_{\mu p,\mathrm{elas}} = \left(\frac{d\sigma}{dq^2}\right)_{ep,\mathrm{elas}} \tag{13}$$

But suppose the muon is not a point particle; suppose the muon, like the proton, has a form factor $G_\mu(q^2)$. Then Eq. (13) becomes

$$\left(\frac{d}{dq^2}\right)_{\mu p,\mathrm{elas}} = \left(\frac{d}{dq^2}\right)_{ep,\mathrm{elas}} G_\mu^2(q^2) \tag{14}$$

Of course the most general modification[28] of Eq. (13) would require the introduction of two form factors corresponding to G_E and G_M. But our very primitive knowledge of the structure of the muon does not warrant such a refinement. We have no theoretical guidance to what $G_\mu(q^2)$ might be. But the data reviewed in Section III show that with great precision the static properties of the muon are those of a point Dirac particle. Therefore at $q^2 = 0$ we must have $G_\mu(0) = 1$. Conventionally we take a form analogous to the proton form factor and write

$$G_\mu(q^2) = 1/\left[1-q^2/\Lambda_\mu^2\right]. \tag{15a}$$

When q^2 is spacelike, and hence negative in our metric, we write Eq. (15a) in the form

$$G_\mu(q^2) = 1/\left[1+|q^2|/\Lambda_\mu^2\right]. \tag{15b}$$

Note however that unlike Eq. (10), only the first power of $\left[1+|q^2|/\Lambda_\mu^2\right]$ appears in the denominator. Λ_μ is a sort of inverse measure of the deviation of the muon from a point particle. The smaller Λ_μ, the greater the deviation. The form of Eq. 15 is actually not as restrictive as it might appear to be. As we shall see later in this paper, all experiments have led to values of Λ_μ^2 which are much larger than the $|q^2|$ values occuring in the experiment. Therefore Eq. (15b) is well approximated by

$$G_\mu(q^2) \approx 1 - |q^2|/\Lambda_\mu^2 \tag{15c}$$

Therefore we are actually allowing $G_\mu(q^2)$ to differ from 1 by a term linear in $|q^2|$; this is certainly a simple enough assumption.

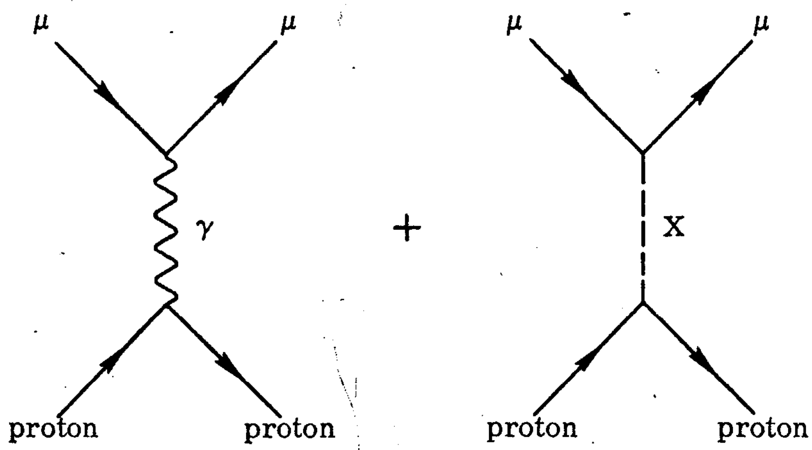
Comparing Eq. (15c) with Eq. (12b) we are tempted to make the identification $\langle r^2\rangle_\mu/6 = 1/\Lambda_\mu^2$

or

$$\sqrt{\langle r^2\rangle_\mu} = \sqrt{6}/\Lambda_\mu = (.48/\Lambda_\mu)\times 10^{-13}\text{ cm} \tag{16}$$

where Λ_μ is in GeV/c. But this identification is only rigorous if $|q^2| \ll (\text{mass}_\mu)^2$. We shall see that in the high energy experiments $|q^2| \gg (\text{muon mass})^2$. Therefore for these experiments we shall have to be cautious in our interpretation of $1/\Lambda_\mu$ as an indication of the size of the muon.

But a muon might differ in other ways from an electron. There might be a special particle, the X particle, that couples to muons and hadrons but not to electrons. Then muon-proton elastic scattering would be the result of two amplitudes whose diagrams are



This would produce some deviation from Eq. (13), but the nature of the deviation cannot be determined because we do not know what X is. Therefore we continue to use $G_\mu(q^2)$ in Eq. (14) to express the deviation of muon-proton elastic scattering from electron-proton elastic scattering. In doing so we are making an assumption to which I shall return at the end of the article. We are assuming that the deviation between muon-proton and electron-proton elastic scattering will increase as $|q^2|$ increases.

Nonrelativistic quantum mechanics provides some insight into the relation between the anomalous interaction concept and the form factor concept.

Nonrelativistically the form factor of Eq. (15b) is the three-dimensional Fourier transform of

$$\rho_\mu(\underset{\sim}{r}) = \frac{1}{4\pi r}\, e^{-r/\Lambda_\mu} \tag{15d}$$

Thus the discovery of a form factor for the muon of the type of Eq. (15b) could also be interpreted as the discovery of an anomalous interaction of the Yukawa form with range $1/\Lambda_\mu$.

In all of this we have assumed that the electron is a pure Dirac point particle. There is no need for this assumption. We can ascribe a form factor $G_e(q^2) = 1/\left[1 + |q^2|/\Lambda_e^2\right]$ to the electron. Then to order $|q^2|$

$$\frac{G_\mu(q^2)}{G_e(q^2)} = \frac{1 + |q^2|/\Lambda_e^2}{1 + |q^2|/\Lambda_\mu^2} \approx \frac{1}{1 + |q^2|/\Lambda_d^2} \tag{17}$$

where

$$\frac{1}{\Lambda_d^2} = \frac{1}{\Lambda_\mu^2} - \frac{1}{\Lambda_e^2}$$

Then Λ_d simply measures a difference in behavior between the electron and the muon. From now on I shall use Λ_d. Defining

$$\rho_{elastic}(q^2) = G_\mu^2(q^2)/G_e^2(q^2) = 1/\left[1 + |q^2|/\Lambda_d^2\right]^2 \tag{18a}$$

Eq. (14) becomes

$$\rho_{elastic}(q^2) = (d\sigma/q^2)_{\mu p, elas}/(d\sigma/dq^2)_{ep, elas} \tag{18b}$$

Eq. (18b) is not exactly true, there is a slight correction due to the muon mass which is not explictly exhibited. Eq. (18a) is the exact definition of $\rho_{elastic}(q^2)$.

Muon-electron universality predicts that $\rho_{elastic}(q^2) = 1$. Two muon-proton elastic scattering experiments have been performed[29,30] to test this prediction. Figure 2 shows the results of the most recent experiment, that of Camilleri et al.[29]

$\rho_{elastic}(q^2)$ is always close to 1, but usually it is a little less than 1. If we set $\rho_{elastic}(q^2) = .92$ we get a good fit. This looks like a normalization problem between the two experiments. To allow for this it is usual to rewrite Eq. (18a) in the form

$$\rho_{elastic}(q^2) = \frac{N^2}{\left[1 + |q^2|/\Lambda_d^2\right]^2} \tag{18c}$$

The authors of this experiment give as a best fit

$$1/\Lambda_d^2 = .064 \pm .056 (\mathrm{GeV}/c)^2, \; N^2 = .95 \pm .035 . \tag{19a}$$

As noted above, the results of the experiment can also be fit with

$$1/\Lambda_d^2 = 0, \quad N^2 = .92 \pm .01 \tag{19b}$$

With 95% confidence, regardless of any normalization uncertainties, the lower limit on Λ_d is

$$\Lambda_d > 2.4 \; \mathrm{GeV}/c \tag{19c}$$

The earlier experiment of Ellsworth et al.[30] found a similar lower limit on Λ_d. If a fit like that given in Eq. (19b) is made to the earlier experiment, one finds

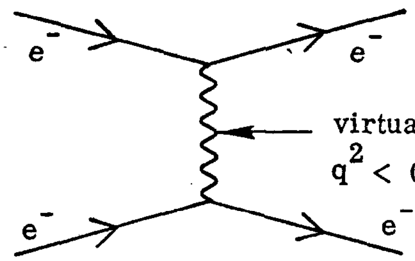
$$1/\Lambda_d^2 = 0, \; N^2 = .88 \pm .04 \tag{19d}$$

I shall discuss the significance of N<1 in Section IX. For the present I note that the significance of finding only a lower limit on Λ_d is that the muon and the electron exhibit, to the precision of the experiments, the same form factors, in charged lepton-proton elastic scattering.

