Vafa's Approach: F-Theory and GUTs

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Abstract. After a short mention of String- and M- and F-Theory, we introduce the new methods of Vafa (2008), which aim to understand some features of the Standard Model (without gravity), by following a "bottom-up" approach in F-Theory. We recall briefly the role of the 7-Branes, which encode the gauge groups and can generate chiral matter in their intersections. Prospects for a realistic Grand Unified Theory (GUT) are also pointed out.

1. Review of String Theory

The *aim* of String Theory is to deduce the Standard Model (SM) of particles and forces from scratch, including gravitational interactions; the fundamental objects are *strings*, not point particles. Starting with an inglorious birth in 1968, String Theory was proposed first as a possible theory for strong interactions; but by 1972 we had Quantum Chromodynamics (QCD) for hadrons, and String Theory was proposed then as a theory of everything (essentially because it included gravitation), (Yoneda; Scherk and Schwarz; 1973/75). A bright moment arose in 1984, when Green and Schwarz proved that the absence of anomalies selected dimensions and gauge groups, to the extent that there were few viable so-called SuperString theories. By early 1985, the situation was: there were only FIVE SuperString theories, all living in 10 dimensions, all included gravitation, all were supersymmetric on the worldsheet (2 dimensions) *and* in the target space (10 dimensions):

(1) Type IIA: [closed, nonchiral, $\mathcal{N} = 2$ Susy, no gauge group].
(2) Type IIB: [closed, chiral, $\mathcal{N} = 2$ Susy, no gauge group].
(3) Heterotic Exceptional H-E: [closed, $\mathcal{N} = 1$ Susy, $E_8 \times E_8$ gauge group].
(4) Heterotic Orthogonal H-O: [closed, $\mathcal{N} = 1$ Susy, $O(32)$ gauge group].
(5) Type I: [open, $\mathcal{N} = 1$, $O(32)$ gauge group].

Here $\mathcal{N}$ is the number of Supercharges. Notice the exotics: besides working only (!) in $10 = (9, 1)$ dimensions, the gauge groups were also very peculiar (the largest exceptional Lie group E_8 by duplicate, or $SO(32)$, which has the same dimension (496) and rank(16)).

The favourite *girl* was the Heterotic Exceptional SuperString, as E_8^2 could give rise, in principle, to E_6 (with another hidden E_8 sector) and other Grand Unified Theory Groups, down to the Standard Model (SM) gauge group $SU(3) \times SU(2) \times U(1) \approx$ "$3, 2, 1$". The method was

compactification (dimensional reduction) from 10 to our mundane 4 dimensions, by a compact complex manifold of type Calabi-Yau (CY_3), as to provide only $\mathcal{N} = 1$ Supersymmetry in four dimensions:

10-dimensional Heterotic-Exceptional Theory [$\mathcal{N} = 1 \Leftrightarrow 16$ elementary Supercharges, group E_8^2] descending to 4-dim (Minkowski space), $\mathcal{N} = 1$; 4 elementary Supercharges, gauge group "3, 2, 1". This might be achieved by means of a Calabi-Yau space with holonomy $SU(3)$, allowing a surviving spinor (as $SU(3) \subset SU(4) = Spin(6),\ 6 = 10 - 4$), so to mantain Susy $\mathcal{N} = 1$ down to 4 dimensions to allow for chiral couplings.

No model was realistic, but Candelas and coll. in Austin, TX, discovered a new phenomenon, *Mirror Symmetry*, of enormous interest in pure mathematics [4]. However, this heterotic model will play a role in our story.

Best reference for this "classical" period of SuperStrings is the book [3].

2. M- and F- Theories

In 1995 Paul Townsend, Ed Witten and Joe Polchinski established "M-Theory", linking the 10-dimensional five SuperStrings among themselves and also with Supergravity in 11-dimensions and allowing for extended objects other than strings:

1) Supergravity in 11 dimensions was known since 1978 (Cremmer, Julia and Scherk), and its particle content

$$h \text{ graviton, } 44, \quad \Psi \text{ gravitino}, 128, \quad C \text{ (3}-\text{form}, 84) \tag{1}$$

became the particle content of the low-energy limit of the IIA theory by descending to ten dimensions: $h \to (h + A + \Phi)$, or $44 \to 35 + 8 + 1$; $C \to (B + C)$, etc. Besides, the membrane "$M2$" seen in M-theory became the fundamental string in 10-dimensions, after wraping a circle S^1. *Viceversa*, and that was really the big breakthrough, Witten showed explicitly (March, 1995) how the strong limit of the IIA-theory in 10 dimensions generated a 11-th one, becoming M-Theory by definition.

