

**A PLANE-SYMMETRIC MAGNETIZED INHOMOGENEOUS
COSMOLOGICAL MODEL OF PERFECT FLUID DISTRIBUTION WITH
VARIABLE MAGNETIC PERMEABILITY**

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A plane-symmetric magnetized inhomogeneous cosmological model of perfect fluid distribution with variable magnetic permeability is investigated. The source of the magnetic field is due to an electric current produced along the z -axis. Thus F_{12} is the only non-vanishing component of electromagnetic field tensor F_{ij} . To get a unique determined solution, we assume the free gravitational field is Petrov type-II non-degenerate. For our derived model we obtain the deceleration parameter $q = -1$ as in the case of de Sitter universe. Some physical and geometric properties of the model are also discussed.

Key words: cosmology, variable magnetic permeability,
inhomogeneous Universe.

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1. INTRODUCTION

The standard Friedman-Robertson-Walker (FRW) cosmological model prescribes a homogeneous and an isotropic distribution for its matter in the description of the present state of the universe. At the present state of evolution, the universe is spherically symmetric and the matter distribution in the universe is on the whole isotropic and homogeneous. But in early stages of evolution, it could have not had such a smoothed picture. Close to the big bang singularity, neither the assumption of spherical symmetry nor that of isotropy can be strictly valid. So we consider plane-symmetric, which is less restrictive than spherical symmetry and can provide an avenue to study inhomogeneities. Inhomogeneous cosmological models play an important role in understanding some essential features of the universe such as the formation of galaxies during the early stages of evolution and process of homogenization. The early attempts at the construction of such models have done by Tolman [1] and Bondi [2] who considered spherically symmetric models. Inhomogeneous plane-symmetric models were considered by Taub [3, 4] and later by Tomimura [5], Szekeres [6], Collins and Szafron [7, 8], Szafron and Collins [9]. Recently,

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Senovilla [9] obtained a new class of exact solutions of Einstein's equation without big bang singularity, representing a cylindrically symmetric, inhomogeneous cosmological model filled with perfect fluid which is smooth and regular everywhere satisfying energy and causality conditions. Later, Ruis and Senovilla [11] have separated out a fairly large class of singularity free models through a comprehensive study of general cylindrically symmetric metric with separable function of r and t as metric coefficients. Dadhich *et al.* [12] have established a link between the FRW model and the singularity free family by deducing the latter through a natural and simple in-homogenization and anisotropization of the former. Recently, Patel *et al.* [13] presented a general class of inhomogeneous cosmological models filled with non-thermalized perfect fluid by assuming that the background space-time admits two space-like commuting killing vectors and has separable metric coefficients. Bali and Tyagi [14] obtained a plane-symmetric inhomogeneous cosmological models of perfect fluid distribution with electro-magnetic field. Recently, Pradhan *et al.* [15]–[19] have investigated plane-symmetric inhomogeneous cosmological models in various contexts.

The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zeldovich *et al.* [20]. Also Harrison [21] has suggested that magnetic field could have a cosmological origin. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model [22]. The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors [23]–[32]. Strong magnetic fields can be created due to adiabatic compression in clusters of galaxies. Large-scale magnetic fields give rise to anisotropies in the universe. The anisotropic pressure created by the magnetic fields dominates the evolution of the shear anisotropy and it decays slower than if the pressure was isotropic [33, 34]. Such fields can be generated at the end of an inflationary epoch [35]–[39]. Anisotropic magnetic field models have significant contribution in the evolution of galaxies and stellar objects. Bali and Ali [40] had obtained a magnetized cylindrically symmetric universe with an electrically neutral perfect fluid as the source of matter. Pradhan *et al.* [41]–[45] have investigated magnetized viscous fluid cosmological models in various contexts.

Maxwell considered the magnetic permeability ($\bar{\mu}$) to be a constant for a given material. Maxwell considered the spatial gradient of the magnetic field intensity in the steady state to be exclusively determined by a variation in the velocity of the molecular vortices within the magnetic lines of force. To this day, it is assumed that the magnetic permeability is a constant for a given material. But from 'The Double Helix Theory of the Magnetic Field' [46], we must look to a variable magnetic

permeability in order to account for variations in magnetic flux density in the steady state, and if we look at the solenoidal magnetic field pattern around a bar magnet, this is not very difficult to visualize. The magnetic field lines are clearly more concentrated at the poles of the magnet than elsewhere. It should be quite obvious that the density of the vortex sea, as denoted by the quantity $\bar{\mu}$, is a variable quantity and that this density visibly varies according to how tightly the magnetic lines of force are packed together [47].