VI. CHARGED LEPTON FORM FACTORS IN COLLIDING BEAM EXPERIMENTS

A. Elastic Electron-Electron Scattering: $e^- + e^- \rightarrow e^- + e^-$

This reaction occurs through the diagram



To obtain large values of $|q^2|$ it is necessary to use an electron-electron colliding beam apparatus, and such an experiment has been carried out at the Princeton-Stanford storage rings[30]. If we ascribe an electron form factor of the form of Eq. (15a) to each vertex, the differential cross section predicted by pure quantum electrodynamics will be multiplied by the factor $1/\left[1-q^2/\Lambda_{ee}\right]^4$. The Λ_{ee} parameter in this form factor has the subscript ee to distinguish it from the Λ_e parameter which occurs in Eq. (17). The latter applies to electron-proton elastic scattering. An anomalous electron-proton interaction could lead to an electron form factor differing from unity in electron-proton elastic scattering, but would not effect electron-electron scattering. Therefore Λ_{ee} and Λ_e need not be the same.

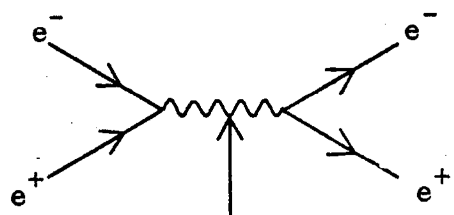
Barber et al.[31] find with 95% confidence the lower limit

$$\Lambda_{ee} > 6.1 \text{ GeV/c} \qquad (20)$$

Thus this experiment agrees with the assumption that the electron is a point particle. Assuming $\Lambda_e = \Lambda_{ee}$ and combining this experiment with the previously discussed muon-proton elastic scattering leads to the conclusion that the muon is also a point particle within the precision of both experiments.

B. Bhabha Scattering: $e^- + e^+ \rightarrow e^- + e^+$

This process takes place through the diagram just given above and through the diagram



Here the photon is timelike and, in our metric q^2 is positive. Until this point we have been considering only spacelike form factors, that is, form factors corresponding to spacelike values of q^2. But now we must also consider timelike form factors, that is, form factors corresponding to timelike values of q^2. The latter form factors, even for small values of q^2, do not have the physical interpretation associated with Eq. (12). Thus Bhabha scattering allows one to study both spacelike and timelike form factors for the electron.

Several electron-positron colliding beam experiments on Bhabha scattering have been carried out.[32,33,34] They all agree with the assumption that the electron is a point particle in both the spacelike and timelike region. I will give the result for one of the experiments[33] in which the form factor $1/(1 - q^2/\Lambda_e^2)$, Eq. (15a), was used for both the spacelike and timelike regions. The lower limit on Λ_{ee} is

$$\Lambda_{ee} > 6.0 \text{ GeV/c}$$

with 95% confidence. In this experiment and in the electron-electron experiment somewhat different limits are obtained if Λ_e^2 is allowed to be negative as well as positive. But I feel this very unphysical additional degree of freedom is not a useful concept for this analysis.

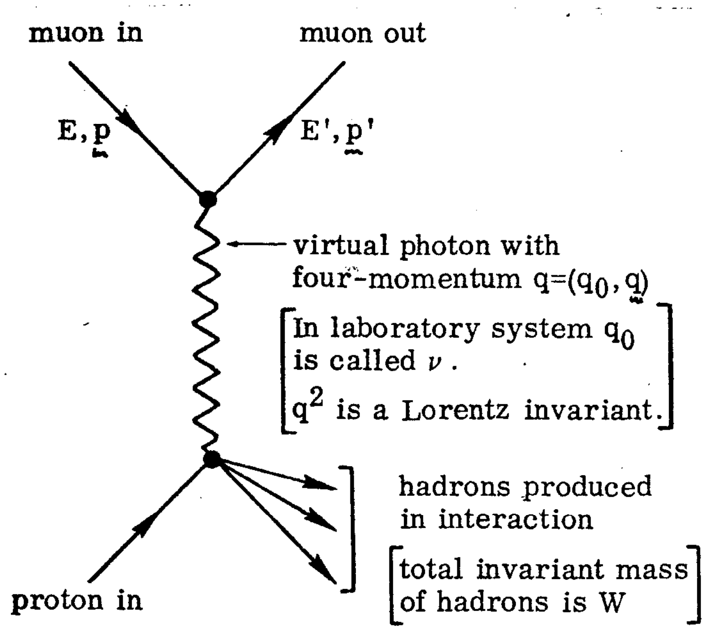
C. Muon Pair Production: $e^- + e^+ \rightarrow \mu^- + \mu^+$.

This process takes place only through the diagram given just above and thus involves only timelike form factors. Several colliding beam studies of this reaction have been made.[32,35,36] They all agree with the assumption that the muon is pointlike in the timelike region. For example if the form factor of Eq. (15a) is used, the 95% confidence lower limit[36] on $\Lambda_{\mu e}$ is

$$\Lambda_{\mu e} > 6.0 \text{ GeV/c} \tag{21}$$

VII. MUON-PROTON INELASTIC SCATTERING

I shall now describe the results of a muon-proton inelastic scattering experiment we[37] recently carried out at the Stanford Linear Accelerator Center. The general diagram for charged lepton-proton inelastic scattering, with one photon exchange is



KINEMATICS
(in laboratory system)

$\nu = q_0 = E - E'$

$\underset{\sim}{q} = \underset{\sim}{p} - \underset{\sim}{p}'$

$q^2 = q_0^2 - (\underset{\sim}{q})^2$

$|q^2| < 2M\nu$

M is proton mass

$W^2 = 2M\nu + M^2 - |q^2|$

Muon-proton or electron-proton inelastic reactions comprise a vast field whose outlines are just now being experimentally determined. I shall describe results from the simplest experiment in that field — an experiment in which only the inelastically scattered charged lepton is detected. No attempt is made to detect any of the hadrons produced. This inelastic scattering experiment then sums experimentally over the different hadronic states which can be produced. As may be deduced from the foregoing diagram the reaction is then completely described by three independent kinematic quantities. These we take to be E (the initial lepton's energy), q^2 (the square of the four-momentum transferred from the lepton vertex) and ν (the laboratory energy of the virtual photon given by E-E').

The experiment consists of the measurement of the double differential cross section of the inelastically scattered muon. This differential cross section, $d^2\sigma/dq^2\,d\nu$, is a function of E, ν and q^2. The one photon exchange property of the diagram above allows one to analyze $d^2\sigma/dq^2\,d\nu$ in more detail. Just as in elastic scattering (Eq. 9) where the differential cross section depends on two independent experimentally determined quantities, $G_E(q^2)$ and $G_M(q^2)$, so the inelastic differential cross section $d^2\sigma/dq^2\,d\nu$ also depends on two independent quantities — quantities which must be experimentally determined. From the above diagram we see that inelastic scattering may be regarded as the production of a virtual photon by the charged lepton, and the subsequent reaction of that virtual photon with the proton leading to the production of all sorts of hadrons. Indeed one set of quantities $\sigma_T(q^2, K)$ and $\sigma_S(q^2, K)$, introduced by Hand,[38] may be thought of as the total cross section for the interaction of transverse and scalar virtual photons with protons. Here $K = \nu - |q^2|/2M$, and is the equivalent energy that a real photon must have to give the same total energy in the photon-proton center-of-mass system.[38] Also $(d^2\sigma/dq^2\,d\nu) = d^2\sigma/dq^2\,dK)$ and I shall use the latter from now on. $\sigma_T(q^2, K)$ and $\sigma_S(q^2, K)$ are defined by

$$\begin{aligned} d^2\sigma/dq^2dK &= \Gamma_T(q^2, K, p)\,\sigma_T(q^2, K) + \Gamma_S(q^2, K, p)\,\sigma_S(q^2, K) \\ &= \Gamma_T(q^2, K, p)\left[\sigma_T(q^2, K) + \epsilon\,(q^2, K, p)\,\sigma_S(q^2, K)\right] \end{aligned}$$

Γ_T and Γ_S are the virtual photon fluxes for transverse and scalar photons, respectively, and ϵ is the ratio of these fluxes as shown in the next two equations.

$$\Gamma_T = \left(\frac{\alpha}{2\pi|q^2|}\right)\left(\frac{K}{p^2}\right)\left(1 - \frac{2m^2}{|q^2|} + \frac{2EE' - |q^2|/2}{(E-E')^2 + |q^2|}\right)$$

$$\epsilon = \Gamma_S/\Gamma_T = \left(\frac{2EE' - |q^2|/2}{(E-E')^2 + |q^2|}\right) \Bigg/ \left(1 - \frac{2m^2}{|q^2|} + \frac{2EE' - |q^2|/2}{(E-E')^2 + |q^2|}\right) \leq 1$$

Here $\underset{\sim}{p}$ ($\underset{\sim}{p}'$) and E(E') are the momentum and energy in the laboratory system of the incident (scattered) lepton; m is the lepton mass and α is the fine structure constant. As q^2 goes to zero, $\sigma_S(q^2, K)$ goes to zero and $\sigma_T(q^2, K)$ goes to $\sigma_{\gamma p}(K)$ — the total cross section for the interaction of a real photon of energy K with a proton. In our muon experiment we could not separate σ_T from σ_S; therefore we use only the combination

$$\sigma_{exp,\ell}(q^2, K) = \sigma_T(q^2, K) + \epsilon\, \sigma_S(q^2, K) \tag{22a}$$

There are three reasons why lepton-proton inelastic scattering is a good way to search for muon-electron differences.

