Analysis of CP conserving Higgs bosons self couplings in SM×S(3)

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Abstract. We carry out a detailed analysis of a minimal S_3-invariant extension of the Standard Model, with an extended S_3-Higgs sector. Within this extended $S(3)$-Standard Model, we study the trilinear Higgs couplings and its dependence on the details of the model, even when the lightest Higgs boson mass is taken to be a fixed parameter. We study quantitatively the trilinear Higgs couplings, and compare these couplings to the corresponding Standard Model trilinear Higgs coupling in some regions of the parameter space. A precise measurement of the trilinear Higgs self coupling will also make it possible to test this extended $S(3)$-Standard Model with the Standard Model trilinear Higgs coupling. We present analytical expressions for the trilinear Higgs couplings.

1. Introduction

In these days when it is seeing a new boson with a mass of 125 GeV, observed in CMS at the LHC, corresponding to the scalar of the Standard Model (SM), is of fundamental importance to analyze the mechanism of spontaneous symmetry breaking with models that have more number of scalar bosons [1]. The Higgs boson is a fundamental part of the SM providing mass to the gauge bosons and fermions upon the spontaneous breaking of the electroweak symmetry $SU(2)_L \times U(1)_Y$, and thus preserving the renormalisability of the theory. In the SM, only one $SU(2)_L$ doublet Higgs field is included, and although their existence is a fundamental piece of

the theory and the Higgs potential is very simple and sufficient to describe a realistic model of mass generation, this may not be the final form of the theory [2]. In the SM each family of fermions enters independently, in order to understand the replication of generations and to reduce the number of free parameters, usually more symmetry is introduced in the theory. In this direction interesting work has been done with the addition of discrete symmetries to the SM [3]. Many interesting features of masses and mixing of the SM can be understood using a minimal discrete group, namely the permutational group S_3 [4].

Prior to the introduction of the Higgs boson, the SM is chiral and invariant with respect to any permutation of the left and right quark and lepton fields. After the introduction of the Higgs boson in the theory, this field may be treated as an S_3 singlet H_S, but then, only one fermion in each family can acquire mass. To give mass to all fermions and, at the same time, preserve the S_3 flavour symmetry of the theory, an extended flavoured Higgs sector is required with three Higgs $SU(2)$ doublets, one in a singlet and the other two in a doublet irreducible representation of S_3 [5].

Up to now, the particle observed at the LHC is a particle in the physical spectrum of the Higgs boson of the SM . It is not known if there is one or many Higgs bosons. An indication of the presence of one Higgs boson or an extended Higgs sector, as the one proposed in the S_3-invariant extension of the SM, could be found at the Large Hadron Collider [6]. Models with more than one Higgs doublet, with or without supersymmetry, have been studied extensively. For a review of supersymmetric and two Higgs-doublet models [7], as well as models with and without discrete symmetries [8, 9, 10].

We study the trilinear Higgs couplings and its dependence on the details of the model, with an extended S_3-Higgs sector, even when the lightest Higgs boson mass is taken to be a fixed parameter. We study quantitatively the trilinear Higgs couplings, and compare these couplings to the corresponding Standard Model trilinear Higgs coupling in some regions of the parameter space.

2. The $S(3)$ extended Higgs doublet model
The Lagrangian $\mathcal{L}_\Phi$ of the Higgs sector is given by

$$\mathcal{L}_\Phi = [D_\mu H_S]^2 + [D_\mu H_1]^2 + [D_\mu H_2]^2 - V(H_1, H_2, H_S), \tag{1}$$

where D_μ is the usual covariant derivative. The scalar potential $V(H_1, H_2, H_S)$ is the most general Higgs potential invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y \times S_3$. The analysis of the stability properties of the potential V is of great relevance to study the phenomenological implications of this model. In here we will concentrate on the analysis of the potential V following the lines of previous work done in the vacuum stability of multi-Higgs models [9]. In our case, the discrete flavour symmetry S_3 simplifies the analysis.