Recently Bali [48] has obtained Bianchi type V magnetized string dust universe with variable magnetic permeability. Motivated by the above, in this paper, we have obtained a new plane-symmetric magnetized inhomogeneous cosmological model of perfect fluid distribution with variable magnetic permeability. To get a unique determined solution, we consider the free gravitational field is Petrov type-II non-degenerate in which the distribution is that of perfect fluid. The paper is organized as follows. The metric and the field equations are presented in Section 2. In Section 3, we deal with the solution of the field equations. Section 4 includes the physical and geometric features of the models. Finally the results are discussed in Section 5.

2. THE METRIC AND FIELD EQUATIONS

We consider the metric in the form of Marder [49]

$$ds^2 = A^2(dx^2 - dt^2) + B^2dy^2 + C^2dz^2, \quad (1)$$

where the metric potential A , B and C are functions of x and t . The energy momentum tensor is taken as

$$T_i^j = (\rho + p)v_i v^j + p g_i^j + E_i^j, \quad (2)$$

where E_i^j is the electro-magnetic field given by Lichnerowicz [50] as

$$E_i^j = \bar{\mu} \left[h_l h^l (v_i v^j + \frac{1}{2} g_i^j) - h_i h^j \right]. \quad (3)$$

Here ρ and p are the energy density and isotropic pressure respectively and v^i is the flow vector satisfying the relation

$$g_{ij} v^i v^j = -1. \quad (4)$$

$\bar{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}} {}^*F_{jiv^j}, \quad (5)$$

where ${}^*F_{ij}$ is the dual electro-magnetic field tensor defined by Synge [51]

$${}^*F_{ij} = \frac{\sqrt{-g}}{2} \epsilon_{ijkl} F^{kl}. \quad (6)$$

F_{ij} is the electro-magnetic field tensor and ϵ_{ijkl} is the Levi-Civita tensor density. The coordinates are considered to be comoving so that $v^1 = 0 = v^2 = v^3$ and $v^4 = \frac{1}{A}$. We consider that the current is flowing along the z-axis so that $h_3 \neq 0$, $h_1 = 0 = h_2 = h_4$. The only non-vanishing component of F_{ij} is F_{12} . The Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \quad (7)$$

and

$$\left[\frac{1}{\bar{\mu}} F^{ij} \right]_{;j} = J^i \quad (8)$$

require that F_{12} be function of x alone. We assume that the magnetic permeability as a function of x and t both. Here the semicolon represents a covariant differentiation.

The Einstein's field equations (in gravitational units $c = 1$, $G = 1$) read as

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -8\pi T_i^j, \quad (9)$$

for the line element (1) has been set up as

$$8\pi A^2 \left(p + \frac{F_{12}^2}{2\bar{\mu}A^2B^2} \right) = -\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A} \left(\frac{B_4}{B} + \frac{C_4}{C} \right) + \frac{A_1}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right) + \frac{B_1 C_1}{B C} - \frac{B_4 C_4}{B C} - \Lambda A^2, \quad (10)$$

$$8\pi A^2 \left(p + \frac{F_{12}^2}{2\bar{\mu}A^2B^2} \right) = -\left(\frac{A_4}{A} \right)_4 + \left(\frac{A_1}{A} \right)_1 - \frac{C_{44}}{C} + \frac{C_{11}}{C} - \Lambda A^2, \quad (11)$$

$$8\pi A^2 \left(p - \frac{F_{12}^2}{2\bar{\mu}A^2B^2} \right) = -\left(\frac{A_4}{A} \right)_4 + \left(\frac{A_1}{A} \right)_1 - \frac{B_{44}}{B} + \frac{B_{11}}{B} - \Lambda A^2, \quad (12)$$

$$8\pi A^2 \left(\rho + \frac{F_{12}^2}{2\bar{\mu}A^2B^2} \right) = -\frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{A_1}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right) + \frac{A_4}{A} \left(\frac{B_4}{B} + \frac{C_4}{C} \right) - \frac{B_1 C_1}{B C} + \frac{B_4 C_4}{B C} + \Lambda A^2, \quad (13)$$

$$0 = \frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_1}{A} \left(\frac{B_4}{B} + \frac{C_4}{C} \right) - \frac{A_4}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right), \quad (14)$$

where the sub indices 1 and 4 in A, B, C and elsewhere indicate ordinary differentiation with respect to x and t , respectively.

