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Gravitational waves in hybrid quintessential inflationary models

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Abstract. The generation of primordial gravitational waves is investigated within the hybrid quintessential inflationary model. Using the method of continuous Bogoliubov coefficients, we calculate the full gravitational-wave energy spectrum. The post-inflationary kination period, characteristic of quintessential inflationary models, leaves a clear signature on the spectrum, namely, a sharp rise of the gravitational-wave spectral energy density $\Omega_{\rm GW}$ at high frequencies. For appropriate values of the parameters of the model, $\Omega_{\rm GW}$ can be as high as 10^{-12} in the MHz–GHz range of frequencies.

In this article, we investigate the generation of gravitational waves within the hybrid quintessential inflationary model [1]. Gravitational waves of cosmological origin [2] are of the utmost importance, since they have the potential of providing us with unique information about the very early Universe, not directly available by any other means. In particular, the detection of these waves would allow for a better understanding of several issues relevant both for cosmology and high-energy physics, like inflation, preheating and reheating mechanisms, post-inflationary phase transitions, topological defects of grand unified theories and string theory.

The hybrid quintessential inflationary model, proposed recently by Bastero-Gil *et al.* [3], is a modification of the original quintessential inflationary model of Peebles and Vilenkin [4]. In the original model, the unification of inflation and dark energy within a single framework is achieved with a scalar field ϕ playing both roles of inflaton and quintessence. Since, in such model, the potential of the scalar field ϕ has no minimum, reheating cannot proceed in the usual way, namely, by the decay of the scalar field into quanta of other fields. Instead, reheating proceeds by gravitational particle production, a quite inefficient mechanism which may lead to cosmological problems associated with large isocurvature fluctuations and overproduction of gravitinos and moduli fields [5]. Within the hybrid model, the usual reheating mechanism is recovered by introducing another scalar field χ, coupled to the original inflaton/quintessence field ϕ, with a hybrid-like potential [3]

$$U(\phi,\chi)=\frac{1}{2}g^2\chi^2(\phi^2-m^2)+\frac{1}{4}\lambda_\chi\chi^4+\left\{\begin{array}{lll}\lambda_\phi(\phi^4+M^4), & \text{for} & \phi<0,\\ \lambda_\phi M^8(\phi^4+M^4)^{-1}, & \text{for} & \phi\geqslant 0,\end{array}\right. , \qquad (1)$$

where g, m, λ_χ, λ_ϕ and M are constants. Recent measurements of the cosmic microwave background radiation and large-scale structure constrain the value of λ_ϕ to be of the order of

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10^{-13} [6], while agreement with the measured value of today's dark-energy density [7] requires M to be of the order of $10^{-14}\, m_{\rm p}$. The requirement that the scalar field χ responds quickly enough to changes in the potential $U(\phi,\chi)$ and that it does not influence significantly the evolution of the inflaton/quintessence field ϕ imposes restrictions on the values of g, m and λ_χ, namely,

$$g\left(\frac{m}{m_{\rm p}}\right)^2 \gtrsim 0.2\,\lambda_\phi^{1/2} \quad \text{and} \quad \frac{g^4}{\lambda_\chi}\left(\frac{m}{m_{\rm p}}\right)^4 \lesssim 0.001\,\lambda_\phi, \tag{2}$$

where $m \ll m_{\rm p}$ [1, 3].

For $|\phi| \geqslant m$ the potential $U(\phi,\chi)$ has just one minimum at $\chi = 0$, while for $|\phi| < m$ it has two minima located at $\chi = \pm\sqrt{g^2(m^2-\phi^2)/\lambda_\chi}$.

Within the hybrid quintessential inflationary model the evolution of the Universe can be divided into four stages. During the first stage ($\phi \leqslant -m$), the potential energy of the scalar field ϕ dominates the evolution, yielding a period of chaotic inflation, while the field χ remains at rest at the minimum of the potential U, $\chi_{\rm min} = 0$. Enough inflation is guaranteed by choosing, as initial condition, $\phi_i \lesssim -5m_{\rm p}$. Inflation is followed by a period of kination, during which the scalar field ϕ behaves as stiff matter with equation of state $p = \rho$. During the second stage of evolution, which takes place for $-m < \phi < m$, the origin of the potential becomes unstable for the scalar field χ. As a result, this field rolls down toward one of the new minima of the potential, $\chi_{\rm min} = \pm\sqrt{g^2(m^2-\phi^2)/\lambda_\chi}$, and starts to oscillate around it, acquiring kinetic energy. The Universe continues to undergo a period of kination. During the third stage of evolution ($\phi \geqslant m$), the scalar field χ returns to the minimum of the potential, located at $\chi = 0$, and oscillates around it. The transference of energy from this field to a radiation fluid is achieved by the introduction of a phenomenological dissipative coupling, proportional to the mass of χ, namely [1, 8],

$$\Gamma_\chi = \mu g\sqrt{\phi^2 - m^2}. \tag{3}$$

In the above expression, μ is the proportionality constant, which is constrained to be [1, 3]

$$0.2\,\lambda_\phi^{1/6}\left(\frac{m_{\rm p}}{m}\right)^{2/3}\frac{g^{5/3}}{\lambda_\chi} \ll \mu < \mu_{\rm crit}, \tag{4}$$

where $\mu_{\rm crit}$ is a critical value above which the oscillatory motion of the scalar field χ becomes over damped. The scalar field χ is assumed to decay away completely during this stage of evolution, reheating the Universe to a temperature of the order of 10^{14} GeV. Gradually, kination gives place to a radiation-dominated Universe. Finally, during the fourth stage of evolution, the Universe undergoes, successively, a period of radiation-domination, matter-domination and accelerated expansion, the latter being induced by the field ϕ, which also played the role of inflaton in the first stage of evolution.

