The use of many-body methods for the calculation of meson-like spectra.

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Abstract. The study of properties for the low-energy regime of hadronic spectra is a challenging task; e.g. meson-like states require the introduction of effective degrees of freedom, a condition resulting from confinement. This work presents the results of adopting a non-perturbative scheme where quarks are treated as quasiparticles. These quasiparticles interact and by employing many-body techniques, we can describe a spectrum of meson-like states as a collective superposition of quasiquark-pairs followed by calculating their widths. The result of the calculations shows that this scheme is a suitable one to describe meson states up to energies of the order of a couple of GeV.

1 Introduction

The identification of effective degrees of freedom in a physical theory, such as the many-body problem in the quantum regime, served for many years as the main guide by describing physical systems in the QCD low-energy regime, which is a highly non-perturbative theory. In previous works, The SO(4) symmetry model was proposed in [1], which allowed a description of meson-like particles' effective degrees of freedom in addition to calculating their widths.

In recent years, the Cornell-like potential, composed by a Coulomb interaction ($\frac{1}{r}$) plus a linear potential (r), has been considered a good choice to describe the quark confinement [2, 3]. In this work, motivated by previous efforts [4, 5], we aimed to provide a global description of the meson-like spectra properties in the nuclear many-body framework from a single-fit of parameters. We constructed an effective Hamiltonian starting from the QCD Hamiltonian in the Coulomb gauge [6] considering a pure quark-confined interaction. The harmonic oscillator basis was used to describe the particle states, afterwards a transformation of particle basis to a quasiparticle representation and the BCS method were used to obtain the mass. Finally, the description of properties of the meson-like states by the quasiparticles interaction is carried out with the Random Phase Approximation (RPA) method [7], and if relevant, the calculations of their widths.

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2 Effective Hamiltonian and canonical transformation.

The proposed effective Hamiltonian for the QCD low-energy regimen restricted to the quark sector with a confined interaction $V(|\mathbf{x}-\mathbf{y}|) = -\frac{\alpha_c}{V|\mathbf{x}-\mathbf{y}|} + \beta_L V|\mathbf{x}-\mathbf{y}|$ is:

$$\begin{aligned}\mathcal{H}_{QCD}^{Eff} &= \int \{\bar{\psi}(-i\alpha\cdot\nabla + \beta m)\psi\} d\mathbf{x} - \frac{1}{2}\int \rho_c(\mathbf{x}) V(|\mathbf{x}-\mathbf{y}|)\rho^c(\mathbf{y}) d\mathbf{x} d\mathbf{y}. \\ &= K + \mathcal{H}_{Coul}, \end{aligned} \tag{1}$$

where the fermionic field $\psi^\dagger(\mathbf{x})$ is expanded in terms of the harmonic oscillator basis, α_c is the associated parameter to Coulomb interaction and β_L is the parameter of string tension.

As a result of writing the fields, the effective kinetic term of the Hamiltonian $\mathcal{H}_{QCD}^{Eff}$ in the Eq. (1) is no longer diagonal, so we must diagonalize it. We will refer to this process as *prediagonalization*. Also, using this basis creates a new way of writing the terms of the Hamiltonian for effective quark $(b^\dagger, b)$ and antiquark $(d^\dagger, d)$ degrees of freedom. The kinetic term is rewritten as

$$\mathbf{K} = \sum_{\pi k\gamma} \epsilon_{\lambda\pi k\gamma} \sum_{\mu} \left(\mathbf{b}^\dagger_{\pi k\gamma\mu} \mathbf{b}_{\pi k\gamma\mu} - \mathbf{d}^{\pi k\gamma\mu} \mathbf{d}^\dagger_{\pi k\gamma\mu} \right), \tag{2}$$

where the short-hand notation $\gamma = (J, c, (Y, T))$ and $\mu = (m, m_c, T_z)$ is used and $\epsilon_{\lambda\pi k\gamma}$ represents the eigenvalue of the effective quark energies [8].