3. SOLUTION OF THE FIELD EQUATIONS

Equations (10) - (12) lead to

$$\left(\frac{A_4}{A} \right)_4 - \frac{B_{44}}{B} + \frac{A_4}{A} \left(\frac{B_4}{B} + \frac{C_4}{C} \right) - \frac{B_4 C_4}{B C} =$$

$$\left(\frac{A_1}{A} \right)_1 + \frac{C_{11}}{C} - \frac{A_1}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right) - \frac{B_1 C_1}{B C} = k \text{ (constant)} \quad (15)$$

and

$$\frac{8\pi F_{12}^2}{\bar{\mu} B^2} = \frac{B_{44}}{B} - \frac{B_{11}}{B} + \frac{C_{11}}{C} - \frac{C_{44}}{C}. \quad (16)$$

Eqs. (10) - (14) represent a system of five equations in seven unknowns A, B, C, ρ, p, F_{12} and Λ . For the complete determination of these unknowns two more conditions are needed. As in the case of general-relativistic cosmologies, the introduction of inhomogeneities into the cosmological equations produces a considerable increase in mathematical difficulty: non-linear partial differential equations must now be solved. In practice, this means that we must proceed either by means of approximations which render the non-linearities tractable, or we must introduce particular symmetries into the metric of the space-time in order to reduce the number of degrees of freedom which the inhomogeneities can exploit. In the present case, we assume that the metric is Petrov type-II non-degenerate. This requires that

$$\begin{aligned} \left(\frac{B_{11} + B_{44} + 2B_{14}}{B} \right) - \left(\frac{C_{11} + C_4 + 2C_{14}}{C} \right) = \\ \frac{2(A_1 + A_4)(B_1 + B_4)}{AB} - \frac{2(A_1 + A_4)(C_1 + C_4)}{AC}. \end{aligned} \quad (17)$$

Let us consider that

$$\begin{aligned} A &= f(x)\lambda(t), \\ B &= g(x)\mu(t), \\ C &= g(x)\nu(t). \end{aligned} \quad (18)$$

Using (18) in (14) and (17), we get

$$\left[\frac{\frac{g_4}{g} - \frac{f_1}{f}}{\frac{g_1}{g}} \right] = \left[\frac{\frac{2\lambda_4}{\lambda}}{\frac{\mu_4}{\mu} + \frac{\nu_4}{\nu}} \right] = b \text{ (constant)} \quad (19)$$

and

$$\frac{\frac{\mu_{44}}{\mu} - \frac{\nu_{44}}{\nu}}{\frac{\mu_4}{\mu} - \frac{\nu_4}{\nu}} - \frac{2\lambda_4}{\lambda} = 2 \left(\frac{f_1}{f} - \frac{g_1}{g} \right) = L \text{ (constant)}. \quad (20)$$

Equation (19) leads to

$$f = n g^{(1-b)} \quad (21)$$

and

$$\lambda = m(\mu\nu)^{\frac{b}{2}}, \quad (22)$$

where m and n are constants of integration. Equations (15), (18) and (20) lead to

$$\left(\frac{b}{2} - 1 \right) \frac{\mu_{44}}{\mu} + (b-1) \frac{\mu_4 \nu_4}{\mu \nu} = k, \quad (23)$$

and

$$(2-b)\frac{g_{11}}{g} + (3b-4)\frac{g_1^2}{g^2} = k. \quad (24)$$

Let us assume

$$\mu = e^{U+V}, \quad (25)$$

and

$$\nu = e^{U-V}. \quad (26)$$

Equations (20), (25) and (26) lead to

$$V_4 = M e^{Lt+2(b-1)U}, \quad (27)$$

where M is constant. From equations (23), (25), (26) and (27), we have

$$(b-1)U_{44} + 2(b-1)U_4^2 - 2bM e^{Lt+2(b-1)U}U_4 - M L e^{Lt+2(b-1)U} = k. \quad (28)$$

If we put $e^{2U} = \xi$ in equation (28), we obtain

$$\frac{(b-1)}{2} \frac{d^2\xi}{dt^2} - M \frac{d}{dt}(e^{Lt}\xi^b) = k\xi. \quad (29)$$

If we consider $\xi = e^{qt}$, then equation (29) leads to

$$\frac{(b-1)}{2} g^2 e^{qt} - M \frac{d}{dt}(e^{Lt} e^{qbt}) = k e^{qt}, \quad (30)$$