There are many different ways to write the Higgs potential for this model, but for the purpose of this work the best basis is

$$H_1 = \begin{pmatrix} \phi_1 + i\phi_4 \\ \phi_7 + i\phi_{10} \end{pmatrix}, \; H_2 = \begin{pmatrix} \phi_2 + i\phi_5 \\ \phi_8 + i\phi_{11} \end{pmatrix}, \; H_S = \begin{pmatrix} \phi_3 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}, \tag{2}$$

the numbering of the real scalar ϕ fields is chosen for convenience by writing the mass matrices for the scalar particles, and the subscript S is the flavour index for the Higgs field singlet. Now a simple manner to write down the potential is

$$\begin{aligned} V = \;& \mu_1^2 \left(x_1 + x_2 \right) + \mu_0^2 x_3 + a x_3^2 + b \left(x_1 + x_2 \right) x_3 + c \left(x_1 + x_2 \right)^2 \\ & -4dx_7^2 + 2e \left[\left(x_1 - x_2 \right) x_6 + 2x_4 x_5 \right] + f \left(x_5^2 + x_6^2 + x_8^2 + x_9^2 \right) \\ & + g \left[\left(x_1 - x_2 \right)^2 + 4x_4^2 \right] + 2h \left(x_5^2 + x_6^2 - x_8^2 - x_9^2 \right), \end{aligned} \tag{3}$$

where the $\mu_{0,1}^2$ parameters have dimensions of mass squared, the real couplings $a, \cdots, h$ are dimensionless and the invariants x_i, the potential V depends on the fields ϕ_i through x_i, considering our assignment as

$$\begin{aligned} x_1 &= H_1^\dagger H_1, & x_4 &= \mathcal{R}\left(H_1^\dagger H_2 \right), & x_7 &= \mathcal{I}\left(H_1^\dagger H_2 \right), \\ x_2 &= H_2^\dagger H_2, & x_5 &= \mathcal{R}\left(H_1^\dagger H_S \right), & x_8 &= \mathcal{I}\left(H_1^\dagger H_S \right), \\ x_3 &= H_S^\dagger H_S, & x_6 &= \mathcal{R}\left(H_2^\dagger H_S \right), & x_9 &= \mathcal{I}\left(H_2^\dagger H_S \right). \end{aligned} \tag{4}$$

If the S_3 invariant Higgs potential, Eq. (3), is bounded from below being a quartic polynomial function, it will certainly have a global minimum somewhere. We can two types of minima: the trivial one for which the Higgs acquires zero VEV's, and the usual one, where electroweak symmetry breaking occurs, away from the origin, for

$$\phi_7 = v_1, \; \phi_8 = v_2, \; \phi_9 = v_3, \; \phi_i = 0, \; i \neq 7, 8, 9, \tag{5}$$

defined as the normal minimum, with VEV's which do not have any complex relative phase. So, v_1, v_2 and v_3 are the VEV's of the neutral Higgs fields components.

The minimization constraints must be determined by demanding the vanishing of $\partial V / \partial \phi_i$, then we get the mass parameter by

$$\mu_1^2 = -\left(b + f + 2h \right) v_3^2 - 2 \left(c + g \right) \left(v_1^2 + v_2^2 \right) + \frac{3e(v_1^2 - 2v_1 v_2 - v_2^2)v_3}{v_1 - v_2}, \tag{6}$$

and

$$\mu_0^2 = -\left[2a v_3^2 + \left(b + f + 2h \right) \left(v_1^2 + v_2^2 \right) - e \left(\frac{3v_1^2 - v_2^2}{v_3} \right) v_2 \right]. \tag{7}$$

Accordingly, we have $v_1 = \sqrt{3} v_2$.