The pairlike terms in the Hamiltonian acquire a form

$$\begin{aligned}\mathcal{F}_{\lambda_1\mathbf{q}_1,\lambda_2\mathbf{q}_2;\gamma_{f_0},\mu_{f_0}} &= \frac{1}{\sqrt{2}}\left\{ \delta_{\lambda_{1,\frac{1}{2}}}\delta_{\lambda_{2,\frac{1}{2}}} \left[\mathbf{b}^\dagger_{\mathbf{q}_1} \otimes \mathbf{b}_{\bar{\mathbf{q}}_2}\right]^{\gamma_{f_0}}_{\mu_{f_0}} - \delta_{\lambda_{1,-\frac{1}{2}}}\delta_{\lambda_{2,-\frac{1}{2}}} \left[\mathbf{d}_{\mathbf{q}_1} \otimes \mathbf{d}^\dagger_{\bar{\mathbf{q}}_2}\right]^{\gamma_{f_0}}_{\mu_{f_0}} \right\} \\ \mathcal{G}_{\lambda_1\mathbf{q}_1,\lambda_2\mathbf{q}_2;\gamma_{f_0},\mu_{f_0}} &= \frac{1}{\sqrt{2}}\left\{ \delta_{\lambda_{1,-\frac{1}{2}}}\delta_{\lambda_{2,\frac{1}{2}}} \left[\mathbf{d}_{\mathbf{q}_1} \otimes \mathbf{b}_{\bar{\mathbf{q}}_2}\right]^{\gamma_{f_0}}_{\mu_{f_0}} - \delta_{\lambda_{1,\frac{1}{2}}}\delta_{\lambda_{2,-\frac{1}{2}}} \left[\mathbf{b}^\dagger_{\mathbf{q}_1} \otimes \mathbf{d}^\dagger_{\bar{\mathbf{q}}_2}\right]^{\gamma_{f_0}}_{\mu_{f_0}} \right\}. \end{aligned} \tag{3}$$

We also have used a short-hand notation $1 = \lambda_1\pi_1 k_1 J_1 Y_1 T_1$ (same for index 2) for the quantum numbers of the intermediate coupling. With this, the Coulomb interaction acquires the following structure

$$\begin{aligned}\mathcal{H}_{Coul} = -\frac{1}{2}\sum_L \sum_{\lambda_i \mathbf{q}_i} \mathbf{V}^L_{(\lambda_i\mathbf{q}_i)} \Big(&\left[\mathcal{F}_{\lambda_1\mathbf{q}_1,\lambda_2\mathbf{q}_2;\gamma_{f_0}} \mathcal{F}_{\lambda_3\mathbf{q}_3,\lambda_4\mathbf{q}_4;\bar{\gamma}_{f_0}}\right]^{\gamma_0}_{\mu_0} + \left[\mathcal{F}_{\lambda_1\mathbf{q}_1,\lambda_2\mathbf{q}_2;\gamma_{f_0}} \mathcal{G}_{\lambda_3\mathbf{q}_3,\lambda_4\mathbf{q}_4;\bar{\gamma}_{f_0}}\right]^{\gamma_0}_{\mu_0} \\ &+ \left[\mathcal{G}_{\lambda_1\mathbf{q}_1,\lambda_2\mathbf{q}_2;\gamma_{f_0}} \mathcal{F}_{\lambda_3\mathbf{q}_3,\lambda_4\mathbf{q}_4;\bar{\gamma}_{f_0}}\right]^{\gamma_0}_{\mu_0} + \left[\mathcal{G}_{\lambda_1\mathbf{q}_1,\lambda_2\mathbf{q}_2;\gamma_{f_0}} \mathcal{G}_{\lambda_3\mathbf{q}_3,\lambda_4\mathbf{q}_4;\bar{\gamma}_{f_0}}\right]^{\gamma_0}_{\mu_0} \Big), \end{aligned} \tag{4}$$

The following section describes the transformations at the quasiparticle basis in terms of the effective quark operators and the BCS equations and their solutions and, results for the energy and gap for each quasiparticle type.

3 Bogoliubov transformations basis and BCS equations

The pair of terms of the Eq. (3) ought to be written in the basis of quasiparticle operators by a Bogoliubov transformation, which leads for the creation operators

$$\begin{aligned} B^\dagger_{k_i\pi_i,\gamma_i\mu_i} &= u_{k_i\pi_i,\gamma_i} \mathbf{b}^\dagger_{k_i\pi_i,\gamma_i\mu_i} - v_{k_i\pi_i,\gamma_i} \mathbf{d}_{k_i\pi_i,\gamma_i\mu_i} \\ D^{\dagger k_i\pi_i,\gamma_i\mu_i} &= u_{k_i\pi_i,\gamma_i} \mathbf{d}^{\dagger k_i\pi_i,\gamma_i\mu_i} + v_{k_i\pi_i,\gamma_i} \mathbf{b}^{k_i\pi_i,\gamma_i\mu_i}, \end{aligned} \tag{5}$$