1. In elastic scattering, $\nu = |q^2|/2M$, whereas in inelastic scattering, ν and q^2 may be varied independently. Inelastic scattering therefore explores a much larger kinematic region.

2. Measurements of inelastic lepton scattering in which only the scattered lepton is detected, place no restrictions upon the nature of the final hadronic state. It is conceivable that a violation of muon-electron universality involving hadrons other than the proton would more easily be seen in inelastic scattering than in elastic scattering.

3. One of the more unexpected results of inelastic muon and electron scattering was the large cross section, compared to elastic scattering, at large q^2. Hence inelastic scattering may provide greater sensitivity to muon-electron differences at large q^2.

To search for muon-electron differences we compared $\sigma_{exp,\mu}(q^2, K)$ from our muon experiment[37] with $\sigma_{exp,e}(q^2, K)$ from electron-proton inelastic scattering. We used the extensive and precise electron-proton inelastic scattering data obtained at the Stanford Linear Accelerator Center by the Stanford Linear Accelerator Center and Massachusetts Institute of Technology electron scattering groups.[39] We interpolated from their kinematic values to our kinematic values. This interpolation in principle depends upon knowing the ratio σ_S/σ_T. But we found that varying that ratio from 0 to 1 produced less than a 1% change in the comparison. This is so because our ϵ is close to 1. We used an average of 0.18 for that ratio in making the comparison.[39]

Figure 3 presents some of our data and comparable electron data. The error bars show only the statistical errors. Both sets of data have been corrected for radiative effects. In looking at these comparisons, we see that there is no obvious q^2 dependent difference between the muon data and the electron data. But on the average $\sigma_{exp,\mu}$ seems to be a little smaller than $\sigma_{exp,e}$. It is clear that we must use a more quantitative comparison method. Therefore we define

$$\rho_{inelastic}(q^2,K) = \sigma_{exp,\mu}(q^2,K)/\sigma_{exp,e}(q^2,K) \tag{23a}$$

in analogy to $\rho_{elastic}(q^2)$ in Eq. (18).

We see in Fig. 3 that within the errors the ratio ρ is always about 1.0, <u>but</u> on the average ρ seems to be a little less than one. In comparing these two experiments we must also consider the possibility of relative overall normalization errors. We have done so, and we estimate that the overall relative normalization error due to systematic uncertainties may be as large as 7%. With this consideration we see that none of the individual deviations of $\rho_{inelastic}(q^2,K)$ from unity are significant.

To combine the data to search for less obvious differences and to quantify the observations I have just made, we assign the <u>inelastic</u> form factors $G_\ell'(q^2,K)$ to the charged lepton. This is in analogy to the elastic form factor $G_\ell(q^2)$. Formally

$$G_\ell'(q^2,0) \equiv G_\ell(q^2)$$

but physically $K > 0$ and there is no direct relation between $G_\ell'(q^2,K)$ and $G_\ell(q^2)$

Equation (22a) becomes

$$\sigma_{\exp,\ell}(q^2,K) = \left[\sigma_T(q^2,K) + \epsilon\,\sigma_S(q^2,K)\right] G_\ell'^{\,2}(q^2,K) \tag{22b}$$

and Eq. (23a) becomes

$$\rho_{\text{inelastic}}(q^2,K) = G_\mu'^2(q^2,K) \Big/ G_\ell'^2(q^2,K) \tag{23b}$$

As I stated when discussing elastic scattering we have no profound theoretical guidance as to how to select leptonic form factors. For inelastic scattering, where the dynamics are even more complicated, we have even less guidance. Therefore I shall first follow convention, use the form factor model of elastic scattering and assume that the charged leptons have a form factor

$$G_\ell'(q^2,K) = \left[1/1 + |q^2|/\Lambda_\ell'^2\right]$$

Then allowing for a normalization problem we obtain

$$\rho_{\text{inelastic}}(q^2,K) = N'^2/\left[1 + |q^2|/\Lambda_d'^2\right]^2 \tag{23c}$$

just as in Eq. (18c). But here we average over K to obtain N' and Λ_d'. Here the parameters are primed to indicate that they apply to inelastic scattering. Again $\Lambda_d'^{-2} = \Lambda_\mu'^{-2} - \Lambda_e'^{-2}$.

Since N'^2 and $\Lambda_d'^2$, are correlated parameters, the best way to present the fit of the data to Eq. (23c) is to use the error contour plot of Fig. 4. The ellipses show the one and two standard deviation contours. The center of the ellipse is at $N^2 = .95 \pm .04$ and $1/\Lambda_d^2 = .021 \pm .021\ (\text{GeV}/c)^{-2}$. If just the statistical error are considered, the point $N^2 = 1$, $1/\Lambda_d^2 = 0$ which is demanded by muon-electron universality lies about three standard deviations from the center of the ellipse. But we must allow the overall normalization to have a systematic error as large as ± 7%. This means that the origin can be shifted vertically by ±.07. If we allow a systematic error in the normalization to go to its lower limit of -7%, then the center of the ellipse is only about one standard deviation from the prediction of muon-electron universality. Obviously smaller vertical shifts of the origin will give almost as good agreement.

Now regardless of the normalization problem we can set a limit on how small Λ_d' can be in Eq. (23c). No matter how we shift the origin vertically, the largest value of $1/\Lambda_d'^2$, for two standard deviations, does not change. With 97.7% confidence

$$\Lambda_d' > 4.1 \text{ GeV/c} \tag{24}$$

Thus we have found no strong, statistically significant, deviation from muon-electron universality in inelastic scattering. The comparison of the inelastic scattering results with the elastic scattering results of Camilleri et al. is shown below.

		Muon-Proton Inelastic Scattering	Muon Proton Elastic Scattering
Best fit	N^2 =	.95 ± .04	.95 ± .04
	$(1/\Lambda_d^2)(\text{GeV/c})^{-2}$ =	.021 ± .02	.064 ± .06
Λ_d (GeV/c) is greater than		4.1	2.4
with a confidence level of		97.7%	95%

VIII. SPECULATIONS ON MUON FORM FACTORS

Although no violations of muon-electron universality have been found in these scattering experiments, or for that matter in any experiment; I will use the results of the scattering experiments as a stimulus for some speculations on how such violations might be found. In our inelastic experiment and in the elastic experiment, there is no indication of any q^2 dependent difference between the muon and the proton. But in these experiments the muon cross sections turn out to be lower than the electron cross sections. I emphasized that in our experiment this difference is not significant because the overall normalization uncertainty is about 7%. In the elastic experiments the authors give a smaller normalization uncertainty for the muon data, but the combined overall normalization uncertainty of the muon and electron data might be as large as our 7%. Thus the low muon cross section in any one experiment is not significant. But perhaps, and this is a very weak perhaps, the two experiments together are telling us something. Perhaps they are saying that we have been looking for the wrong kind of muon-electron difference. We have used the elastic and inelastic form factors

$$G_\mu(q^2) = 1/\left[1 + |q^2|/\Lambda_\mu^2\right] \tag{25}$$

to describe the difference. As I have already stated, this form factor is in accord with the belief that any muon-electron difference will increase steadily with $|q^2|$.

But if we look at the muon-proton inelastic and elastic experiments, with no preconceived notions as to how the muon-electron difference might behave with q^2 we would not use that form factor. We would simply observe that all we have seen is a roughly q^2 independent difference in the cross sections. Thus experimentally we would choose the form factor

$$G_\mu(q^2) = N$$

where N is a constant less than 1. But this contradicts $G_\mu(0) = 1$.

