ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-THPL060

IMPACT OF DIPOLE COMPONENT CHANGE ON QUADRUPOLE BEAM-BASED ALIGNMENT ACCURACY FOR CIRCULAR ACCELERATORS*

Y. Hidaka†, J. Choi, G. Wang, Brookhaven National Laboratory, Upton, USA

Abstract

Beam-based alignment (BBA) for quadrupoles is a routine process for circular accelerators to steer beam orbit through the magnetic centers such that the orbit is unperturbed when the strengths of quadrupoles are varied. The random errors associated with BBA are well known, but a type of systematic error appears to be neglected by the community. A standard measurement procedure involves variation of the quadrupole gradient. This systematic error is introduced when there is a non-zero dipole component after quadrupole strength is changed. This dipole component can be also interpreted as a shift in the magnetic center. The analytical formulas for this error and its amplification factor with respect to the magnetic center motion have been derived and confirmed with simulations. We demonstrate the significance of this error, potentially on the order of hundreds of microns, through both simulations and recent experimental results at NSLS-II. In addition, a special term in this error that is not extractable from orbit measurements alone will be discussed in detail.

INTRODUCTION

Beam-based alignment (BBA) for accelerators is a well-established process in which electron beam is steered to pass through the centers of quadrupoles. This alignment is performed first during commissioning of an accelerator, and repeated afterwards as needed, whenever the electronics of beam position monitors (BPMs) are modified, or accelerator components such as BPMs and quadrupoles are physically moved whether intentionally or not. There are many varieties of BBA techniques, good summaries of which can be found in [1, 2]. Both model-dependent and model-independent approaches are available. The latter appears to be more commonly used when BPMs are located close to independently-powered quadrupoles, as they do not require precise knowledge on the actual accelerator lattice. When those favorable conditions do not exist, the first approach is taken.

In this paper, we focus on the model-independent BBA techniques such as the one implemented at ALS [3]. What is common to all these types is that they are all "nulling" techniques. The goal is to move around the beam orbit until we find a beam position at a quadrupole such that the orbit change with variation of quadrupole strength is minimized. The main topic of this paper is the systematic error in the center estimates these methods provide, when a change in the dipole component of the quadrupole, or equivalently, a motion of the quadrupole center, accompanies its gradient change.

* Work supported by U.S. DOE under Contract No. DE-SC0012704.
† yhidaka@bnl.gov

This systematic error was first noticed and reported by ALS in [4]. Their experimental BBA estimates varied significantly for different amounts of gradient change. This was explained by a simple hypothesis that the dipole component was changing nonlinearly with respect to its gradient. The source of this nonlinear change was attributed to the asymmetry of C-shaped magnets common to light sources that require photon beam extraction. However, this report appeared to have been largely unnoticed by the accelerator community roughly for the last few decades. In [1], the authors discussed and derived a formula for this systematic error, but were seemingly unaware of [4]. Reasons for this may include that they only analyzed in terms of center motion, instead of dipole component variation, and more importantly limited the discussions only to linear accelerators (linacs), while the earlier report was for a storage ring. It was not obvious whether the same formula would apply to circular accelerators, as their derivation involved the trajectory response matrix for a transport line, not the closed orbit response matrix for a ring. As a result, the relevance and impact of the formula in [1] have not been recognized by the circular accelerator community. We will mainly use the so-called "bow-tie" method (an example data shown in Fig. 1) to demonstrate how serious this systematic error can be.

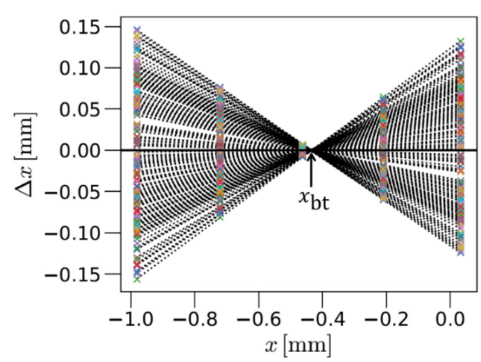

Figure 1: A typical bow-tie plot. From each BPM, a set of difference orbit values Δx are obtained by changing quadrupole strength. Each cluster of points at 5 discrete x values corresponds to a different swing corrector setpoint (Different colors indicate different BPMs). Linear fitting to each set generates a line for each BPM. The horizontal value of the zero-crossing point is x_{bt}.