The Higgs-bosons masses in this model are obtained by diagonalizing the 12×12 matrix, with the 3×3 sub-matrices $M_I^2 = M_{II}^2$ and M_{IJ}^2, $I, J = C, \; S, \; P$:

$$\left(\mathcal{M}_H^2 \right)_{ij} = \frac{1}{2} \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \bigg|_{\text{mim}}, \tag{8}$$

where $i, j = \overline{1, 12}$. We have that $\mathcal{M}_H^2 = \text{diag}(M_C^2, M_C^2, M_S^2, M_P^2)$, the zeros correspond to 3×3 sub-matrices with zeros in its entirety. For example, the mass matrix for the CP-even Higgs scalars is given by

$$M_S^2 = \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ & s_{22} & s_{23} \\ & & s_{33} \end{pmatrix}, \tag{9}$$

where

$$
\begin{aligned}
s_{11} &= 12(c+g)v_2^2, & s_{12} &= 2\sqrt{3}v_2\left(2(c+g)v_2 + 3ev_3\right), \\
s_{13} &= 2\sqrt{3}v_2\left(3ev_2 + (b+f+2h)v_3\right), & s_{22} &= 4v_2\left((c+g)v_2 - 3ev_3\right), \\
s_{23} &= 2v_2\left(3ev_2 + (b+f+2h)v_3\right), & s_{33} &= 4av_3^2 - 8ev_2^3/v_3.
\end{aligned}
$$

After diagonalized mass matrices, the masses of the physical scalars/pseudoscalars are obtained. In our analysis we are not taking into account the parameter space with negative eigenvalues solutions for the squared masses of the physical Higgs fields. Of the original twelve scalar degrees of freedom, three Goldstone bosons ($G^\pm$ and G) are absorbed by $W^\pm$ and Z. The remaining nine physical Higgs particles are three $CP-$even scalar (h and H_1, H_2, with $m_h \leq m_{H_1} \leq m_{H_2}$), two CP-odd scalar (A_1, and A_2, with $m_{A_1} \leq m_{A_2}$), and two charged Higgs pair ($H_{1,2}^\pm$, mass degenerate). Their physical masses and the respective Higgs-scalar mixing are related to the weak parameters of the Higgs potential as follows. We start considering the mass matrix for the CP-even Higgs scalars Eq. (9).

Defining the physical mass eigenstates m_h^2, $m_{H_1}^2$, and $m_{H_2}^2$, the masses are found from the diagonalization process $\mathcal{M}_{diag}^2 = R^T \mathcal{M}_{even}^2 R = \text{diag}(m_h^2, m_{H_1}^2, m_{H_2}^2)$. The masses for the CP-even Higgs scalars are:

$$m_h^2 = -18ev_2v_3, \quad m_{H_1^0, H_2^0}^2 = (\mathcal{M}_a^2 + \mathcal{M}_c^2) \pm \sqrt{(\mathcal{M}_a^2 - \mathcal{M}_c^2)^2 + (\mathcal{M}_b^2)^2}, \tag{10}$$

where

$$
\begin{aligned}
\mathcal{M}_a^2 &= v_2(8(c+g)v_2 + 3ev_3), \\
\mathcal{M}_b^2 &= 4v_2\left(3ev_2 + (b+f+2h)v_3\right), \\
\mathcal{M}_c^2 &= -\frac{4ev_2^3}{v_3} + 2av_3^2,
\end{aligned}
\tag{11}
$$

and mixing angles are defined as $\tan 2\alpha_3 = \dfrac{\mathcal{M}_b^2}{\mathcal{M}_a^2 - \mathcal{M}_c^2}$, as well as

$$R = OPQ = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\theta & s_\theta \\ 0 & -s_\theta & c_\theta \end{pmatrix}, \tag{12}$$

where

$$\tan\theta = \frac{s_\theta}{c_\theta} = \frac{2s_{13}}{\sqrt{3}(m_{H_1}^2 - s_{33})} = \frac{\mathcal{M}_b^2}{2\mathcal{M}_a^2 - m_{H_2}^2}, \tag{13}$$

s_{13} and s_{33} are given in Eq. (9).