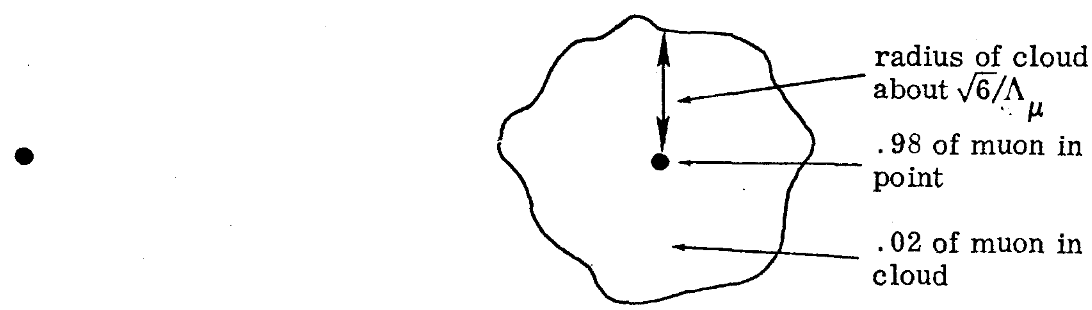
Thus it seems reasonable to select a form factor which is close to $G_\mu(q^2) = N$ but does not violate $G_\mu(0) = 1$. Such a form factor is

$$G_\mu(q^2) = (1-b) + b/(1 + |q^2|/\Lambda_\mu^2) \tag{26}$$

$$= 1 - (b\ |q^2|)/(|q^2| + \Lambda_\mu^2) \qquad 0 \le b \le 1$$

Then in the scattering experiments as $|q^2|$ increases $\rho_{\text{inelastic}}(q^2, K) = \rho_{\text{elastic}}(q^2) \rightarrow (1-b)^2$. If, for example, b = .02, then all that would be observed even at very high $|q^2|$ values would be a normalization difference of 4%. Such a difference in normalization would be masked by the systematic uncertainties of the scattering experiments under discussion. Thus b = .02 may be taken as an example of a possible muon-electron difference. The form factor of Eq. (26) could come from the following model. Take the electron to be a point charge. Take the muon to have 98% of its electric charge concentrated in a point and just 2% spread out in a halo whose average radius is given by $r_\mu \approx \sqrt{6}/\Lambda_\mu$ as in Eq. (16). The picture is

$r_\mu = \hbar c/\Lambda_\mu$. The picture is

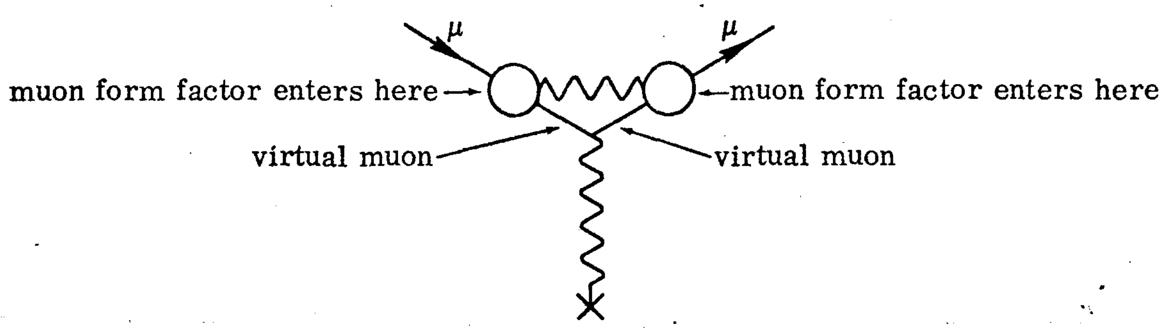


Point electron

Mostly point muon

I remind you that this is just speculation. At present all measurements agree, within their errors, with the assumption that the muon is a point particle. We cannot distinguish between the hypothesis of a diffuse, but very small, muon represented by Eq. (25) and the hypothesis of a mostly point particle muon represented by Eq. (26). But the planning of future high energy scattering experiments designed to search for muon-electron differences does depend upon which hypothesis lies closest to the heart of the experimenter. Thus if the experimenter believes in a mostly point particle muon, it would be best to plan an experiment at moderate q^2 values where high statistical precision and low systematic uncertainties can be most easily achieved.

Until now I have been concerned with the limits set on possible muon form factors by the high energy elastic and inelastic scattering experiments. I shall now extend the discussion to the limits set on the muon form factor by other experiments. The limit set by the reaction $e^+ + e^- \rightarrow \mu^+ + \mu^-$ has already been given in Eq. (21). The measurement of the gyromagnetic ratio of the muon (g_μ) also sets limits on the muon form factor through the diagram



The q^2 which enters the muon form factor in this diagram comes from the virtual muons, and the important q^2 values are those whose magnitude is smaller than $m_\mu^2 d^2$; m_μ is the muon mass. Hence the measurement of g_μ determines the muon form factor with great precision for $|q^2|$ values less than $.01\,(\mathrm{GeV}/c)^2$. Now the experimental value of g_μ is completely explained by quantum electrodynamics, as discussed earlier in this article. Therefore the muon form factor is unity at small values of q^2. In the calculation of the effect of a muon form factor on g_μ both spacelike and timelike values of q^2 must be considered. Hence a form factor for both the spacelike and timelike regions must be selected. It is conventional to select the form factor given by Eq. (15a). The g_μ measurement requires[1,40] that in Eq. (15a)

$$\Lambda_{\mu g} > 7.0 \text{ GeV/c} \tag{27}$$

With 95% confidence. The subscript g indicates that this Λ_μ parameter derives from the g_μ measurement.

As I discussed in Section IV, the mu-mesic atom can be used to set a limit on the size of the muon. Iachello and Lande[20] have considered the effect of non-pointlike muon on the mu-mesic atom energy levels. Using muonic X-ray data from ^{206}Pb and the muon form factor of Eq. (15a), they find that with 95% confidence

$$\Lambda_{\mu A} > 1.4 \text{ GeV/c} \tag{28}$$

While this lower limit is not as large as that set by the g_μ experiment or the high experiments, it is certainly an interesting limit. It is independent confirmation that the muon behaves like a point Dirac particle in this very low energy region. Incidentally, this limit is capable of quite some improvement.[20]

I now come to the question of the impact of the $\Lambda_{\mu e}$, $\Lambda_{\mu g}$ and $\Lambda_{\mu A}$ limits on the speculative equation, Eq. (26), for the muon form factor in muon-proton elastic and inelastic scattering. One may adopt either of two opposite points of view. First, one may suppose that there is an anomalous muon-proton interaction. Then $\Lambda_{\mu e}$, $\Lambda_{\mu g}$ and $\Lambda_{\mu A}$ have no direct relation to the parameters b and Λ_μ in Eq. (26). This point of view is developed in Section IX. Second, one may suppose that Eq. (26) and these limits only concern the muon-photon-muon electromagnetic vertex. Then there is an intimate connection between Eq. (26) and these limits. This connection is discussed in the remainder of this section.

I first note that the limit on $\Lambda_{\mu g}$ includes the limit on $\Lambda_{\mu A}$ so that the latter need no longer be discussed. Now for small values of $|q^2|$

$$b/\Lambda_\mu^2 = 1/\Lambda_{\mu g}^2$$

Therefore the g_μ experiment sets an upper limit on (b/Λ_μ^2) in Eq. (26). But this limit turns out to be incompatable with the "normalization difference" which I would like to explain using Eq. (26). Briefly, this limit does not allow a large enough "normalization difference". (Of course, I am speculating here that the "normalization difference is a physically meaningful effect, not an effect caused by experimental errors.) Thus the g_μ measurement does not allow one to adopt Eq. 26 as a speculative form for the muon form factor.