2) M-theory compactified in a segment *could* be equivalent to the H-E theory (Horawa and Witten, 1996).

3) Besides, it was already known that by T-duality (descending to 9 dimensions) the IIA and IIB theories were equivalent (there is no chirality in odd dimensions), and so were the two heterotic strings, H-E and H-O.

4) Also, Polchinsky and Witten proved that the strong limit of the H-O theory corresponds to the weak limit of the Type I (open) string: recall they share the same gauge group, O(32).

This unique and ill-defined theory, encompassing the five superstring theories, and whose low-energy particle limit was coincidental with the particle content of 11-dim SuperGravity, and containing the $M2$ brane and other extended objects, was christened M-Theory, and it was, for a (short) time, the "Gran Esperanza Blanca". It included, besides strings, other extended objects, among them the D-p-Branes, radiating $(p + 1)$-forms, which played an important role as terminal endings of open strings. The best reference here is the preprint book [5]

Now, in mid-2009, M-theory has not lived up the expectations: The Standard Model is not recovered, neither some of its conspicuous features explained, as number of generations, chiral

matter, Grand Unification, the very different mass scales, etc.; neither has been any clarification of cosmological issues, inspite of several attemps.

Before we delve in the extension to 12-dim theories (originally from C. Vafa, 1996), we need to show the particle content of the diverse superstring theories.

The primordial theory is a $\mathcal{N} = 1$ Yang-Mills gauge theory in 10 dimensions; no counting the gauge group, the symmetry is $8_v = 8_s$, namely the irreducible representation vector 8_v of $O(8)$ and the two spinor representations (*irreps*) 8_s, $s = L$ or $s = R$ of $Spin(8)$ are equivalent under triality. From this situation we construct *four* supersymmetric theories:

1) IIA Theory: $(8_v - 8_L) \times (8_v - 8_R) =$ NS sector + RR sector − Fermi sector = 35 (h, graviton) + 28(B, $2 -$ form) + $1(\Phi,$ dilaton) + 56(C, $3 -$ form) + 8(A, $1 -$ form) − fermions

2) IIB Theory: $(8_v - 8_L) \times (8_v - 8_L) = (h + B + \Phi + D^+ + B' + \Phi' -$ fermions)

3) Type I and heterotic: $(8_v - 8_s) \wedge (8_v - 8_s) = (h + B + \Phi, -$ fermions)

The Neveu-Schwarz (NS) sector is the Bose $\times$ Bose product, the Ramond (RR) sector is Fermi $\times$ Fermi product; the fermions come from the Bose $\times$ Fermi product. The symmetrizing symbol ($\wedge$) means: symmetrize the bose product and antisymmetrize the fermi product.

11-dim SuperGravity has the particle content as the IIA theory, but seen from eleven dimensions:

4) 11d Supergravity: 44 (h, graviton) + 84 (C, 3-form) -128 (Ψ, gravitino).

As particles couple to paths, strings couple to surfaces (world-sheet), and in general even dimensional p-*branes* couple to odd (p+1)-forms. In particular, in IIA theory, we have even-dimensional Branes, in IIB odd ones. The 7-Brane of IIB comes from dualizing the axion (or RR scalar, a 0-form), and it will play an important role.

The 3-form C in 11-dim SuGra couples to a 2-Brane (called $M2$-Brane), which becomes the fundamental string of the IIA theory after the descent $11d \to 10d$.

Notice there is *no* direct route to IIB from M-theory: it is only via compactification to 9 dimensions that IIA and IIB are duals of each other. This (and other reasons) prompted C. Vafa in 1996 to introduce F-Theory:

There is a "fundamental" theory in 12 dimensions, becoming M-theory with circle (S^1) compactation, but also it becomes IIB theory with torus $(T^2 = S^1 \times S^1)$ compactation:

$$T^2 \to \text{"F"} - \text{Theory} \to \text{IIB} \quad (\text{elliptic fibration}) \tag{2}$$

The two scalars (a, Φ) in IIB combine to parameterize the Torus T^2, $\chi = a + i\exp(\Phi)$, as they vary along the 10-dim IIB theory. There was much reluctance to accept F-Theory at face value: for one thing, to keep the number of supersymmetries, F-Theory has to include two time dimensions. For another, the whole particle content is unclear. The most conservative approach is to consider just the IIB theory, with the 2-Torus to vary over the surface of the 10-dim theory, allowing even the "fibers" to shrink at some points: this is the difference between fiber bundle and fibration, which is the case here. Notice the Torus T^2 is the only oriented surface with

Euler number $= 0$, hence the only one that can accept a nonzero vector field, that is, it admits a metric with $(1,1)$ signature, as F-Theory would imply perhaps two time dimensions.