which again reduces to

$$q = \frac{L}{1-b}, \quad (31)$$

and

$$k = \frac{L(L+2M)}{2(b-1)}. \quad (32)$$

Thus

$$U = \frac{Lt}{2(1-b)}. \quad (33)$$

Equations (27) and (33) reduce to

$$V = Mt + \log N, \quad (34)$$

where N is an integrating constant. Eq. (24) leads to

$$g = \beta \cosh^{\frac{2-b}{2(b-1)}} (\alpha x + \delta), \quad (35)$$

where

$$\alpha = \frac{\sqrt{2(3b-4)(1-b)}}{(2-b)}, \quad (36)$$

and β, δ being constants of integration. Hence

$$f = n\beta \cosh^{\frac{b-2}{2(b-1)}} (\alpha x + \delta), \quad (37)$$

$$\lambda = m e^{\frac{Lbt}{2(1-b)}}, \quad (38)$$

$$\mu = e^{\frac{Lbt}{2(1-b)} + Mt + \log N}, \quad (39)$$

$$\nu = e^{\frac{Lbt}{2(1-b)} - Mt - \log N}. \quad (40)$$

Therefore, we have

$$A = f\lambda = mn\beta e^{\frac{Lbt}{2(1-b)}} \cosh^{\frac{b-2}{2}}(\alpha x + \delta), \quad (41)$$

$$B = g\mu = N\beta e^{(\frac{L}{1-b} + 2M)\frac{t}{2}} \cosh^{\frac{2-b}{2(b-1)}}(\alpha x + \delta), \quad (42)$$

$$C = g\nu = \frac{\beta}{N} e^{(\frac{L}{1-b} - 2M)\frac{t}{2}} \cosh^{\frac{2-b}{2(b-1)}}(\alpha x + \delta). \quad (43)$$

By using the transformation

$$\begin{aligned} X &= x + \frac{\delta}{\alpha}, \\ Y &= y, \\ Z &= z, \\ T &= t, \end{aligned} \quad (44)$$

the metric (1) reduces to the form

$$\begin{aligned} ds^2 &= K^2 \cosh^{b-2}(\alpha X) e^{\frac{Ltb}{1-b}} (dX^2 - dT^2) + \\ &G^2 \cosh^{\frac{2-b}{b-1}}(\alpha X) e^{(\frac{L}{1-b} + 2M)T} dY^2 + H^2 \cosh^{\frac{2-b}{b-1}}(\alpha X) e^{(\frac{L}{1-b} - 2M)T} dZ^2, \end{aligned} \quad (45)$$

where $K = mn\beta$, $G = N\beta$ and $H = \frac{\beta}{N}$.

Since the magnetic permeability is a variable quantity, we have assumed it as

$$\bar{\mu} = e^{-(\frac{L}{1-b} + 2M)T}. \quad (46)$$

Thus $\bar{\mu} \rightarrow 0$ as $T \rightarrow \infty$ and $\bar{\mu} = 1$ when $T \rightarrow 0$. Zel'dovich [52] has explained that $\rho_s/\rho_c \sim 2.5 \times 10^{-3}$, where ρ_s is the mass density and ρ_c the critical density then the bodies frozen in plasma would change their density like a^{-2} i.e. like t^{-1} in the radiation dominated universe where a is the radius of the universe.

4. SOME PHYSICAL AND GEOMETRIC PROPERTIES

The physical parameters, pressure (p) and density (ρ), for the model (45) are given by

$$\begin{aligned} 8\pi p &= \frac{1}{K^2} e^{\frac{mnbT}{b-1}} \cosh^{2-b}(\alpha X) \left[\frac{(2-b)^2 \alpha^2}{4(b-1)} \left\{ 1 + \frac{2-b}{b-1} \tanh^2(\alpha X) \right\} \right. \\ &\quad \left. - \frac{L^2}{4(1-b)^2} - M^2 \right] - \Lambda, \end{aligned} \quad (47)$$

$$8\pi\rho = \frac{1}{K^2} e^{\frac{mbT}{b-1}} \cosh^{2-b}(\alpha X) \left[\frac{(2-b)\alpha^2}{2(b-1)} \left\{ \frac{b}{b-1} \tanh^2(\alpha X) - 1 \right\} + \frac{L^2(2b-1)}{4(1-b)^2} - M^2 - \frac{ML}{(1-b)} \right] + \Lambda. \quad (48)$$

The non-vanishing component F_{12} of the electromagnetic field tensor is given by

$$F_{12} = \sqrt{\frac{ML}{4\pi(1-b)}} G \cosh^{\frac{2-b}{2(b-1)}}(\alpha X), \quad (49)$$

which is function of x alone. So it is consistent as the Maxwell's equations (7) and (8) require F_{12} to be function of x alone.

