SIMPLEST DYNAMIC MODELS OF COMPOSITE PARTICLES

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In our talk we wish to give a short review on recent work done by N. N. Bogoliubov and co-workers[1] concerning the study of electromagnetic and weak vertices of mesons and baryons in the simplest relativistic quark model.

As is well known these problems were investigated with success in the non-relativistic quark model. By the help of this model one could obtain some interesting results in connexion with the explanation of magnetic moments of mesons and baryons, radiative and weak decays, etc. It would be interesting to give an answer to the question concerned with the nature of the relativistic correction in the quark model.

Our paper consists of two parts, the first of which treats the semi-relativistic quark model, the second one being devoted to the method of construction of relativistic models.

1. Semi-relativistic model of quasi-independent quarks

In this model quarks in mesons or in baryons are considered as particles moving in a certain self-consistent field.

In this case the individual single-particle wave functions of each quark are meaningful. These functions are described by the Dirac equation for a particle in an external field

$$[\gamma_0 E - i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} - m - V(r)]\psi(r) = 0 \tag{1.1}$$

where m indicates the free quark mass.

We assume that the potential $V(r)$ is spherically symmetric and that the lower energy level is in the S-state. Other properties of the potential are not essential for the model considered. Let us calculate the magnetic moment and the ratio of the axial and vector weak interaction constants of the nucleon.

Note first of all that baryons in this scheme are thought of as a three-quark system in the S-state. It is also required that the baryon wave function should be completely symmetric in spin and unitary spin indices. Owing to this requirement the baryon wave function transforms as the

56-plet of the $SU(6)$ group. Here, unlike the purely group approach, the orbital momentum of the quark is considered.

For the calculation of the magnetic moment of the nucleon the magnetic moment of the quark has to be determined. Postulating the minimal electromagnetic interaction for a free quark we get the following value for a renormalized magnetic moment of the quark

$$\mu_q = \frac{e_q}{2E_0}(1 - \delta) \tag{1.2}$$

Here e_q denotes the charge of the quark; E_0 is the bound state energy, which by definition is equal to one third of the nucleon mass

$$E_0 = \tfrac{1}{3}m_N \tag{1.3}$$

δ is always positive-definite and is equal to the mean value of the L_z component of the orbital momentum in the states with total angular momentum $I_z = \frac{1}{2}$.

This formula is remarkable because the effective quark mass appears in the definition of the magnetic moment. Therefore an enhancement of the magnetic moment occurs in the scalar field.

Having obtained the formula for the magnetic moment of the quark the proton and neutron magnetic moments could be easily calculated. In terms of Bohr magnetons these quantities read

$$\begin{aligned} \mu_p &= 3(1 - \delta) \\ \mu_n &= -2(1 - \delta) \end{aligned} \tag{1.4}$$

Now we pass to calculate the weak interaction constants of the nucleon.

If the axial and vector constants of the free quarks are equal, for the renormalized values of these quantities we have

$$\begin{aligned} g_{\mathrm{V}} &= g_0\tau^+ \\ g_{\mathrm{A}} &= g_0(1 - 2\delta)\tau^+\sigma_z \end{aligned} \tag{1.5}$$

Here g_0 is the nonrenormalized weak interaction constant, while τ and σ are the isotopic and spin Pauli matrices. Hence, the ratio of the vector and axial constants for the nucleon reads

$$G_{\mathrm{A}}/G_{\mathrm{V}} = \tfrac{5}{3}(1 - 2\delta) \tag{1.6}$$

We see that the expression for the magnetic moments and weak interaction constants involves the quantity δ. Note that δ vanishes in the approximate nonrelativistic description when the lower components of the Dirac spinors are neglected. In this case for the proton and neutron magnetic moments we have

$$\mu_p = 3 \qquad \mu_n = -2 \tag{1.7}$$

while for the ratio of the axial and vector constants we obtain the well-known results of the $SU(6)$-symmetry:

$$G_A/G_V = \tfrac{5}{3} \tag{1.8}$$

For a tentative estimate we give here a value of δ which was calculated in a scalar potential well:

$$\begin{aligned} V(r) + m &= 0 \qquad r < r_0 \\ V(r) &= 0 \qquad r > r_0 \end{aligned} \tag{1.9}$$

when the large mass of the quark is completely compensated inside it.