and the annihilation operators

$$B^{k_i\pi_i,\gamma_i\mu_i} = u^*_{k_i\pi_i,\gamma_i}\mathbf{b}^{k_i\pi_i,\gamma_i\mu_i} - v^*_{k_i\pi_i,\gamma_i}\mathbf{d}^{\dagger k_i\pi_i,\gamma_i\mu_i}$$
$$D_{k_i\pi_i,\gamma_i\mu_i} = u^*_{k_i\pi_i,\gamma_i}\mathbf{d}_{k_i\pi_i,\gamma_i\mu_i} + v^*_{k_i\pi_i,\gamma_i}\mathbf{b}^{\dagger}_{k_i\pi_i,\gamma_i\mu_i}. \tag{6}$$

In both cases, the elements $u_{k_i\pi_i,\gamma_i}$ and $v_{k_i\pi_i,\gamma_i}$ are the factors of the Bogoliubov transformations between quarks and quasiquarks operators. It's worth mentioning the bare vacuum of the quark will be replaced by a correlated vacuum of quasiparticles which fulfills the condition $B|BCS\rangle = 0$ and $D|BCS\rangle = 0$

The next step consists of transforming both terms of the Eq. (2) and Eq. (4) of the Hamiltonian to the quasiquark basis in order to write and solve the BCS equations for occupation factors and gaps, for each of the channels entering in these expressions. That step is quite long and we will omit it here, the details are disclosed in the main work at [9]. The BCS procedure requires that the expectation value of the terms with two creation or two annihilation quasiquark operators should vanish when acting on the correlated vacuum. Together with the condition that the term with one quasiquark creation and annihilation operators should be diagonal when acting on the correlated vacuum. These requirements lead to the determination of the values of the gap and occupation factor for each quark flavor. The resulting set of equations, using the short-hand notation $\kappa = k\pi\gamma$, is

$$\Sigma_{\kappa_1} Z_{\kappa_1} + \Delta_{\kappa_1} W_{\kappa_1} = E_{\kappa_1}$$
$$-\Delta_{\kappa_1} Z_{\kappa_1} + \Sigma_{\kappa_1} W_{\kappa_1} = 0 \tag{7}$$

where the E_{κ_1} are the quasiquark energies and

$$\begin{aligned}
\Sigma_{\kappa_1} &= \epsilon_{\kappa_1} + \bar{V}^{\Sigma}_{\kappa_1\kappa_2\kappa_3\kappa_4}(u^2_{\kappa_2} - v^2_{\kappa_2}) \\
\Delta_{\kappa_1} &= \bar{V}^{\Delta}_{\kappa_1\kappa_2\kappa_3\kappa_4}(u_{\kappa_2} v_{\kappa_2}) \\
E_{\kappa_1} &= \sqrt{\Sigma^2_{\kappa_1} + \Delta^2_{\kappa_1}} \\
Z_{\kappa_1} &= \left\{u^2_{\kappa_1} - v^2_{\kappa_1}\right\} \\
W_{\kappa_1} &= \left\{2u_{\kappa_1} v_{\kappa_1}\right\}.
\end{aligned} \tag{8}$$

The last step which will lead us to the meson-like spectrum construction is, the application of the many-body method Random Phase Approximation (RPA) and, in case the state's identification might not be clear, use of the Bohr-Mottelson method to calculate the decay with of some particular states.

4 The random phase approximation procedure

Once we have transformed the Hamiltonian with the terms of the Eq. (2) and Eq. (4) to the quasiparticle basis, there remain some terms with the creation and annihilation of one and two pairs of coupled quasiquarks. This Hamiltonian, which will be called $\mathcal{H}^{QCD}_{RPA}$ contains a term of creation and annihilation of quasiparticle pairs $\mathcal{H}_{22}$ and a term of creation or annihilation of two quasiparticle pairs $\mathcal{H}_{40}, \mathcal{H}_{04}$, respectively, besides a number quasiparticle operator $\mathcal{H}_{11}$. The RPA method consists of the diagonalization of the Hamiltonian

$$\mathcal{H}_{RPA}^{QCD} = \mathcal{H}_{11} + \mathcal{H}_{22} + \mathcal{H}_{40} + \mathcal{H}_{04}, \tag{9}$$