Integrating numerically the equations of motion, corresponding to the above mentioned stages of evolution, with appropriate initial conditions and for the allowed values of the parameters of the model, one obtains the scale factor a and its first and second time derivatives as functions of time (see, for more details, Ref. [1]). The knowledge of these quantities, from the beginning of the inflationary period to the present time, allows us to determine the full gravitational-wave energy spectrum, i.e., the gravitational-wave spectral energy density parameter, $\Omega_{\rm GW}$, as a function of today's value of the angular frequency of the gravitational waves, ω_0. The method used to obtain this spectrum is the method of continuous Bogoliubov coefficients, first applied to particle production in an expanding Universe [9] and then extended to the case of gravitons [10]. For applications of this method to several cosmological models see Ref. [11].

The gravitational-wave spectral energy density parameter is given by

$$\Omega_{\rm GW} = \frac{8\hbar G}{3\pi c^5 H_0^2}\omega_0^4\beta_0^2, \tag{5}$$

where $\hbar$ is the reduced Planck constant, G is the Newton constant, c is the speed of light, H is the Hubble parameter and β is a Bogoliubov coefficient. The subscript "0" denotes quantities evaluated at the present time. The squared Bogoliubov coefficient β^2, which gives the number of gravitons, can be expressed as $\beta^2 = (X-Y)^2/4$, where $X(t)$ and $Y(t)$ are functions of time, determined from the system of differential equations

$$\dot{X} = -i\omega_0\frac{a_0}{a}Y, \tag{6}$$

$$\dot{Y} = -\frac{i}{\omega_0}\frac{a}{a_0}\left[\omega_0^2\left(\frac{a_0}{a}\right)^2 - \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2\right]X. \tag{7}$$

In the above equations, a_0 is today's value of the scale factor and a dot denotes a derivative with respect to the cosmic time t.

The gravitational-wave spectrum for the hybrid quintessential inflationary model, obtained applying the above mentioned procedure, is shown in Fig. 1. The sharp rise of the gravitational-wave spectral energy density parameter $\Omega_{\rm GW}$ in the high-frequency region of the spectrum is due to the existence of a kination period, occurring between the end of the inflationary era and the beginning of the radiation-dominated era. This peak, which is firmly located in the MHz–GHz range of frequencies, is an unavoidable feature of quintessential inflationary models [12].

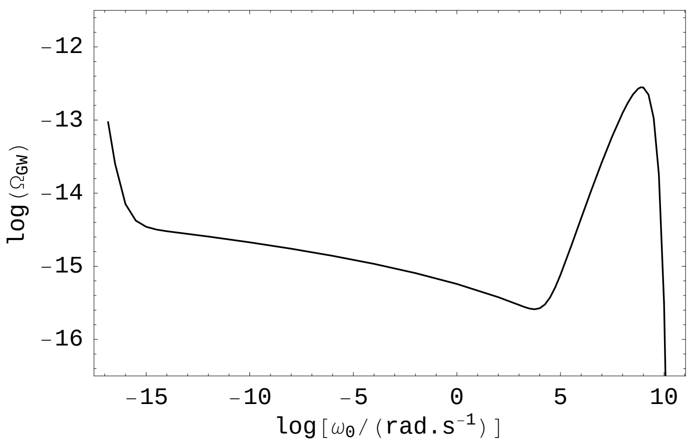

Figure 1. Gravitational-wave spectrum for $\lambda_\phi = 10^{-13}$, $M = 3.16\times10^{-14}\,m_{\rm p}$; $g = 6.3\times10^{-4}$, $m = 0.01\,m_{\rm p}$, $\lambda_\chi = 1$ and $\mu = 0.1$.

The duration of the kination period, and hence the height of the high-frequency peak, depends on the values of the parameters of the hybrid-like potential g, m and λ_χ, as well as on the value of the dissipation parameter μ [1]. Namely, if g and m increase (λ_χ decreases), the kination period becomes shorter and the peak smaller. This can be understood as follows. Let us consider, for example, the parameter g. An increase of g (for fixed values of the other parameters) leads to an increase of the value of χ at the minimum of the potential during the second stage of evolution. This implies that more energy is acquired by this field as it oscillates around the minimum of the potential. Consequently, more energy is available to be transferred, during the third stage of evolution, from the scalar field χ to the radiation fluid, leading to a higher reheating temperature. This, in turn, implies that, after the complete decay of χ, the period of time required for the energy density of radiation to become greater than the kinetic energy of the scalar field ϕ is shorter. As a result, the radiation-dominated era begins earlier, the kination period is shorter and the high-frequency peak of the spectrum is smaller. For the dissipation parameter μ, the duration of the kination period, as well as the height of the peak, increases as μ increases. Indeed, for higher values of μ, the energy transfer from the scalar field χ to the radiation fluid proceeds faster. As a consequence, the energy density of radiation begins to decrease as a^{-4} sooner, implying that the energy density of radiation is smaller after the complete decay of the field χ. Therefore, the time it takes for this energy density to dominate the dynamics of the Universe is longer, implying that the kination period is also longer and the peak bigger.

For the allowed values of the parameters of the model, the high-frequency peak of the gravitational-wave spectrum can be as high as $\Omega_{\rm GW} \simeq 10^{-12}$. Microwave-cavity and interferometric detectors, presently under development [13], are far from being sensitive enough to detect the primordial gravitational waves predicted by the hybrid quintessential inflationary model. However, one can hope that, in a not-so-distant future, a detection would be possible, allowing us to dramatically improve our understanding about processes taking place in the very early Universe.

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