There are several additional reasons to believe in F-theory [1]:

1) There is a Chern-Simons "boundary" term $(dC) \cdot (dC) \cdot C$ in the Lagrangian of 11d SuGra, which is best understood as coming from a conventional term $(dC)^3$ in 12 dimensions.

2) There is $(2,2)$ Brane extant in the Brane scan, which lives naturally in $12 = (10,2)$ dimensions, and generates the $M2$ Brane in eleven and the string in ten dimensions.

3) The algebra of 32 elementary supercharges of 11d SuGra still operates in 12 dimensions, as $2^{12/2-1} = 32$, and the $(10 - 2 = 8)$ type is real, so there is no need to enlarge the supersymmetries of the system.

4) We remarked above that $|8_v - 8_s|^2$ gives the particle content of 11-dim Sugra. Well, it turns out that another squaring $(44(h) - 128(\Psi) + 84(C))^2$ [6] gives the content of a putative 27-plet of particles of F-Theory. Details can be found in [1].

5) Compactification from 10d to 4d via CY_3 is complemented with compactation from 11 to 4 via manifolds with exceptional holonomy G_2, but also (and finally!) compactation from 12 to 4 can occur either with CY_4 manifolds, or $Spin(7)$ exceptional holonomy manifolds [7]. It is remarkable the (hidden) relation to octonions: $Spin(7)$ leaves the S^7-sphere of unit octonions invariant, G_2 leaves the S^6-sphere of unit imaginary octonions fixed, and $SU(3)$ leaves the equator of the later (S^5) also invariant.

Best reference for this "modern" period of strings are the Lectures [2].

3. The New Theory of Vafa (2008)

The philosphy of the new approach of Cumrum Vafa is based on two main ideas:

1) *Bottom-up* method: do not get trapped in the labyrinth of infinite vacua landscapes ($\approx 10^{500}$!!) which occur in the latest versions of string theories, and try instead to retrieve some of the features of the Standard Model (SM) model.

2) Please decouple *gravitation*: totally insignificant for microscopic physics; even the possible GUT scale, 10^{16} GeV, is far from the scale of Planck, 10^{19} GeV. This philosophy is totally contrary to the old wisdom, in which the automatic presence of gravitation was one of the atractiveness of string models (see our Sect.1). One of the present reasons to neglect gravity with respect to the other forces is this: people are getting more and more convinced that the extreme UV limit of gravitation would require new physics (for example, non-commutative geometry), in order to cope with exotic phenomena, like black holes and that like.

Vafa addresses directly the Grand Unification (GUT) issue: how the gauge groups of the standard model $SU(3) \times SU(2) \times U(1)$ would fit in a higher gauge group. He observes the "3, 2, 1" group is close to the E_3 group ($SU(3) \times SU(2)$), which appears naturally in the E_8 to $SU(2) = A_1 = E_1$ chain. He also addresses another important feature of the SM: the origin of chiral matter, a conspicuous aspect of nature, implying that matter multiplets are not invariant under parity; we even observe maximal parity violation in the beta decay sector of the

electroweak theory.

There are three basic assumptions in Vafa's theory:

1) Low-scale 4-dim $\mathcal{N} = 1$ *Susy* is present at energies around the TeV (hence testable soon at the forthcoming accelerator LHC). Is this evidence for the Minimal Supersymmetric Standard Model (MSSM)?

2) At high energies, the matter content of the MSSM unifies into a GUT (Grand Unified Theory) scheme: either $SU(5)$, the original Georgi-Glashow invention (1974), or the offsprings $SO(10)$ (Fritsch and Minkowski) or E_6 (Gürsey, Ramond) [16]. Partial evidence for this stems from high-energy coupling constant unification, pointing out to $E_{GUT} \approx 10^{16}\ GeV$, well below the $10^{19}\ GeV$ of Plancks scale. This leads to the third assumption:

3) There is a well-separated limit

$$M_{GUT}/M_{Pl} \to 0 \tag{3}$$

which is the numerical reason for the decoupling of gravity (Cfr. remarks above).

The starting point to concrete calculations in this new Vafa's theory is the following: Take F-Theory, in the above sense, as a variable fibration of the 10-dimensional IIB theory by a 2-Torus. The main ingredient is *strong duality*, implying for example equivalence with the heterotic string suitably compactified. For example

$$F - Theory/K3 \quad \approx Het - Excep/T^2 \tag{4}$$

We recall $K3$ is the (topologically) unique manifold with $SU(2)$ holonomy, i.e. CY_2. How on earth the gauge group E_8^2 of the heterotic string is hidden in this compactation of F-theory? The answer is that it is hidden in the $K3$ manifold: the Hirzebruch signature of $K3$ is $(19, 3)$, and this 16 is related to the two E_8 groups!