In this case $\delta = 0.17$ and

$$\mu_p = 2.49$$

$$G_A/G_V = 1.1$$

to be compared with experiment

$$\begin{aligned} (\mu_p)_{\text{exp}} &= 2.79 \\ (G_A/G_V)_{\text{exp}} &= 1.18 \end{aligned} \tag{1.10}$$

2. Attempts at relativistic generalization

Now we will be concerned with the problem of finding the vector and axial vertices of composite particle in the relativistic quark model.

Consider first a simplified model of mesons as a bound state of two spinless quarks. To describe this system it is possible to start from the relativistic invariant equation[1]

$$\begin{gathered} [\tfrac{1}{4}p^2 + q^2 - m^2]\varphi_p(q) - \int V_p(q, q')\,\delta(n \cdot q')\varphi_p(q')\,dq' = 0 \\ p \cdot q = 0 \qquad p^2 > 0 \end{gathered} \tag{2.1}$$

Here m is the quark mass, p is the four-momentum of the system, n_μ is the unit four-vector

$$n_\mu = \frac{1}{\sqrt{p^2}}\, p_\mu \tag{2.2}$$

The four-momentum q characterizes the intrinsic motion of the quarks.

The wave functions $\varphi_p(q)$ are normalized by the relativistic-invariant condition

$$\int \varphi_{p'}(q)\varphi_p(q)\,\delta(n\cdot q)\,dq = \delta_{MM'}$$
$$M = \sqrt{p^2} \qquad M' = \sqrt{p'^2} \tag{2.3}$$

In the absence of the interaction $V = 0$ it follows from (2.1) that for each quark the Klein–Gordon equation holds.

In the centre-of-mass system $\mathbf{p} = 0$ Eqn. (2.1) looks like

$$[\tfrac{1}{4}E^2 - \mathbf{q}^2 - m^2]\varphi_E(\mathbf{q}) - \int V_E(\mathbf{q}, \mathbf{q}')\varphi_E(\mathbf{q}')\,d\mathbf{q}' = 0 \tag{2.4}$$

In this case the normalization condition reads

$$\int \varphi_{E'}(\mathbf{q})\varphi_E(\mathbf{q})\,d\mathbf{q} = \delta_{EE'} \tag{2.5}$$

As we can see the wave function φ_E in the centre-of-mass system does not depend on the relative energy q_0 or in the x-space it does not depend on the relative time t.

Later on, to determine in quantum field theory the vertex of the interacting quarks we use the following approach. Consider the one-time two-particle Bethe-Salpeter amplitude

$$\tilde{\varphi}_p(x, y) = \langle 0|\,\varphi(x)\varphi(y)\,|p\rangle_{x_0=y_0} = e^{ip(x+y/2)}\tilde{\varphi}_p(\mathbf{x} - \mathbf{y})\,\big|_{x_0=y_0} \tag{2.6}$$

$\varphi(x)$ denotes the second-quantized Heisenberg operator for the scalar field. It was shown[1] that the Fourier-transform of one-time wave function $\tilde{\varphi}_p(\mathbf{q})$ satisfies in the centre-of-mass system an equation of type (2.4) for the definite potential $\tilde{V}_E(\mathbf{q}, \mathbf{q}')$.

It is essential to emphasize that there is a method of constructing this potential by means of Feynman graphs of the two-particle Green function.

Indeed, for the one-time wave function we have

$$\int d\mathbf{q}'\tilde{G}_E^{-1}(\mathbf{q}, \mathbf{q}')\varphi_E(\mathbf{q}') = 0 \tag{2.7}$$

where the operator $\tilde{G}$ is the two-time four-particle Green function in the centre-of-mass system. By definition

$$\tilde{G}_E(\mathbf{q}, \mathbf{q}') = \int dq_0\,dq_0'G_E(q, q') \tag{2.8}$$

Expanding the Green function as follows

$$G_i = G_0 + G_0KG_0 + \cdots \tag{2.9}$$

where G_0 denotes a free Green function of two particles, and K is the Bethe-Salpeter kernel, for the inverse operator of the two-time Green function the relation is valid:

$$\tilde{G}^{-1} = \tilde{G}_0^{-1} + \tilde{G}_0^{-1}\widetilde{G_0KG_0}\tilde{G}_0^{-1} + \cdots. \tag{2.10}$$

The tilde denotes the integration as in (2.8).