The scalar of expansion (θ) calculated for the flow vector (v^i) is given by

$$\theta = \frac{L(b+2)}{2K(1-b)} e^{\frac{LbT}{2(b-1)}} \cosh^{\frac{(2-b)}{2}}(\alpha X) \quad (50)$$

The shear scalar (σ^2), acceleration vector (\dot{v}_i), deceleration parameter q and proper volume (V^3) for the model (45) are given by

$$\sigma^2 = \frac{(L^2 + 12M^2)}{12K^2} e^{\frac{LbT}{(b-1)}} \cosh^{(2-b)}(\alpha X), \quad (51)$$

$$\dot{v}_i = \left(\frac{\alpha(b-2)}{2} \tanh(\alpha X), 0, 0, 0 \right), \quad (52)$$

$$q = -1, \quad (53)$$

$$V^3 = \sqrt{-g} = K^2 G H e^{\frac{L(b+1)T}{(1-b)}} \cosh^{\frac{(b-2)(b-3)}{(b-1)}}(\alpha X). \quad (54)$$

From equations (50) and (51), we have

$$\frac{\sigma^2}{\theta^2} = \frac{(L^2 + 12M^2)(1-b^2)}{3L^2(b+2)^2} = \text{constant}. \quad (55)$$

The rotation ω is identically zero and the non-vanishing component of conformal curvature tensor are given by

$$C_{(1212)} = \frac{1}{6K^2} e^{\frac{LbT}{(b-1)}} \cosh^{(2-b)}(\alpha X) \left[b\alpha - \frac{L^2}{4b} + 3ML - 2M^2 \right], \quad (56)$$

$$C_{(1313)} = \frac{1}{6K^2} e^{\frac{LbT}{(b-1)}} \cosh^{(2-b)}(\alpha X) \left[b\alpha - \frac{L^2}{4b} - 3ML - 2M^2 \right], \quad (57)$$

$$C_{(2323)} = \frac{1}{3K^2} e^{\frac{LbT}{(b-1)}} \cosh^{(2-b)}(\alpha X) \left[-b\alpha + \frac{L^2}{4b} + \frac{ML}{(1-b)} + 2M^2 \right], \quad (58)$$

$$C_{(1224)} = \frac{ML}{2K^2} e^{\frac{LbT}{(b-1)}} \cosh^{(2-b)}(\alpha X). \quad (59)$$

The rate of expansion H_i in the direction of x, y and z are given by

$$H_x = \frac{A_4}{A} = \frac{Lb}{2(1-b)}, \quad (60)$$

$$H_y = \frac{B_4}{B} = \frac{1}{2} \left(\frac{L}{1-b} + 2M \right), \quad (61)$$

$$H_z = \frac{C_4}{C} = \frac{1}{2} \left(\frac{L}{1-b} - 2M \right). \quad (62)$$

The dominant energy condition is given by Hawking and Ellis [53]

$$(i) \quad \rho - p \geq 0$$

$$(ii) \quad \rho + p \geq 0$$

lead to

$$e^{\frac{mnbT}{(b-1)}} \left[\frac{(2-b)\alpha^2}{2(b-1)} \left\{ \frac{(b^2-6b+4)}{2(1-b)} \tanh^2(\alpha X) + \frac{b-4}{2} \right\} + \frac{L^2b}{2(1-b)^2} - \frac{ML}{(1-b)} \right] + 2K^2\Lambda \cosh^{(b-2)}(\alpha X) \geq 0 \quad (63)$$

and

$$\frac{(2-b)\alpha^2}{4(b-1)^2} [(b^2-2b+4) \tanh^2(\alpha X) + b(1-b)] \geq \frac{L^2}{2(1-b)} + 2M^2 + \frac{ML}{(1-b)}, \quad (64)$$

respectively.

5. CONCLUSION

We have obtained a new plane-symmetric inhomogeneous cosmological model of electro-magnetic perfect fluid as the source of matter with variable magnetic permeability. Generally the model represents expanding, shearing, non-rotating and Petrov type-II non-degenerate universe in which the flow vector is geodesic. We find that the model starts expanding at $T = 0$ and goes on expanding indefinitely. However, if $b < 0$ the process of contraction starts and at $T = \infty$ the expansion stops. For large values of T , the model is conformally flat and Petrov type-II non-degenerate otherwise. Since $\frac{\sigma}{\theta} = \text{constant}$, hence the model does not approach isotropy. The electromagnetic field tensor does not vanish when $L \neq 0$, $M \neq 0$, and $G \neq 0$.

In spite of homogeneity at large scale our universe is inhomogeneous at small scale, so physical quantities being position-dependent are more natural in our observable universe if we do not go to super high scale. Our derived model shows this kind of physical importance. From equation (53), it follows that our model of the universe is consistent with recent observations which reveal that the present universe is in accelerating phase and the deceleration parameter lies somewhere in the range $-1 < q \leq 0$. For our model $q = -1$ as in the case of de Sitter universe.

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