How F-Theory (seen as a IIB Theory in 10 dimensions with a variable complex coupling χ) generates the physics can be seen in the following general scheme:

Gravitation (that we neglect) is extant in ten dimensions. Reduction to 8 makes it appear the all-important 7-Brane, with the time as the 8-th direction. This brane can be seen as dual to the RR scalar, the axion a.

In the main body of the 7-Brane the gauge groups emerge as singularities of the fibration: the membrane wraps around some cycles of the compactifying manifold; when the cycles collapse, the brane becomes massless, and hence supports gauge groups, which are also massless: there is a well-known relation between ADE singularities and the emergence of the ADE Lie groups (Arnold). Here A are the unitary groups, D the even orthogonal, E the exceptional E-groups.

Now intersection of 7-Branes gives rise, in 6 dimensions, to the presence of chiral matter (we do not enter into the details). Finally, descent to realistic, 4-dimensions makes it appear the Yukawa couplings among the scalars and fermi matter.

Here are some references for this new Vafa's theory: [8, 9, 10, 11].

4. Descent by even-dimensional manifolds

This is a synoptic scheme of the several phases on the descent from the hypothetical F-Theory living on 12 dimensions to the realistic, 4-dimensional spacetime; the process is better understood in steps of *two* dimensions each:

1) In twelve dimensions we have the realm of F-Theory. Here gravitation is supposed to dominate, but indeed the particle content is obscure. Best proposal seems to be a supersymmetric 27-plet of particles associated to the symmetric space $E_6/Spin^c(10)$ [1], which is the complexification of $F_4/Spin(9)$, the determinant of the trio of particles in 11-dim SuGra [12]. For the future.

2) Ten dimensions. The realm of the original IIB theory. Descent by an elliptic fibration, that is, F-Th/Ell $\approx IIB$, where "Ell" is a Torus (an elliptic curve): this is close to conventional string theory. The theory is self-dual: there is a discrete $SL_2(Z)$ symmetry, with the half-4-form and the graviton fixed, and with varying complex parameter $z = a + i\ exp(\Phi)$ undergoing an homographic transformation.

3) Eight dimensions. The realm of the space of the D-7-Brane. It could be understood directly as a $K3$ compactation from 12 dimensions, with $K3$ itself an elliptic fibration. When the fibration degenerates, as a membrane wraps around one of the 22 two-cycles of $K3$, there appear ADE-type singularities, and the cycles collapse so there are massless excitations, signalling the generation of gauge groups: this is the correspondence between singularity types and ADE gauge groups. Exceptional groups are favoured, as well as spin representations. (For example, the E_7 group may appear by combining the $SU(8)$ group with its four-form representation: $8^2 - 1 + 70 = 133$. Also a dual version consists on a stack of collapsing D-7 Branes on top of each other, with the connecting strings shrinking: this duality has been shown by Sen [14].

4) Six dimensions. Here the string duality dominates, one of the possible forms being similar to the IIB-Heterotic duality mentioned above, $IIB/K3 \approx HE/T^4$. In particular, the E_8^2 gauge group in the HE theory emerges from the hidden homology of $K3$; see above. From the physical point of view, two $D7$-Branes intersect also in six dimensions, and this generates, by an obscure procedure, chiral matter with bi-fundamental representations, another genuine feature of the standard model SM.

5) In the final descent to realistic four dimensions, we witness the appearance of Yukawa couplings, completing the ingredients of the SM. Hopefully one should describe a $\mathcal{N}$=1 Supersymmetric theory, at a scale of the order of the TeV: this is a prediction for the forthcoming experiments with the LHC at CERN. So we expect $1 + 4$ Higgses and plenty of Susy partners... Finally, even Susy is broken, and we have the bare SM at energies of 100 GeV, with no trace of Susy.

6) There are still many open problems, besides the problematic character of some of the above steps; after excluding gravitation, the most sensible absent feature is perhaps the unexplained existence of the three generations of quarks and leptons; recently some progress is being made on this issue also, but by a totally different approach: finite nonabelian groups as remnants of another broken gauge symmetry [15].

It should be clear to the reader that this is a wishful thinking development, although each step exhibits a well-defined mathematical structure. The general scheme seems clear, with the main ingredients fairly well understood: generation of gauge groups, presence of chiral matter

and Yukawa couplings, similar to hoped-for Higgs couplings. Details will keep theoreticians busy for (many?) years to come.

Acknowledgements

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