SYSTEMATIC ERRORS OF BOW-TIE MEASUREMENTS

We derive the formula for the systematic error in the bow-tie estimates in the presence of dipole component change when the strength of a quadrupole is changed. We will refer to this error as SED (Systematic Error induced by Dipole component change) for bow-tie measurements. There are other sources of systematic errors such as large

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orbit angles with respect to the magnet central axis and large distances between the BPM and the quadrupole being measured. But we will ignore them in this paper.

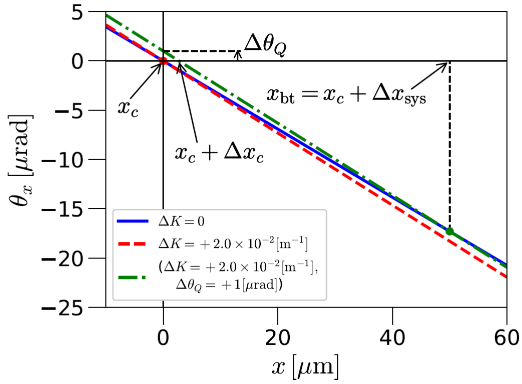

Figure 2: Kicks from a quadrupole at a reference state Q_r (blue solid), and at a gradient-modified state Q_m without (red dash) and with (green dash-dot) dipole component change $\Delta\theta_Q$.

We assume the quadrupole whose center is being searched has zero length. The center of a thin quadrupole is defined to be the magnetic center where $B_y = 0$ (we only consider the horizontal plane, but the vertical case is the same except the change in sign for K and ΔK). For a quadrupole with the integrated strength of K_r at a reference state Q_r, the horizontal kick θ_x from it when displaced can be expressed as:

$$\theta_x = -K_r \cdot (x - x_c), \tag{1}$$

if $B_y = 0$ (and hence $\theta_x = 0$) at $x = x_c$, where x_c is the true magnetic center we aim to extract from the BBA measurement. This line is shown as the blue solid line in Fig. 2. If we modify the quadrupole from the reference state to another state Q_m, with its integrated strength changed by ΔK, but assume there is no dipole component change yet, the blue line moves to the red dashed line in Fig. 2:

$$\theta_x = -K_m \cdot (x - x_c), \tag{2}$$

where $K_m = K_r + \Delta K$. Note that θ_x is still zero at $x = x_c$. Thus, the magnetic center has not moved in this case, and the blue and red lines still cross at x_c.

Now suppose this state transition also induces a dipole component change $\Delta\theta_Q$. The red line is simply shifted vertically up to the green dash-dot line as shown in Fig. 2, and Eq. (2) changes into:

$$\theta_x = -K_m \cdot (x - x_c) + \Delta\theta_Q. \tag{3}$$

Notice that θ_x is no longer zero at x_c. The zero-crossing point has moved to $x = x_c + \Delta x_c$. So, the magnetic center has moved in this case. A dipole component change with its magnetic center fixed is equivalent to a magnetic center motion with its dipole component fixed (at zero).

We may naively think that the bow-tie BBA method should tell us the estimate of this magnetic center motion. However, this is true only if $\Delta\theta_Q = 0$. What the method actually yields is the beam position at which the kick imparted by the quadrupole in the state Q_r is exactly the same kick applied in the state Q_m, as it is only trying to find a position that disturbs orbit between the two different states by the least amount. Graphically interpreted, the bow-tie method attempts to find where the blue and green lines cross each other in Fig. 2, denoted by x_{bt}, which is shifted by Δx_{sys} from x_c. From Eqs. (1) and (3), we can obtain the expression for x_{bt} as

$$x_{bt} = x_c + \Delta x_{sys}, \tag{4}$$

where

$$\Delta x_{sys} = \frac{\Delta\theta_Q}{\Delta K}. \tag{5}$$

We can further calculate the amplification factor κ of the quadrupole center motion when seen by the bow-tie BBA method:

$$\kappa := \frac{\Delta x_{sys}}{\Delta x_c} = \frac{K_m}{\Delta K} = 1 + \frac{K_r}{\Delta K}. \tag{6}$$