in a new basis of phonons conformed by a linear combination of quasiquark pair-creation and quasiquark pair-annihilation operators. These phonons are defined as

$$\hat{\Gamma}^{\dagger}_{n;\Gamma\mu} = \sum_{\mathbf{a},\mathbf{b}} X^{n}_{\mathbf{ab};\Gamma}[B^{\dagger}_{\mathbf{a}}D^{\dagger}_{\bar{\mathbf{b}}}]^{\Gamma}_{\mu} - Y^{n}_{\mathbf{ab};\Gamma}(-1)^{\phi_{\Gamma\mu}}[D^{\bar{\mathbf{b}}}B^{\mathbf{a}}]^{\bar{\Gamma}}_{\bar{\mu}}, \tag{10}$$

where the coefficients $X^{n}_{\mathbf{ab};\Gamma}$ and $Y^{n}_{\mathbf{ab};\Gamma}$ are the amplitudes of the creation-annihilation pairs, respectively. The dynamic equation can be written as the double commutator

$$\langle RPA| \left[\hat{\Gamma}^{n';\Gamma\mu}, \left[H^{QCD}_{RPA}, \hat{\Gamma}^{\dagger}_{n;\Gamma\mu}\right]\right] |RPA\rangle = E^{RPA}_{n;\Gamma}\delta_{n,n'}. \tag{11}$$

The quantities $E^{RPA}_{n;\Gamma}$ are the *nth* one-phonon eigenvalues to be associated with the mesonic states. In the same way we did it in the BCS, we have to define a new vacuum state that fulfills the condition of $\hat{\Gamma}^{n';\Gamma\mu}|RPA\rangle = 0$.

The general solution of the RPA method requires the solving of two sets of equations from the amplitudes $X^{n}_{\mathbf{ab};\Gamma}$ and $Y^{n}_{\mathbf{ab};\Gamma}$. These can be written in matrix form as

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}\begin{pmatrix} X^n \\ Y^n \end{pmatrix} = E^{RPA}_{n}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} X^n \\ Y^n \end{pmatrix}, \tag{12}$$

the matrices A and B are the so-called forward and backward matrices and can be written, in the quasiboson approximation scheme, as

$$\begin{aligned} A_{a'b';\Gamma'\mu';\, a,b;\Gamma\mu} &= \langle\tilde{0}| \left[[D^{\bar{\mathbf{b}}'}B^{\mathbf{a}'}]^{\Gamma'}_{\mu'}, \left[H^{QCD}_{RPA}, [B^{\dagger}_{\mathbf{a}}D^{\dagger}_{\bar{\mathbf{b}}}]^{\Gamma}_{\mu}\right]\right] |\tilde{0}\rangle\,, \\ B_{a'b';\Gamma'\mu';\, a,b;\Gamma\mu} &= -\langle\tilde{0}| \left[[D^{\bar{\mathbf{b}}'}B^{\mathbf{a}'}]^{\Gamma'}_{\mu'}, \left[H^{QCD}_{RPA}, (-1)^{\phi_{\Gamma\mu}}[D^{\bar{\mathbf{b}}}B^{\mathbf{a}}]^{\Gamma}_{\bar{\mu}}\right]\right] |\tilde{0}\rangle, \end{aligned} \tag{13}$$

5 Results and discussion

The calculations have been performed using the set of parameters shown in the table 1, which gives the values of the quark masses and strengths of the Coulomb and linear interactions and the values of the quasiparticle energy and gaps, for up and down quarks (subindex q) and, s quarks (subindex s).

$m_q[GeV]$	$m_s[GeV]$	α_C	$\beta_L[GeV^2]$	$E_q[GeV]$	$\Delta_q[GeV]$	$E_s[GeV]$	$\Delta_s[GeV]$
0.05	0.140	1.501	0.152	0.799	0.6368	0.798	0.6223

Table 1. Values of the mass parameter m_q, quasiparticle energy E_q and gap Δ_q (q = u,d quarks), m_s, E_s and Δ_s (s-quarks), α_c and β_L for Coulomb and linear parameters.

It's worth remembering that the calculations have been performed using a single set of parameters from the table 1, since the main purpose of the work is to compare the general structure of the calculated meson spectra with the experimental ones. The meson-like spectra have been calculated for the subspaces $J^P = 0^{\pm}, 1^-$ and $J^P = 1^{++}, 1^{+-}$, the present work only shows the case of J^P=0^- and 1^- illustrated in the Fig 1 and 2 respectively.