Calculating $\tilde{G}_0$ in the centre-of-mass system we have

$$[\tilde{G}_0(\mathbf{q}, \mathbf{q}')]^{-1} = \frac{\pi}{i} \frac{\delta(\mathbf{q} - \mathbf{q}')}{\sqrt{\mathbf{q}^2 + m^2}} \cdot [\tfrac{1}{4}E^2 - \mathbf{q}^2 - m^2] \tag{2.11}$$

Hence, for the wave function $\tilde{\varphi}_E$, we get the equation

$$[\tfrac{1}{4}E^2 - \mathbf{q}^2 - m^2]\tilde{\varphi}_E(\mathbf{q}) - \frac{1}{\sqrt{\mathbf{q}^2 + m^2}} \cdot \int V_E(\mathbf{q}, \mathbf{q}')\tilde{\varphi}_E(\mathbf{q}')\, d\mathbf{q}' = 0 \tag{2.12}$$

The quasi-potential $\tilde{V}_E$ is given by the expansion

$$\tilde{V}_E = \tilde{G}_0^{-1}\widetilde{G_0KG_0}\tilde{G}_0^{-1} \tag{2.13}$$

In the framework of perturbation theory it is possible to check that the spectrum of bound states and the scattering matrix obtained with the help of Eqn. (2.12) coincides with similar quantities calculated on the basis of the Bethe–Salpeter equation.

Note that Eqn. (2.1) may be considered as a relativistic generalization of (2.12), provided that the following condition is fulfilled:

$$V_E(\mathbf{q}, \mathbf{q}') = \frac{1}{\sqrt{m^2 + \mathbf{q}^2}} \cdot \tilde{V}_E(\mathbf{q}, \mathbf{q}') \tag{2.14}$$

Let us now proceed to the case when the quarks have spin $\frac{1}{2}$. Here the quasipotential equation for the meson is written as

$$(\gamma_0{}^{(1,2)}E - \hat{M}_E)\psi = 0 \tag{2.15}$$

where $\hat{M}_E$ denotes the mass operator of the two interacting quarks.

$$\hat{M}_E\psi = 2\sqrt{m^2 + \mathbf{q}^2}\, \psi_E(\mathbf{q}) + \int \tilde{V}_E(\mathbf{q}, \mathbf{q}')\varphi_E(\mathbf{q}')\, d\mathbf{q}' \tag{2.16}$$

The normalization condition is

$$\int d\mathbf{q}'\,\psi_{E'}(\mathbf{q}')\psi_E(\mathbf{q}') = \delta_{EE'} \tag{2.17}$$

In finding the vertex functions of the bound states in terms of quasi-potential wave functions one faces difficulties. In the presence of the external fields the momentum of the initial and final states are not, generally speaking, conserved. So it is impossible to define the rest system. Therefore, we restrict ourselves to constructing the vector and axial charges, that is, the vertex functions for the zero momentum transfer. For this purpose it is convenient to use the following method.

Let us switch on the external field which does not carry energy and momentum, but can carry other quantum numbers. Such fields were treated by Wentzel and they are usually called spurion fields. The interaction of quarks with spurion fields are chosen as

$$\Gamma^{(i)} = \begin{cases} \gamma_\mu^{(i)}\lambda_\alpha^{(i)}a^{\mu_\alpha} & \text{for vector spurions} \\ \gamma_5^{(i)}\gamma_\mu^{(i)}\lambda_\alpha^{(i)}a_5^{\mu_\alpha} & \text{for axial spurions} \end{cases} \tag{2.18}$$

a^{μ_α}, $a_5^{\mu_\alpha}$ denote the constant C-numbers which define the amplitudes of the vector and axial spurion fields with quantum numbers of the unitary octet.

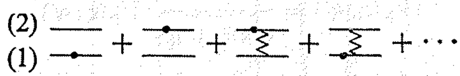
Then the quasipotential equation in the spurion field takes the following form

$$[\gamma_0^{(1,2)}E - \hat{M} + \delta\hat{M}]\psi = 0 \tag{2.19}$$

where $\hat{M}$ is the mass operator in the centre-of-mass system. The operator $\delta\hat{M}$ represents the renormalization of the bound state mass in the spurion field. It can be found by the perturbation theory expansion:

$$\delta\hat{M} = \overline{G}^{-1}\,\overline{\delta G_\alpha}\,\overline{G}^{-1} \tag{2.20}$$