To generate the correct $W^{\pm}$ and Z^0 masses, one must consider that $v_1^2 + v_2^2 + v_3^2 = v^2$, with $v = 246$ GeV and $v_1 = \sqrt{3}v_2$. Also, the v_i are taken real, *i. e.*, we assume that spontaneous CP violation does not occur. The squared-mass parameters μ_0^2 and μ_1^2 can be eliminated by minimizing the scalar potential. The resulting squared masses for the CP-odd Higgs pseudoscalars are

$$m_{A_1}^2 = -16(d+g)v_2^2 - 10ev_2v_3 - 4hv_3^2, \qquad m_{A_2}^2 = -\frac{2(ev_2 + 2hv_3)(4v_2^2 + v_3^2)}{v_3},$$

which can be parametrized with $\tan\omega_3 = \frac{2v_2}{v_3}$, where $\sin\omega_3 = 2v_2/v$, $\cos\omega_3 = v_3/v$.

On the other hand, the charged Higgs boson masses are

$$m_{H_1^{\pm}}^2 = -\left(10ev_2 + (f+2h)v_3\right)v_3, \qquad m_{H_2^{\pm}}^2 = -\frac{v^2}{v_3}\left(2ev_2 + (f+2h)v_3\right).$$

As we known, the Higgs boson masses are not determined *a priori* within the theory, it is necessary to investigate the change of mass spectrum with respect to the quadrilinears $a, \ldots, h$ couplings.

3. Trilinear Self-Couplings of Neutral Higgs Bosons

The measurement of the Higgs self-coupling is crucial to determine the Higgs potential. The self-couplings are uniquely determined in the SM by the mass of the Higgs boson, which is related to the quadrilinear coupling λ by $M_H = \sqrt{2\lambda}v$. The trilinear and quadrilinear vertices of the Higgs field H are given by the coefficients:

$$\lambda_{HHH} = \lambda v = \frac{M_H^2}{2v}, \qquad \lambda_{HHHH} = \frac{\lambda}{4} = \frac{M_H^2}{8v^2}. \tag{14}$$

The following definitions are often used:

$$\lambda_{ijk} = \frac{-i\partial^3 V}{\partial H_i \partial H_j \partial H_k}, \tag{15}$$

which are most easily obtained from the corresponding derivatives of V in Eq. (3) with respect to the fields $\{\phi_i\}$ with $i = 1, \cdots, 12$. Then, we can write the trilinear couplings in terms of the derivatives of the potential (3) with respect to ϕ_i and the elements of the rotation matrix R Eq. (12) as

$$\lambda_{ijk} = N \sum_{lmn} R_{il}R_{jm}R_{kn}\frac{\partial^3 V}{\partial\phi_i\partial\phi_j\partial\phi_k}. \tag{16}$$

Here the indices l, m, n refer to the weak field basis, and $l \leq m \leq n = 1, 2, 3$, N is a factor of $n!$ for n identical fields. We now proceed to obtain these couplings in an explicit form. The

trilinear self-couplings a_{lmn} among the neutral even Higgs bosons can be written as

$$
\begin{aligned}
a_{1,1,1} &= 6\sqrt{6}(c+g)v_2, & a_{1,1,2}, &= \sqrt{2}(2(c+g)v_2 + 3ev_3), \\
a_{1,1,3} &= \sqrt{2}(3ev_2 + (b+f+2h)v_3), & a_{1,2,2} &= 2\sqrt{6}(c+g)v_2, \\
a_{1,2,3} &= 3\sqrt{6}ev_2, & a_{1,3,3} &= \sqrt{6}(b+f+2h)v_2, \\
a_{2,2,2} &= 3\sqrt{2}(2(c+g)v_2 - ev_3), & a_{2,2,3} &= \sqrt{2}((b+f+2h)v_3 - 3ev_2), \\
a_{2,3,3} &= \sqrt{2}(b+f+2h)v_2, & a_{3,3,3} &= 6\sqrt{2}av_3.
\end{aligned}
\tag{17}
$$