It is necessary to devise a speculative form factor which varies more rapidly than linearly with $|q^2|$ at small values of $|q^2|$ and yet does decrease very rapidly with $|q^2|$ at very large values of $|q^2|$. Such a form factor might be

$$G_\mu(q^2) = (1-b) + b/(1 + |q^2|^2/\Lambda_\mu^4) \tag{29}$$

which for small values of $|q^2|$ becomes

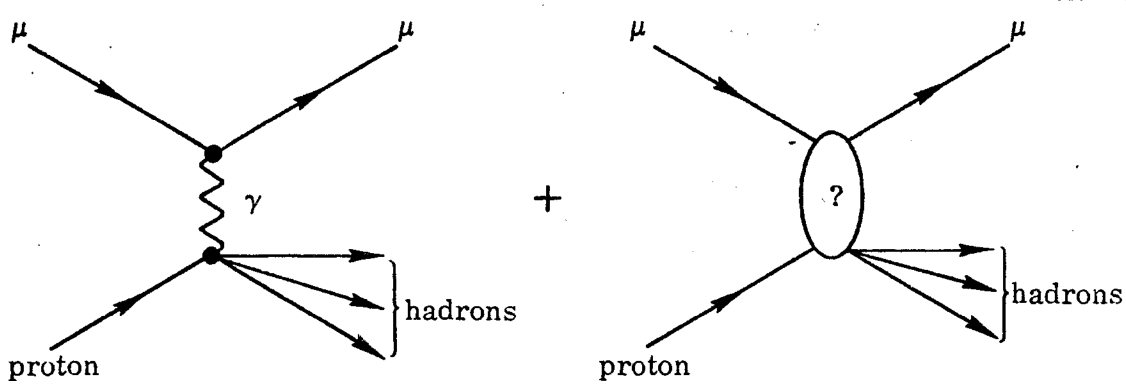
$$G_\mu(q^2) \approx 1 - b|q^2|^2/\Lambda_\mu^4 \tag{30}$$

The form factor of Eq. (29) is compatable with the g_μ measurement and yet can fit the kind of "normalization difference" found in the elastic and inelastic scattering experiments. It is also compatible with experiment results[36] which led to the $\Lambda_{\mu e}$ limit of Eq. (16). Comparing Eq. (30) with Eqs. (12c) we see that the term linear in $|q^2|$ would be missing from this new form factor. I do not know how to interpret the absence of such a term for large values of $|q^2|$.

But for values of $|q^2|$ sufficiently small for Eq. (12b) to be used we have the following interpretation. The absence of the $|q^2|$ term means that $<r^2>$ is zero or very small, and that the $<r^4>$ term dominates. Such values of $<r^2>$ and $<r^4>$ could occur if the muon consisted of a point charge surrounded by two concentric shells with charge densities of alternating sign. Thus if the normalization differences found between the muon and the electron in elastic and inelastic scattering turn out to be true; the muon, and possibly the electron, would turn out to be very complicated objects.

IX. SPECULATIONS ON SPECIAL MUON-HADRON INTERACTIONS

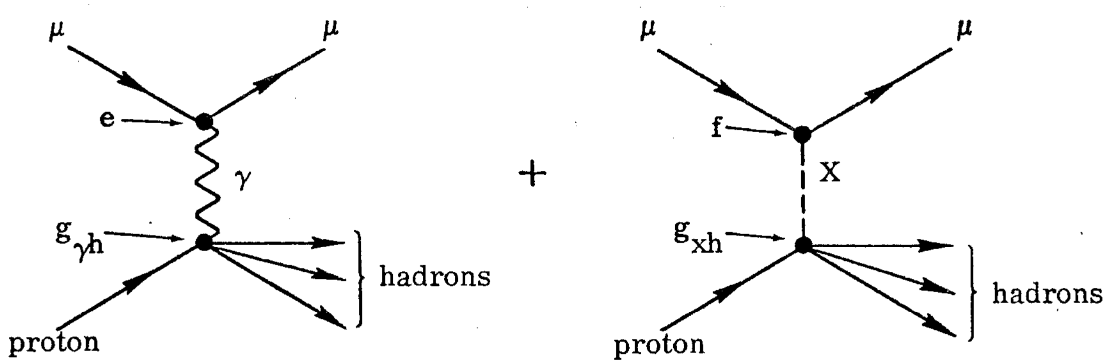
Another way to think about possible muon-electron differences is to speculate that the muon has a special interaction with the hadrons, an interaction not possessed by the electron. I first mentioned this speculation, in the elastic scattering section. Muon-proton inelastic scattering would take place through the sum of two diagrams as follows:



One-photon exchange Special muon-hadron interaction

The second diagram would result in a difference between muon-proton and electron-proton inelastic cross sections, because only the first diagram would enter in electron-proton inelastic scattering.

As an example I shall assume that the muon interacts with the hadrons through the exchange of particle X with spin 1 and mass M_x.



One-photon exchange One-X exchange

The coupling constants are indicated in the diagrams; thus e is the electric charge. Those at the lower vertices are to be regarded only as very crude measures of the strength of the coupling of the virtual photon or the X particle to hadrons. Then[41]

$$\rho_{inelastic}(q^2, K) \approx \left[1 + \left(\frac{f}{e}\right)\left(\frac{g_{xh}}{g_{\gamma h}}\right)\left(\frac{|q^2|}{|q^2| + M_x^2}\right)\right]^2 \qquad (31)$$

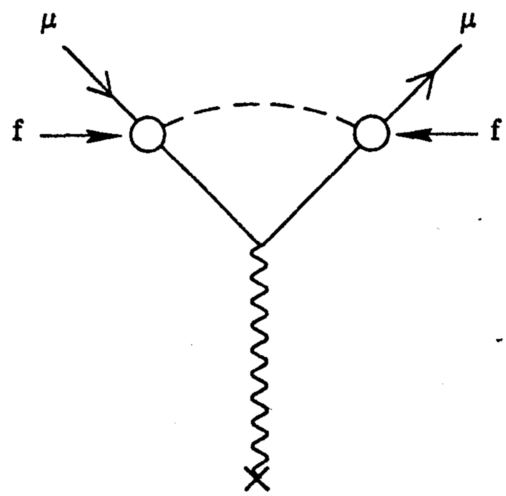
As discussed in relation to Eq. (15d) there is a connection between the assumption of a special muon-hadron interaction and the assumption of a muon form factor. If, in Eq. (31) the product of the coupling constants is a negative real number, Eq. (26) is obtained; $M_x = \Lambda_\mu$ and $b = (f/e)(g_{xh}/g_{\gamma h})$. Thus the assumption of a special muon-hadron interaction which takes place through the exchange of a vector particle leads back to the model of a mostly point particle muon. The radius of the muon is given by the inverse mass of the exchanged particle.

A conventional speculation[41,42] is that the X particle is some undiscovered heavy photon, but I prefer the speculation that the X particle is itself a hadron. More generally the X particle might be taken to represent the summation of the interaction of different kinds of hadrons with the muon. To estimate the present experimental limits on f, the coupling of the muon to the hadron X, I take $(g_{xh}/g_{\gamma h})^2$ to be the ratio of a typical hadron-hadron total cross section (30 mb) to the photon-proton total cross section (0.12 mb). As I discussed above, existing muon-proton scattering measurements easily allow b to be as large as .02. Then

$$f/e \approx .02/\sqrt{250} \approx 1/800$$

Thus in this "X=hadron" model, the coupling of the muon to the hadrons is much weaker than the electromagnetic coupling.

The assumption of such a muon-hadron interaction is compatable with known limits on the parameters $\Lambda_{\mu g}$, $\Lambda_{\mu e}$ and $\Lambda_{\mu A}$. Consider first the g_μ measurement. This muon-hadron interaction contributes to a_μ through the diagram



Because the ratio f/e is so small, this diagram contributes only about 1×10^{-8} to a_μ. This would be completely masked by a_μ^{hadronic} (see Section IIIB) which has yet to be measured. Therefore it is highly unlikely that a g_μ experiment could be sensitive to the muon-hadron interaction about which I have been speculating. Similarly the effect of this muon-hadron interaction on the reaction $e^+ + e^- \rightarrow \mu^+ + \mu^-$ will be very small.

Finally, I come to the relation of this muon-hadron interaction to the mu-mesic atom. Will not such a special interaction perturb the energy levels of the mu-mesic atom from those given by Eq. (8)? The answer is yes. But the problem is how to estimate the perturbation. Here we are faced with the problem of how to go from the high energy behavior of an interaction to the low energy behavior of that interaction. There is no solution to this problem if the nature of the interaction is unknown. We cannot relate the high energy behavior of that interaction to the low energy behavior of that interaction. As an example, the proton-proton and pion-proton interactions have about the same strength at very high energy as measured by their respective total cross sections. But in the very low energy range the proton-proton interactions is several powers of ten stronger than the pion-proton interaction as measured by their respective scattering lengths. Therefore, even a higher lower limit on $\Lambda_{\mu A}$ than that given by Eq. (28) would not directly limit my speculations on a high energy muon-hadron interaction. Certainly then, the relatively low limit given in Eq. (28) has no relation to these speculations.