Here the operator δG_α denotes the first variation of the two-particle Green function $G_p(q, q'; a)$ in the external spurion field and is equal to a sum of the following diagrams



that is, diagrams for the two-particle Green function with one spurion vertex. The upper sign in this formula denotes

$$\overline{\delta G}_E(\mathbf{q}, \mathbf{q}'; a) = \int dq_0\, dq_0' G_E^F(q, q') \tag{2.21}$$

where the integrand is taken in the Foldy–Wouthysen representation. On the basis of Eqn. (2.19) for the effective charges of the composite particle we obtain the following formula

$$\int \psi_E(\mathbf{q})\,\delta\hat{M}\psi_E(\mathbf{q})\,d\mathbf{q} = Q_{\mu_\alpha}a^{\mu_\alpha} + Q_{5\mu_\alpha}a_5^{\mu_\alpha} \tag{2.22}$$

Note that the explicit form of the charges depends essentially on the form of the mass operator $\hat{M}$.

When there is no interaction between quarks it is possible to get the following expression for the vector and axial charges

$$\langle 0| Q_\alpha |0\rangle = \int \psi_E(\mathbf{q})[\lambda_\alpha^{(1)} + \lambda_\alpha^{(2)}]\psi_E(\mathbf{q})\, d\mathbf{q}$$
$$\langle 0| Q_{5\alpha} |0\rangle = \int \psi_E(\mathbf{q})[\lambda_\alpha^{(1)}\Delta_z^{(1)} + \lambda_\alpha^{(2)}\Delta_z^{(2)}]\psi_E(\mathbf{q})\, d\mathbf{q} \tag{2.23}$$

where

$$\Delta_z^{(i)} = \sigma_z^{(i)} \frac{m}{\sqrt{m^2 + \mathbf{q}^2}} + q_z \frac{\boldsymbol{\sigma}^{(i)} \cdot \mathbf{q}}{\sqrt{m^2 + \mathbf{q}^2}(\sqrt{m^2 + \mathbf{q}^2} + m)}$$

One can see from this formula that the vector charge of two non-interacting quarks is not renormalized and is equal to a sum of charge of individual quarks.

The axial charge of a two-quark system is, however, renormalized even in the absence of the interaction between them. For example, in the case of spherical symmetric wave functions ψ from (2.23) we have

$$\langle 0| Q_{5\alpha} |0\rangle = (\lambda_\alpha^{(1)}\sigma_z^{(1)} + \lambda_\alpha^{(2)}\sigma_z^{(2)})\left(1 - \frac{\langle \mathbf{q}^2\rangle}{3m^2} + \cdots\right) \tag{2.24}$$

Now we compare this expression with the renormalized values of the axial charges of the quarks which were calculated earlier in the quasi-independent quark model. Expanding δ in powers of $1/m$ gives

$$\delta = \frac{\langle \mathbf{q}^2\rangle}{6m^2} + \cdots \tag{2.25}$$

It can be easily seen that the first-order calculated values for the renormalized axial constants for both models coincide. The point is that in the expansion (2.25) the first term does not depend on the interaction potential and is only due to the relativistic effects.

The method described is closely connected with the programme suggested by Gell-Mann for constructing representations of the local algebra of currents by means of the relativistic quark model[2].

To describe the meson Gell-Mann[2] uses the Yukawa–Markov type equation of the form

$$(\hat{p} - \hat{M})\psi = 0 \tag{2.26}$$

with the renormalization condition for the wave function

$$\int \bar{\bar{\psi}}_{p'}(q)\psi_p(q)\, dq = \delta_{MM'}$$

Eqn. (2.26) is an analogue of Eqn. (2.1) if quarks have spin $\frac{1}{2}$. From Lorentz-invariance the most general form of the matrix element of the current is written as

$$\mathscr{F}_{\alpha\mu}(p, p') \sim \int dq\, dq' \sqrt{\delta(p \cdot q)\, \delta(p' \cdot q')}\, \bar{\bar{\psi}}_{p'}(q') G_{\alpha\mu} \psi_p(q) \qquad (2.27)$$

where $\hat{G}_{\mu\alpha}$ is a sum over $\lambda_\alpha^{(1)}$ and $\lambda_\alpha^{(2)}$ times all possible independent vector operators such as $\gamma_\mu^{(1)}$, $\gamma_\mu^{(2)}$ etc, times Lorentz-invariant functions

$$F(q \cdot p', p \cdot q' \cdots q^2, q'^2)$$

A requirement is imposed on the invariant functions so that the currents thus determined should satisfy the algebra in the system $p_z \rightarrow \infty$. Such a possibility in the model of non-interacting quarks was illustrated in Gell-Mann's lectures.