Then, we have the self-couplings of three even Higgs bosons by substituting for the elements of the rotation matrix, Eq. (12), one obtains

$$
\begin{aligned}
\lambda_{1,1,1} &= 6v\left(\lambda_1 \sin\omega_3 + \lambda_2 \cos\omega_3\right), & \lambda_{2,2,2} &= 6v\left(\lambda_3 \sin\omega_3 + \lambda_4 \cos\omega_3\right), \\
\lambda_{3,3,3} &= 6v\left(\lambda_5 \sin\omega_3 + \lambda_6 \cos\omega_3\right), & \lambda_{1,1,2} &= 2v\left(\lambda_7 \sin\omega_3 + \lambda_8 \cos\omega_3\right), \\
\lambda_{1,1,3} &= 2v\left(\lambda_9 \sin\omega_3 + \lambda_{10} \cos\omega_3\right), & \lambda_{1,2,2} &= 2v\left(\lambda_{11} \sin\omega_3 + \lambda_{12} \cos\omega_3\right), \\
\lambda_{1,2,3} &= v\left(\lambda_{13} \sin\omega_3 + \lambda_{14} \cos\omega_3\right), & \lambda_{1,3,3} &= 2v\left(\lambda_{15} \sin\omega_3 + \lambda_{16} \cos\omega_3\right), \\
\lambda_{2,2,3} &= 2v\left(\lambda_{17} \sin\omega_3 + \lambda_{18} \cos\omega_3\right), & \lambda_{2,3,3} &= 2v\left(\lambda_{19} \sin\omega_3 + \lambda_{20} \cos\omega_3\right).
\end{aligned}
\tag{18}
$$

$\lambda_1, \ldots, \lambda_{20}$ depend on parameters quadrilinear couplings of the Higgs potential, Eq. (3), and the mixing angle θ Eq. (13). For example

$$
\begin{aligned}
\lambda_1 &= \frac{3\sqrt{3}}{4}(b+f+2h)(c_\theta - 1)s_\theta^2, \\
\lambda_2 &= \frac{2\sqrt{3}}{4}\left(18as_\theta^3 + [(b+f+2h)s_\theta - 3ec_\theta](3c_\theta^2 + 1)\right) \\
&\quad + 2(c+g)(9c_\theta^3 - 3c_\theta^2 + c_\theta - 3) - 3e(3c_\theta(c_\theta + 1) - 1)s_\theta.
\end{aligned}
\tag{19}
$$