To summarize, the speculation in this section on a special muon-hadron interaction is not affected by the known limits on $\Lambda_{\mu g}$, $\Lambda_{\mu e}$ or $\Lambda_{\mu A}$. If such an interaction does exist it can most likely only be found through the study of high energy muon-hadron reactions. In conclusion, I must warn the reader that my speculations have been limited to muon-hadron interactions; they have not included muon-hadron-neutrino or neutrino-hadron interactions. Studies of weak interactions at both high and low energy[3,22] limit the strength of any anomalous neutrino-hadron interactions to less than 10% of the strength of the weak interaction itself. This is a much-lower limit than the limits I have been discussing. Therefore the speculation on the muon which I have presented cannot be extended to interactions involving the muon neutrino.

REFERENCES

1. E. Picasso in High-Energy Physics and Nuclear Structure, 1970 (Plenum Press, New York, 1970), this article also reviews recent tests of muon-electron universality; J. Bailey et al., Nuovo Cimento, to be published; J. Bailey et al. Phys. Letters 28B, 287(1968).
2. S. J. Brodsky in Proceedings of the 4th International Symposium on Electron and Photon Interactions at High Energy, Daresbury Nuclear Physics Laboratory, Daresbury, England (1969); S. J. Brodsky and S. D. Drell, Ann. Rev. Nucl. Sci. 20, 147 (1970). These articles review quantum electrodynamic tests of muon-electron universality.
3. T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 15, 381 (1965). This article reviews weak interaction tests of muon-electron universality.
4. G. Feinberg and L. M. Lederman, Ann. Rev. Nucl. Sci. 13, 431 (1963)
5. E. W. Beier, Letters Nuovo Cimento 1, 1118 (1971)
6. A. K. Mann, Letters Nuovo Cimento 1,486 (1971)
7. J.J. Sakurai, Letters Nuovo Cimento 1,624 (1971)
8. See also the review by K. W. Rothe and A. M. Wolsky, Nucl. Phys. 10B, 241(1969).
9. C. A. Ramm, Nature 227, 1323 (1970).
10. A. Barna et al., Phys. Rev. 173, 1391 (1968); contains references to other lepton searches.
11. H. Behrend et al., Phys. Rev. Letters 15, 900(1965).
    R. Budnitz et al., Phys. Rev. 141, 1313 (1966).
    C. Bétourné et al., Phys. Letters 17, 70(1965).
12. V. Alles-Borelli et al., Letters Nuovo Cimento 4, 1156 (1970);

13. C. S. Wu and L. Wilets, Ann. Rev. Sci. 19, 527 (1969).

14. J. Bernstein, M. Ruderman and G. Feinberg, Phys. Rev. 132,1227(1963)

15. J. C. Wesley and A. Rich, Phys. Rev. Letters 24,1320(1969).

16. P. Langacker and M. Suzuki, University of California Radiation Laboratory Report UCRL-20608 (1971).

17. J. Bailey *et al.*, European Organization for Nuclear Research Report NP 70-13 (1970).

18. D. Kessler *et al.*, paper presented to the Muon Physics Conference, Colorado State University, 1971; M. S. Dixit *et al.*, Phys. Rev. Letters 27,878(1971).

19. C. Petitjean *et al.*, papers presented to the Muon Physics Conference, Colorado State University, 1971; and informal discussion.

20. F. Iachello and A. Lande, Phys. Letters 35B,205(1971); I am indebted to A. Lande for a very interesting conversation on this subject.

21. V. W. Hughes, Ann. Rev. Nucl. Sci. 16, 445(1966).

22. D. H. Perkins in *Proceedings of CERN Topical Conference on Weak Interactions*, CERN, Geneva (1969).

23. M. J. Tannenbaum, paper presented to the Muon Physics Conference, Colorado State University, 1971.

24. J. J. Russell *et al.*, Phys. Rev. Letters 26,46 (1971).

25. J. G. Rutherglen in *Proceedings of the 4th International Symposium on Electron and Photon Interactions at High Energy*. (Daresbury Nuclear Physics Laboratory, Daresbury, England, 1969).

26. R. Hofstadter, Ann. Rev. Nucl. Sci. 7, (1957).

27. S. D. Drell and F. Zachariasen, *Electromagnetic Structure of Nucleons* (Oxford University Press, London, 1961).

28. K. J. Barnes, Nuovo Cimento 27, 228(1963).

29. L. Camilleri et al., Phys. Rev. Letters 23, 153(1969).

30. R. W. Ellsworth et al., Phys. Rev. 165,1449 (1968).

31. W. C. Barber et al., Phys. Rev. D3,2796(1971).

32. J. Perez-Y-Jorba in Proceedings of the 4th International Symposium on Electron and Photon Interactions at High Energy, (Daresbury Nuclear Physics Laboratory, Daresbury, England, 1969); J.E. Augustin et al., Laboratoire de L'Accélérateur Linéaire Report LAL 1221(1969).

33. B. Borgia et al., Phys. Letters 35B,340 (1971).

34. B. Bartoli et al. Nuovo Cimento 70A, 603(1970).

35. G. Barbiellini et al., Laboratori Nazionali di Frascati Report LNF-70/38 (1970).

36. V. Alles Borelli et al., Paper submitted to the International Conference on Elementary Particles, Amsterdam, 1971. I have calculated the value of $\Lambda_{\mu e}$ from the results presented in this paper.

37. W. T. Toner et al., Phys. Letters 36B, 251 (1971). For a description of the experiment see W. T. Toner in Proceedings of the 4th International Symposium on Electron and Photon Interactions at High Energy, Daresbury Nuclear Physics Physics Laboratory, Daresbury, England, 1969.

38. L. N. Hand, Phys. Rev. 129, 1834 (1963). The use of K rather than ν is useful but not necessary. It allows the comparison of virtual photon-proton cross sections and real photon-proton cross sections for fixed values of the total invariant mass of the produced hadrons.

39. E. D. Bloom et al., Phys. Rev. Letters 23, 930 (1969); R. E. Taylor in Proceedings of the 4th International Symposium on Electron and Photon Interactions at High Energy, Daresbury Nuclear Physics Laboratory, Daresbury, England (1969).

40. F. J. M. Farley, unpublished paper entitled, The Status of Quantum Electrodynamics (Royal Military College of Science, Shrivenham, Swindon, England, 1969).

41. For the application of this equation to muon-proton elastic scattering see D. Kiang and S. H. Ng, Phys. Rev. D2, 1964 (1970) and the references contained in that paper.

42. L. Yu. Kobzarev and L. B. Okun', Soviet Physics JETP 14, 859(1962); J. Exptl. Theoret. Phys. (U.S.S.R.) 41, 1205 (1961).