Since κ is determined purely by the measurement setup parameters, this factor stays constant whether $\Delta\theta_Q$ is zero or not. When $\Delta\theta_Q = 0$, no matter what the value of κ is, SED will be zero, as there is nothing to amplify. To minimize κ, it is preferable to choose $|\Delta K|$ as large as possible. However, in storage rings, $|\Delta K|$ usually must stay small to avoid beam loss due to changes in tunes and/or linear optics. As a realistic example, if ΔK is only 1% of K_m, and the actual center motion is 2 μm, the estimate for x_{bt} will shift by 200 μm, i.e., an amplification factor of 100.

Equation (5) can be also derived from closed orbit distortion analysis, including the effect of beta and phase beat caused by the quadrupole strength change [5]. However, this simple formula is true only if the BPM and the quadrupole are at the same location. Fortunately, this equality holds approximately at a BPM elsewhere, as long as the phase advance between them is not close to 90° or 270° and the conditions of $2\pi \cdot \Delta\nu \ll 1$ and $2\pi \cdot \Delta\nu / \sin(2\pi\nu^{(r)}) \ll 1$ are satisfied where $\Delta\nu$ is the tune change and $\nu^{(r)}$ is the tune at the reference state.

UNOBSERVABLE PART OF SED

SED would not be a huge problem if we could estimate it from the measurement data, thereby allowing us to recover the true magnetic center after subtracting this error from x_{bt}. Here we will present an argument that this is impossible with bow-tie measurements alone.

In general, any dipole component change can be described with a polynomial of ΔK: $\Delta\theta_Q = \Sigma_{i=1}^{\infty} p_i \cdot (\Delta K)^i$. Then, Eq. (4) can be expressed as (see [5] for details):

$$x_{bt} = x_c + p_1 + \sum_{i=1}^{\infty} p_{i+1} \cdot (\Delta K)^i. \tag{7}$$

This expression tells us that the value of x_{bt} can depend on the value of ΔK we arbitrarily choose in the case of non-zero dipole component change. This was first observed at ALS [4], and has been recently re-discovered at NSLS-II.

Note that the polynomial summation term is the only dynamic part in Eq. (7) and non-zero only if at least one of the coefficients $p_i (i \geq 2)$ is non-zero. In other words, the variation of x_{bt} with ΔK is detectable only if the dipole component change is a nonlinear function of ΔK. If $\Delta\theta_Q$ changes linearly with ΔK, the value of ΔK we select does not affect x_{bt}. This characteristic has a major ramification.

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As the most problematic case, suppose $\Delta\theta_Q$ is purely linear with respect to ΔK. Then for any ΔK, a bow-tie measurement would produce the same estimate, which can lead us to naively conclude we are obtaining the value of x_c. In truth, however, we are only getting the value of $x_c + p_1$. To the best of our knowledge, there appears to be no way to extract the value of p_1 from bow-tie measurements alone and would need direct magnetic field measurements to acquire such information.

This problem is illustrated with the simulation results of ELEGANT [6] using the NSLS-II lattice and one of the QM2A family magnets as an example. As shown in Fig. 3a, four different hypothetical hysteresis branches of $\Delta\theta_Q$ vs. ΔK were considered. The simulated (circles) and analytical (solid curves) values of y_{bt} for each branch are shown in Fig. 3b. The colors of the circles and curves in Fig. 3b match the colors of the different branches in Fig. 3a. For the blue $\Delta\theta_Q$ curve, there was no linear part ($p_1 = 0$) and nonlinear part. The medians of the y_{bt} estimates (from 180 BPMs) for all the ΔK values stayed constant, close to y_c (the true center value of +5 μm in this example), as expected. The error bars are also plotted, but too small to be distinguishable. The black $\Delta\theta_Q$ curve was the sum of the blue curve and 2 periods of a sinusoidal wave. The median y_{bt} values varied with ΔK due to the nonlinearity of the $\Delta\theta_Q$ curve, but oscillated around y_c, because the linear part was still zero. In contrast, the red $\Delta\theta_Q$ curve had a linear slope of $p_1 = 200$ μm without any nonlinearity, while the magenta curve had an additional sinusoidal modulation on top of the red curve. The resulting y_{bt} values were simply shifted by p_1, as Eq. (7) predicted. These results demonstrate that, without having the information in Fig. 3a, we cannot determine how far the y_{bt} value is from y_c.