4. Numerical results

Here we present numerical results considering a decoupling limit, that is, a CP-even boson Higgs scalar behaves like the Higgs boson of the Standard Model, where the mass of the lightest Higgs scalar is significantly smaller than the masses of the other Higgs bosons of the model. We assume that it is light and its mass is around 125 GeV. In our analysis we proposed H_1^0 as our candidate, it is a free parameter. Consequently, the other Higgs bosons have masses of the order of $\mathcal{O}(500)$ GeV, without loss of generality and for the sake of simplicity, it is considered that they have no equal masses. Figure 1 shows the trilinear couplings by CP-even Higgs bosons in function of the mixing angle. The higher intensity couplings correspond to $\lambda_{h_0 h_0 h_0}$ and $\lambda_{H_2 H_2 H_2}$, whereas that $\lambda_{H_1 H_1 H_1}$ happens to be very small, $\lambda_{h_0 h_0 h_0} \sim \lambda_{H_2 H_2 H_2} > \lambda_{H_1 H_1 H_1}$, followed by $\lambda_{h_0 h_0 H_2}$, $\lambda_{h_0 h_0 H_1}$ $(\lambda_{h_0 H_2 H_2})$ and then the others. We proposed that the lightest is H_1^0 and its trilinear coupling is the lower intensity. Although there are couplings equal in shape, they are different in intensity. Comparing these results with only SM trilinear coupling, we can see that our values are within allowed ranges in the literature and enriched by the Higgs potential analysis, enabling to better understanding of the spontaneous symmetry breaking.
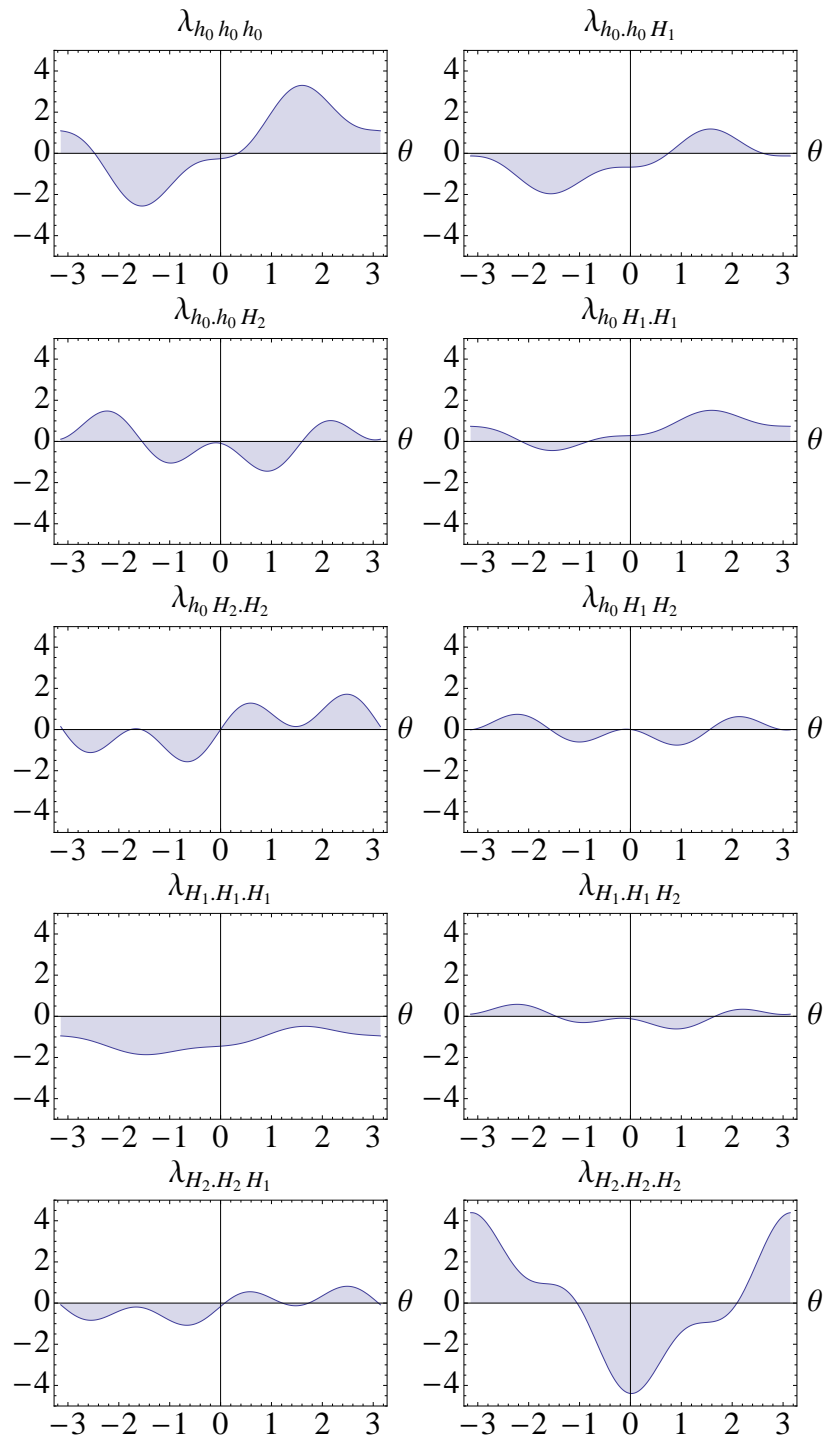
Figure 1. The trilinear couplings of the CP-even Higgs bosons in function on the mixing angle $-\pi \leq \theta \leq \pi$.