Fig 1 shows that the low-lying state of the pion which is quite near the experimental value, but since we have taken it as a reference state, we did not vary the parameters of the model

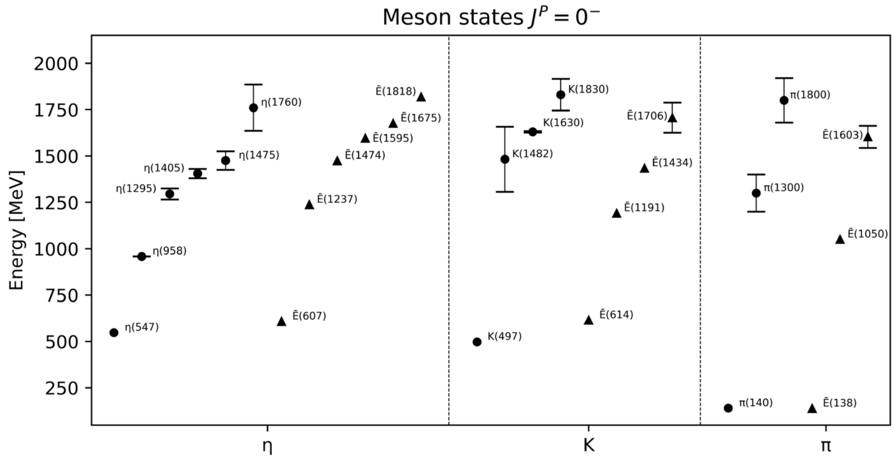

Figure 1. Energy spectrum of $J^P = 0^-$ meson states. Each of the panels shows the experimental values (dots) with the corresponding widths (vertical lines) and the calculated values (triangles). The centroids of the theoretical values are denoted by $\bar{E}$ and the vertical lines on them are results of the calculated decay widths

in order to get a better agreement with the experiment. The calculated spectrum shows two states at intermediate energy and one state at higher energy. The decay widths of these states make them compatible with data. Decay width calculation method has been worked in detail in [9, 10].

The same procedure is applied to other meson states belonging to the different subspaces considered in the calculations. In all cases, we have solved the RPA equations using the same space of quasiquark states to build the configurations restricted by the selection rules of the couplings associated to each mesonic representation.

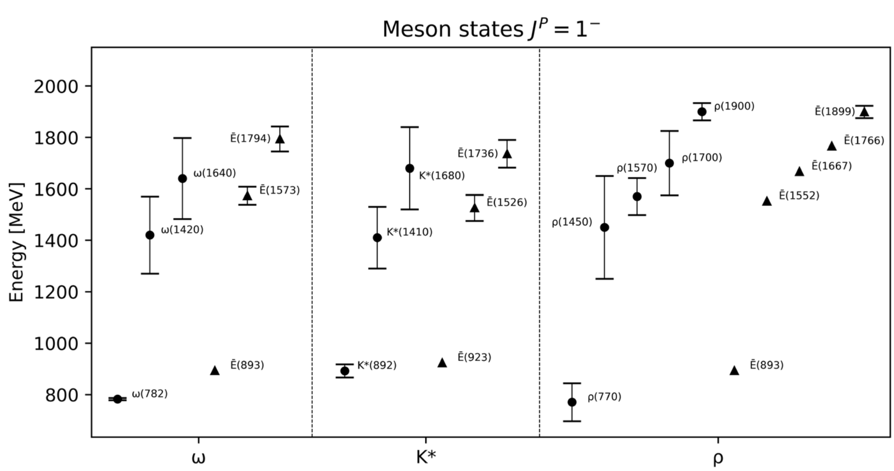

Figure 2. Experimental and theoretical results for mesons with $J^P = 1^-$. See the caption to Fig.(1)

6 Conclusions

In this work, we have presented some of our results for the spectrum of meson-like states with masses up to 2 GeV. From the features exhibited by the results, it could be said that the gross structure of the observed meson spectra can be explained by the RPA method when applied to effective quasiquark degrees of freedom.

The same procedure may be also used to calculate the baryon sector. In this scheme, a three-quark state, like a single nucleon, can be described as the result of the coupling between correlated two-quasiquark states coupled to an extra quasiquark. Work is in progress about it.

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