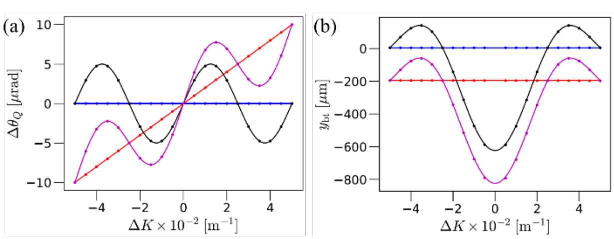

Figure 3: (a) Different hypothetical hysteresis curves $\Delta\theta_Q$ vs. ΔK for QM2A. (b) Comparison between simulated and analytically predicted y_{bt} for each hysteresis curve.

The magnitude of p_1 can be substantial in practice. For example, our NSLS-II magnetic field measurements of QM2A quadrupole family suggest it could add as much as 200 μm unobservable offset to x_c, if the range of K used for bow-tie measurements is between 0.35 and 0.40 m^{-1}, where the dipole component varies by up to 10 μrad.

EXPERIMENTS

At NSLS-II we have recently discovered that measured x_{bt} values have strong dependence on the ΔK values. This triggered the investigation that resulted in the work presented in this paper. The estimate differences ranged from 200 to 500 μm, an example of which is shown in Fig. 4. The pair of BPM C04-P3 and quadrupole QM2G4C04A was used for this experiment. Eight different hysteresis cycles between $\Delta K = -3 \times 10^{-2}$ and $+5 \times 10^{-2}$ m^{-1} are shown. The importance of x_{bt} dependence on not only the values of ΔK themselves but also the hysteresis loops is evident. Even when the same quadrupole and the same ΔK value was used, different hysteresis loops apparently induce different dipole component changes. This clearly demonstrates the need for magnetic field measurements that follow exactly the same hysteresis loop as would be used in a bow-tie BBA measurement.

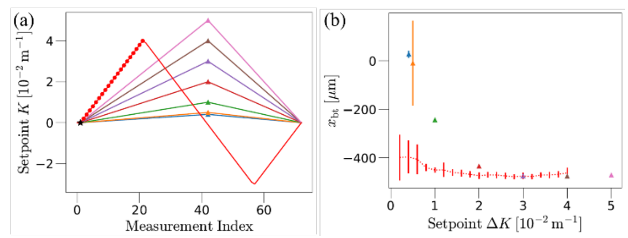

Figure 4: Hysteresis cycle dependence of experimental x_{bt} for C04P3 BPM. (a) A cycle between $\Delta K = -3 \times 10^{-2}$ and $+4 \times 10^{-2}$ m^{-1} in red. The rest of the colors correspond to cycles between $\Delta K = 0$ and the respective maximum ΔK values (up to $+5 \times 10^{-2}$ m^{-1}). Black star is the reference state. (b) x_{bt} estimates that correspond to the hysteresis loops in (a) with the same colors.

CONCLUSION

We have derived a simple, but accurate, formula for SED in the standard bow-tie BBA measurement technique for ring accelerators. The formula for the error amplification factor with respect to the amount of magnetic center motion during its gradient variation was also derived from this SED formula. Even a miniscule magnetic center movement of 1 μm after its strength change can easily result in SED on the orders of hundreds of microns for rings if experimental parameters are not carefully chosen.

If the dipole component changes nonlinearly with respect to the quadrupole strength change, BBA estimates will vary and thus the existence of the dipole component change is detectable from orbit data acquired during BBA measurements. However, if the dipole component change is linear, BBA estimates will stay constant, which makes it indistinguishable from the case when there is no dipole component change. At the moment, the only way to solve this issue is to have an independent magnetic field measurement data for dipole components. Furthermore, this field measurement must be carried out while following the hysteresis curve that would be followed during a BBA measurement.

All the BBA techniques used for circular accelerators that belong to the same family as the bow-tie method are likely to suffer from SED and the same mitigations should work and be utilized, if possible, to minimize the impact of SED.

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