5. Conclusions

We studied only the scalar sector assuming the pseudoscalars to be too heavy and relevant. In this work we have analyzed the complete scalar sector of an S_3 flavor model. We deal with three CP-even, two CP-odd and two sets of charged scalar particles. We have improved our potential minimization technique which enabled us to explore a larger region of the allowed parameter space. We have studied in detail the trilinear couplings of the lightest Higgs boson of this model. Within the allowed domain of the parameters space of the model, the trilinear Higgs couplings have a strong dependence on $\tan\omega_3 = 2v_2/v_3$ and $\tan\theta$. The extended Higgs spectrum in S_3 models gives rise numbers of trilinear couplings. The hhh coupling can be measured in hh continuum production linear colliders as at e^+e^-.

References

[1] Chatrchyan S *et al.* 2012 Phys. Lett. B 716 30

[2] Higgs P W 1964 Phys. Lett. 12 132; 1966 Phys. Rev. 145 1156; Englert F and Brout R 1964 Phys. Rev. Lett. 13 321; Guralnik G S Hagen C R and Kibble T W 1964 Phys. Rev. Lett. 13 585

[3] Ma E 2007 UCRHEP-T431 [arXiv:0705.0327]

[4] Derman E 1979 Phys. Rev. D 19 317; Derman E and Tsao H S 1979 Phys. Rev. D 20 1207; Mondragon A and Rodriguez-Jauregui E 1998 [arXiv:9804267]; 1999 Phys. Rev. D 59 093009 [arXiv:9807214]; 2000 Phys. Rev. D 61 113002 [arXiv:9906429]; Kubo J 2004 Phys. Lett. B 578 156 [arXiv:0309167]; Kubo J Okada H and Sakamaki F 2004 Phys. Rev. D 70 036007 [arXiv:0402089]; Beltran O F Mondragon M and Rodriguez-Jauregui E 2009 J. Phys. Conf. Ser. 171 012028; Morisi S and Peinado E 2010 [arXiv:1001.2265]

[5] Kubo J Mondragon A Mondragon M and Rodriguez-Jauregui E 2003 Prog. Theor. Phys. 109 795 [arXiv:0302196]; Felix O Mondragon A Mondragon M and Peinado E 2007 AIP Conf. Proc. 917 383 [arXiv:0610061]; Mondragon A Mondragon M and Peinado E 2007 Phys. Rev. D76 076003 [arXiv:0706.0354]; Mondragon A Mondragon M and Saldana Salazar U J 2012 J. Phys. Conf. Ser. 378 012027[arXiv:1210.6131; Gonzalez Canales F Mondragon A Mondragon M and Saldana Salazar U 2012 C12-06-30

[6] Barger V Logan H E and Shaughnessy G 2009 Phys. Rev. D 79 115018 [arXiv: 0902.0170]; Gupta R S and Wells J D 2009 [arXiv:0912.0267]

[7] Gunion J F Haber H E Kane G and Dawson S 1990 *The Higgs Hunter's Guide* Addison-Wesley; Quigg C 1999 XXVII Int. Meeting on Fundamental Physics, Sierra Nevada (Granada) [arXiv:9905369]; Spira M and Zerwas P M 1997 Int. Universitätswochen, Schladming [arXiv:9803257]; Ginzburg I F Krawczyk M and Osland P 2002 [arXiv:0211371]; Kanemura S Okada Y Senaha E and Yuan C P 2004 Phys. Rev. D 70 115002 [arXiv:0408364]

[8] Hoang L N 1998 Mod. Phys. Lett. A 13 1865 [arXiv:9711204]; Adler S L 1999 Phys. Rev. D 60 015002 [arXiv:9901449]; Ferreira P M and Silva J 2008 P Phys. Rev. D 78 116007 [arXiv:0809.2788]; Howl R and S. F. King S F 2009 [arXiv:0908.2067]

[9] Barroso A Ferreira P M and Santos R 2006 Afr. J. Math. Phys. 3 103 [arXiv:0507329]; 2006 PoS HEP2005 337 [arXiv:0512037]; Barroso A Ferreira P M Santos R and Silva J P 2006 Phys. Rev. D 74 085016 [arXiv:0608282]

[10] Konisi G and T. Saito T 1995 Prog. Theor. Phys. 93 1093; Toghi A and Moazzen M 2009 Int. J. Theor. Phys. 48 3372