FIGURE CAPTIONS

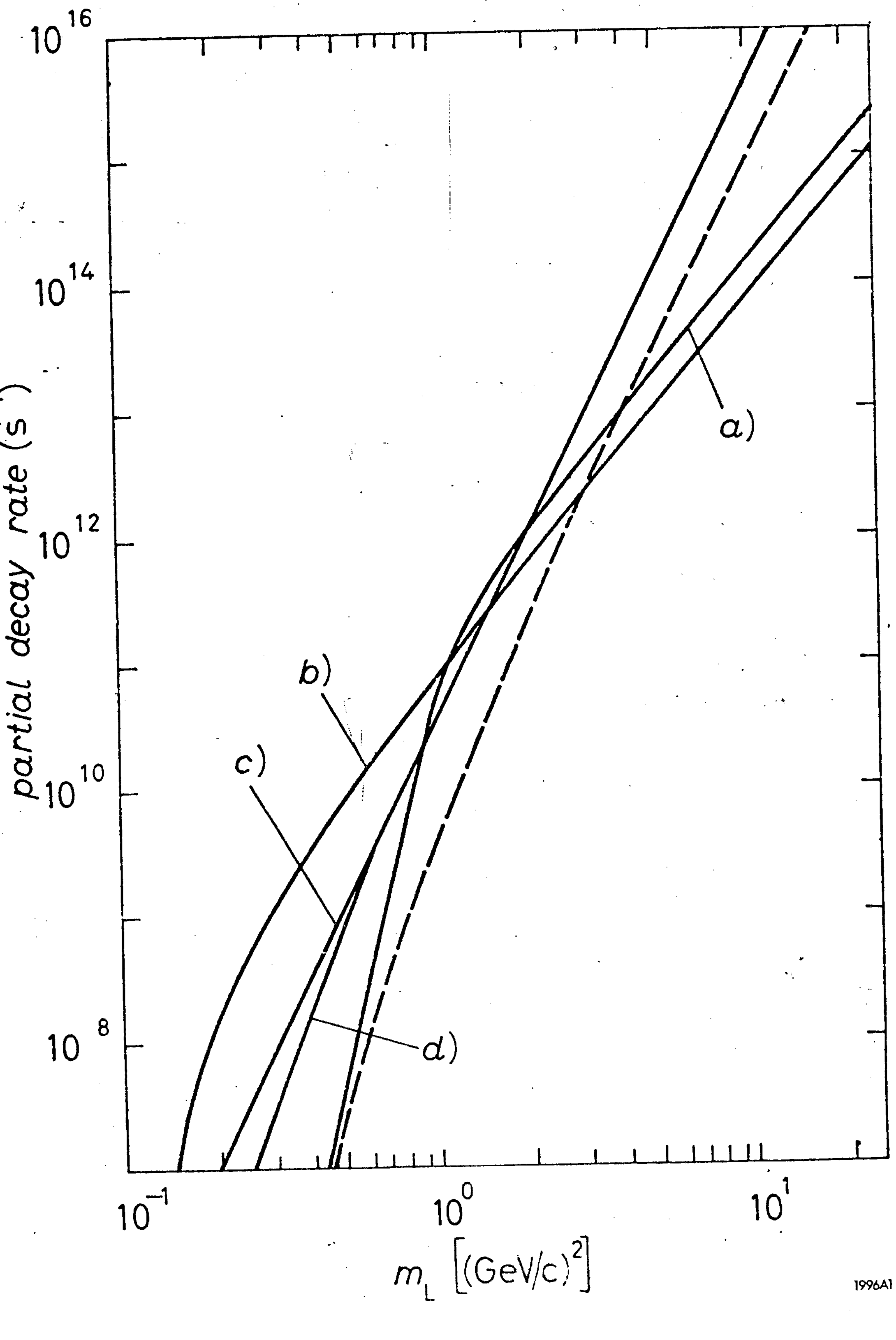
1. Partial decay rates of heavy leptons as a function of the heavy lepton mass, m_L. The solid curves are for the decay modes: a) $\mu'^- \to \pi^- \pi^0 \nu_{\mu'}$ b) $\mu'^- \to \pi^- \nu_{\mu'}$, c) $\mu'^- \to e^- \bar{\nu}_e \nu_{\mu'}$ and d) $\mu'^- \to \mu^- \bar{\nu}_\mu \nu_{\mu'}$. The dashed line is the partial rate for $\mu'^- \to \pi\pi^0 \nu_{\mu'}$ assuming point pions. It is included to show the effect of the pion form factor. This figure is taken from E. W. Beier, Letters Nuovo Cimento 1, 1118(1971).

2. $\rho_{elastic}(q^2)$ is the ratio of the muon-proton elastic differential cross section to the electron-proton elastic differential cross section. The principle of muon-electron universality requires $\rho_{elastic} = 1$ for all values of q^2. The error bars represent only statistical errors; the systematic uncertainties are discussed in the test.

3. For each K interval the upper plot gives the values of $\sigma_{exp,\mu}(q^2, K)$ denoted by a solid circle, $\sigma_{exp,e}(q^2, K)$ denoted by an x and $\sigma_{\gamma p}(K)$ denoted by a triangle. These quantities are defined in the text, but $\sigma_{exp,\mu}$ and $\sigma_{exp,e}$ may be thought of as measures of the respective magnitudes of muon-proton and electron-proton inelastic scattering. q^2 is the square of four-momentum transferred from the lepton. $K = \nu - |q^2|/2M$ where M is the proton mass and ν is the energy lost by the lepton in the laboratory system. The lower plot gives the values of $\rho_{inelastic}(q^2, K) = \sigma_{exp,\mu}(q^2, K)/\sigma_{exp,e}(q^2, K)$. The error bars represent only statistical errors. In most cases the errors in $\sigma_{exp,e}$ are too small to be displayed. The systematic uncertainties are discussed in the text. For the principle of muon-electron universality to be valid $\rho_{inelastic}$ should equal unity for all values of q^2 and K.

4. Contour plots for the parameters N'^2 and $\Lambda_d'^{-2}$ obtained by fitting the experimental values of the ratio $\rho_{inelastic}(q^2, K)$ to the equation

$\rho_{inelastic}(q^2, K) = N'^2/(1.0 + |q^2|/\Lambda_d'^2)^2$. The inner ellipse represents one standard deviation and the outer ellipse represents two standard deviations in the fit.



10^16
10^14
10^12
10^10
10 8
partial decay rate (s^-1)
a)
b)
c)
d)
10^-1
10^0
10^1
m_L [(GeV/c)^2]
1996A1

Fig. 1

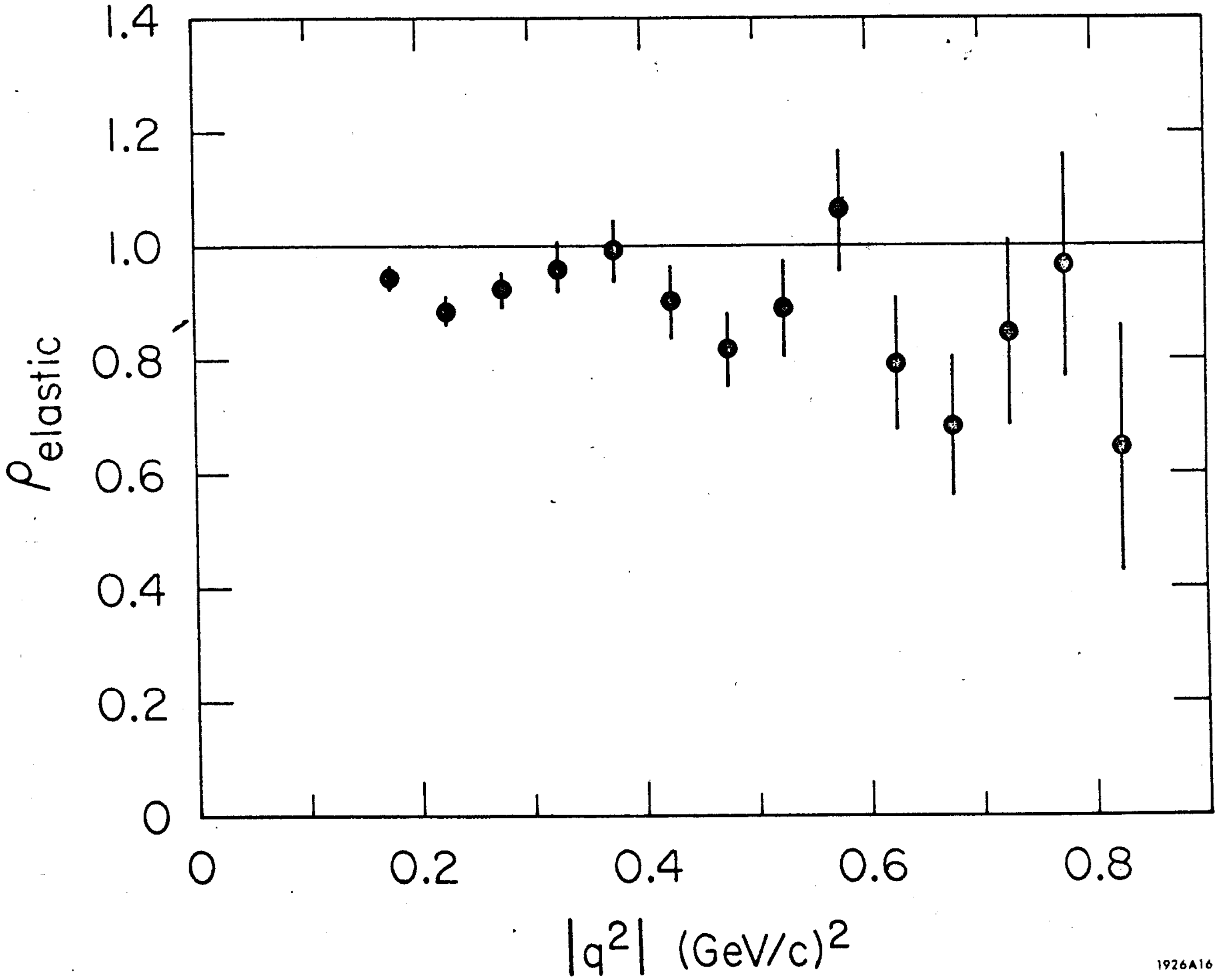
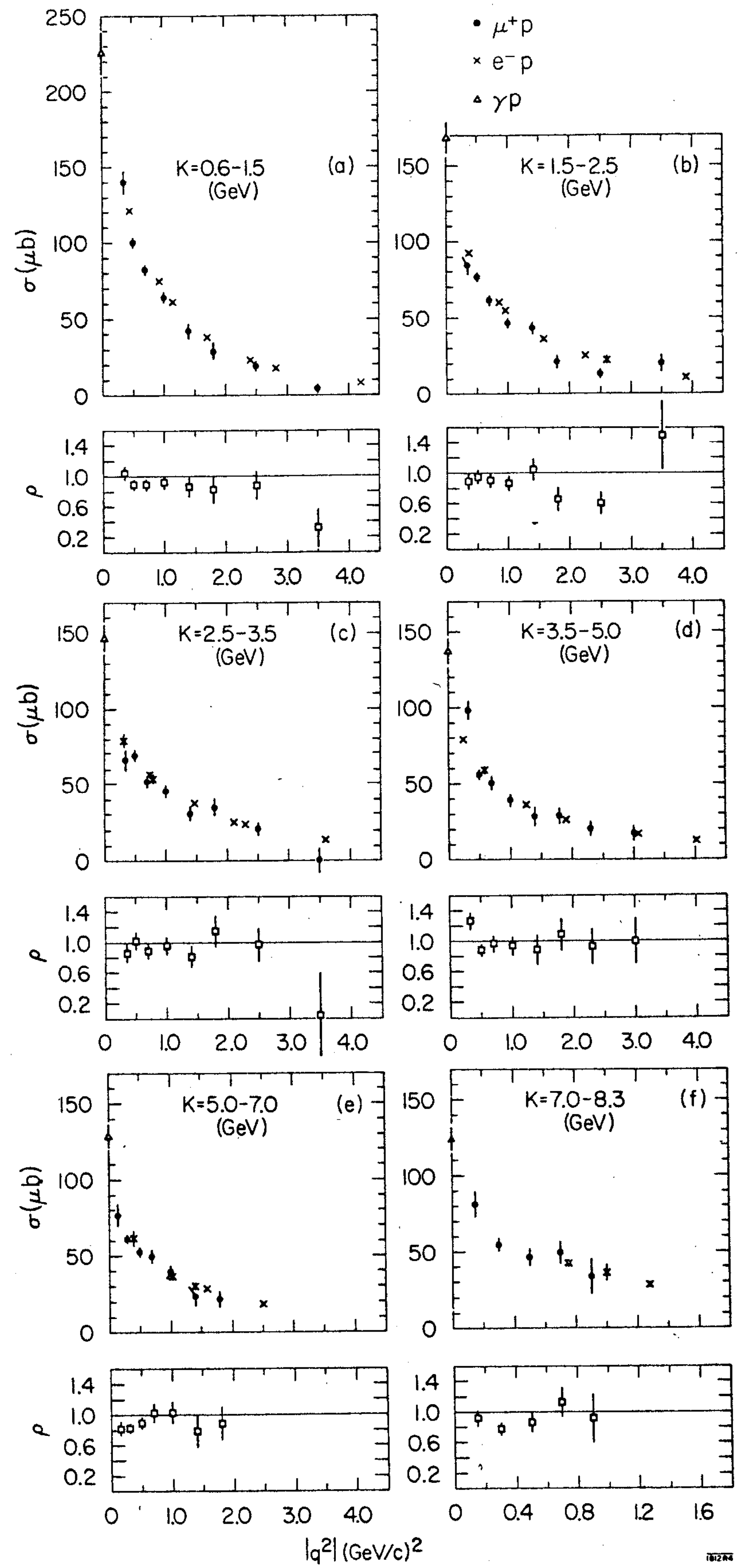