In conclusion we point out that there exists a relationship between the relativistic quark model and the group theoretical approach using the infinite component wave equations.

If we assume that the interaction between quarks takes place by exchange of scalar massless particles then in the instantaneous interaction Eqn. (2.12) becomes a Schrödinger equation in the Coulomb field.

As is known all the solutions of this equation form the basis of one unitary infinite-dimensional representation of the group O (4.1).

Therefore the problem of determining the energy levels of the bound states can be reduced to finding unitary representation of some non-compact group. At present there exists no method for constructing the potential. Therefore, by analogy with the Coulomb problem, attempts are being made to describe the bound states of the system by means of infinite-component wave equations, that is, the problem of determining the potential is replaced by that of finding the symmetry group. The substitution of dynamics by group theory has been one of the recent trends in elementary particle physics. Having seen a general qualitative success of the quark model of mesons and baryons, one wants a more qualitative, but simple and unified description of hadron phenomena without assuming detailed dynamical mechanisms.

References

(1) A. Logunov and A. Tavkhelidze, *Nuovo Cimento*, **29**, (1963).
(2) M. Gell-Mann, Lectures at the International School of Physics 'Ettore Majorana', Erice, Sicily (1966).

Discussion on the report of A. N. Tavkhelidze

H. P. Dürr. There is a point which I have never understood in the dynamical treatment of the quarks: whether they are fermions or bosons. If one succeeds in some way or another in describing the nucleon as a three quark bound state by introducing, for example, a scalar interaction, as you have done, isn't the crucial question here whether the four, five and particularly the six quark state (the deuteron?) etc. have a binding energy compatible with experiment?

A. N. Tavkhelidze. It is obvious that the 3 quark state is of special importance since 2, 4 or 5 quark states have not been observed. The deuteron should be considered most likely to be a system consisting of two 3 quark subsystems. From the dynamical point of view, it is difficult to explain the exceptional role of the 3 quark system.

L. A. Radicati. (1) Did you obtain a particular representation of Gell-Mann's local algebra or a class of representations?

(2) The second question is not specifically directed to you but to anybody in the audience who knows how to answer. Is it obvious that a representation of Gell-Mann's algebra should coincide with a representation of a non-compact group? Is this a general property?

A. N. Tavkhelidze. (1) We have obtained only the simplest representation of the local algebra. The problem of the existence of the representations of other types has not been considered.

(2) The fact that the representation of the local algebra coincides with the representation of the non-compact group is, in may opinion, very interesting and surprising. At least, I do not know any mathematical theorems which would explain this coincidence.

F. E. Low. In the Hartree field model $\Sigma\varepsilon_i \neq E$; one has to add in the potential. Therefore, in your one particle theory, one must explicitly add the potential, which means that E_0 again involves the quark mass. Therefore it is hard to take the specific numerical results seriously.

A. N. Tavkhelidze. Starting from the qualitative arguments about the nature of the forces which bind quarks, and using the independent quark model, it is possible to calculate such quantities as magnetic moments, axial constants and the effective masses of quarks. Collective effects can be taken into account by means of perturbation theory. It is obvious that the equations which describe the residual interactions of bounded independent quarks must include the quark effective mass m_{eff} and not the large real mass M. In any case the calculations performed by means of the

Bethe–Salpeter equation would be extremely useful if we know better the nature of the forces acting between quarks.

W. Heisenberg. What are the symmetry properties of the quarks in your model? Since you assume the spin $\frac{1}{2}$ for the quarks, one should consider Fermi statistics. On the other hand you spoke about the wave function being symmetrical in the quarks. What are your assumptions?

A. N. Tavkhelidze. The totally symmetric spin-unitary spin wave function gives a good description of the main experimental properties of baryons. However, if quarks are fermions such a choice contradicts the Pauli principle. To avoid this difficulty attempts were made to introduce a totally antisymmetric wave function of three quarks in the *s*-state or to ascribe to quarks the parastatistics. There is also a possibility of introducing instead of one triplet of fractionally charged quarks three triplets of integer charged quarks, the main consequences of the $SU(6)$ symmetry being unaffected.