Fig. 2



• μ^+p
× e^-p
▵ γp
K=0.6-1.5 (GeV)
(a)
K=1.5-2.5 (GeV)
(b)
K=2.5-3.5 (GeV)
(c)
K=3.5-5.0 (GeV)
(d)
K=5.0-7.0 (GeV)
(e)
K=7.0-8.3 (GeV)
(f)
$\sigma(\mu b)$
ρ
$|q^2|$ (GeV/c)^2

Fig. 3

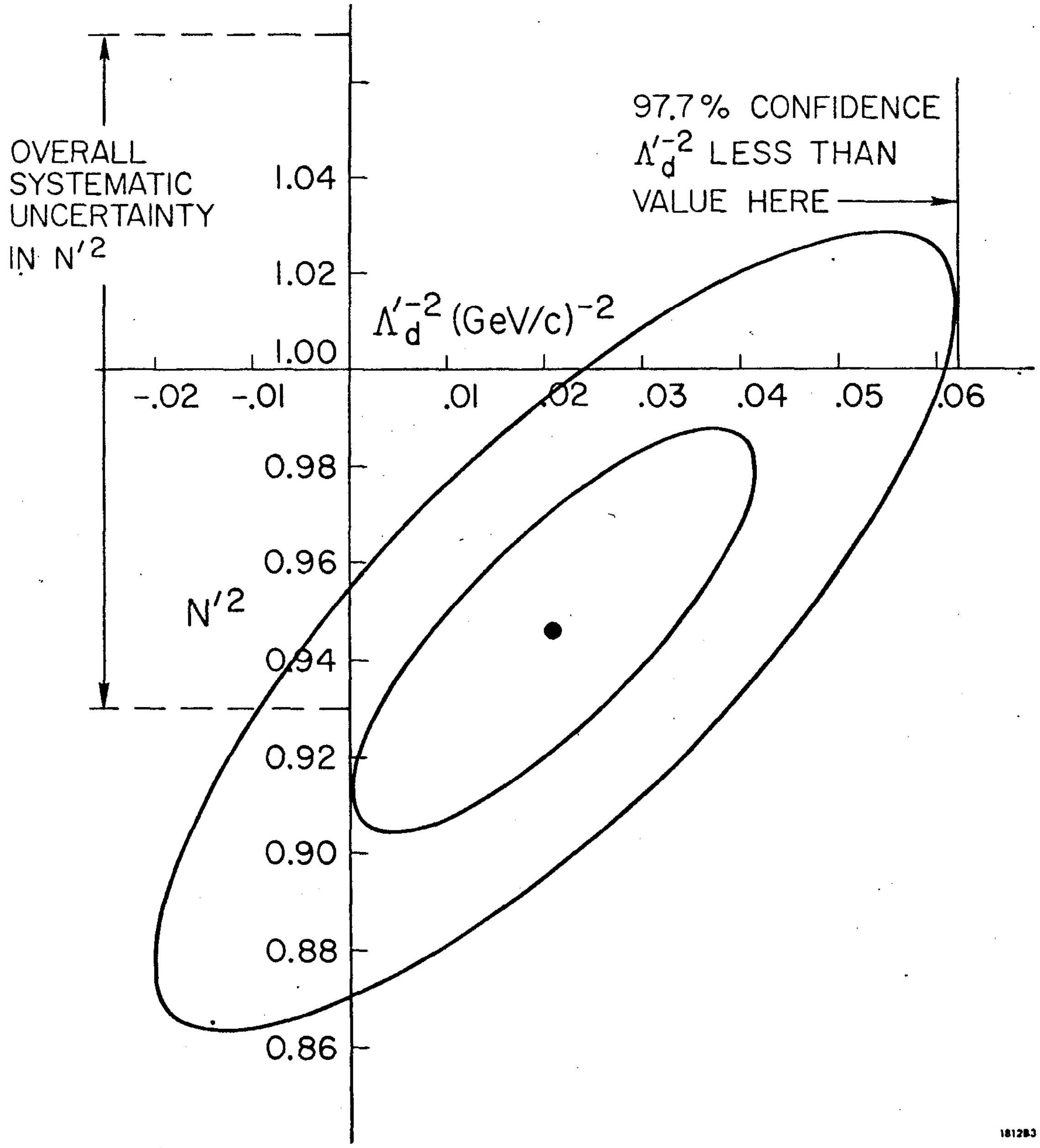

OVERALL SYSTEMATIC UNCERTAINTY IN N′2
97.7% CONFIDENCE
Λ′d-2 LESS THAN
VALUE HERE
1.04
1.02
1.00
Λ′d-2 (GeV/c)-2
-.02
-.01
.01
.02
.03
.04
.05
.06
0.98
0.96
N′2
0.94
0.92
0.90
0.88
0.86
181283

Fig. 4