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A unified formalism to study transverse momentum spectra in heavy-ion collision

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A R T I C L E I N F O

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A B S T R A C T

The study of transverse momentum spectra is crucial to understand the nature of matter produced during heavy-ion collisions. The p_T-spectra in a heavy-ion collision consists of a low p_T-region where soft processes dominate particle production, whereas the high p_T-region is mostly dominated by hard processes. Single and multi-component models based on statistical thermodynamics are extensively used to characterize the spectra. In this work, we have introduced a unified non-extensive statistical approach using the Pearson distribution as a tool to study p_T-spectra. The goodness-of-fit of the proposed distribution as compared to previously used models makes it an interesting method providing strong insights into the underlying physics of heavy-ion collisions. This generalized approach provides a strong correlation with other observables by comparing the predictions of the methods in p_T-distributions with various harmonics of azimuthal distributions.

1. Introduction

Ultra-relativistic nuclear-nuclear collision at RHIC and LHC has opened the door to study the state of matter produced under extreme conditions of temperature and pressure known as Quark Gluon Plasma (QGP). Transverse momentum (p_T) spectra is an important observable as it provides crucial information about the equilibrium dynamics as well as the anisotropy of the system produced in heavy-ion collision [1]. An appropriate theoretical formalism to describe transverse momentum spectra is essential and of great interest within the particle physics community. In this direction, several approaches are being made to find a generalized distribution with an aim to explain the momentum distribution. Due to the asymptotic freedom and very nature of QCD coupling, the coupling strength is extremely strong at low-p_T making it almost impossible to apply perturbative calculations in the region. Thus, statistical approaches become more prominent and turn out to be successful in explaining the p_T-spectra. The idea of using a statistical model to describe particle production was first introduced in 1948 by Heinz Koppe [2,3] and later by Enrico Fermi [4–6]. However, the first systematic application of the statistical model to describe high energy collision was developed by Hagedorn [7,8]. Most of these approaches consider the system under explanation to be of thermal origin. Therefore, considering the system produced in heavy-ion collision as a thermal system of particles, the particle spectra can be well described by standard *Boltzmann-Gibbs* statistics [9], and p_T-spectra in this formalism is given as

$$\frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy} = \frac{gVm_T}{(2\pi)^3} exp\left(-\frac{m_T}{T}\right) \tag{1}$$

where m_T is the transverse mass of particle under consideration given as $\sqrt{p_T^2+m^2}$, g is the spin degeneracy factor, V is the volume and T is the temperature of the system. Boltzmann-Gibbs statistics is a good approximation to study the systems where constituents are independent or weakly correlated, hence entropy will be additive as well as extensive [10]. However, it fail to explain the strongly correlated systems where long-range correlations & interactions are significant [11]. Further, in a strongly correlated system, there might exist the case when entropy is non-extensive or non-additive, and hence Boltzmann-Gibbs statistics will no longer be suitably applicable. On the other hand, the memory effects and long-range color interactions may give rise to non-Markovian processes which in turn affect the dynamical evolution of the fireball produced in heavy-ion collision as described in Ref. [12]. A clear experimental deviation from the Boltzmann-Gibbs distribution function is observed at higher transverse momenta. This deviation can be interpreted as due to the presence of long-range interactions [13] in the produced fireball. The observed transverse momentum spectra looks more like a power-law distribution rather

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than an exponential distribution; this might be due to the intrinsic nature of the Boltzmann-Gibbs statistics. The Tsallis approach overcomes these limitations by incorporating the effects of non-extensivity in the system, and these ideas are discussed in seminal work in Ref. [14]. At the same time, systems with long-range interaction have been nicely explained within Tsallis statistics framework [15–18].

The Tsallis generalization introduces a new parameter, q, which intrinsically takes care of non-extensivity, where the normal exponential is replaced by the $q-$exponential [19]

$$e_q^x = [1 + (q-1)x]^{\frac{1}{q-1}} \tag{2}$$

and Tsallis entropy is given as

$$S_q = k\frac{1-\sum_i p_i^q}{q-1} \tag{3}$$

which in the limit $q \rightarrow 1$ satisfy the Boltzmann-Gibbs entropy

$$S = \sum_i p_i ln(p_i) \tag{4}$$

Since the q parameter explains the deviation from thermal equilibrium, it can be used to quantify temperature fluctuations around the equillibrated value of temperature T_0 and the q parameter can be connected to variance of temperature [20,21]

$$q-1 = \frac{Var(T)}{\langle T\rangle^2} \tag{5}$$

With a negligible chemical potential in LHC energies [22], the Tsallis statistics [23] defines the transverse momentum spectra as

$$\frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy} = \frac{gVm_T}{(2\pi)^3}\left[1+(q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}} \tag{6}$$

where Tsallis distribution preserves Boltzmann-Gibbs distribution in the limit $q \rightarrow 1$. The normalized q-expectation value is usually expanded as a series of the value around $(1-q)$, where the absolute value $|1-q|$ is the measure of the deviation from the Boltzmann-Gibbs statistics [24]. The thermodynamical consistency of Tsallis statistics is proved in Ref. [19], which validates its applicability in explaining p_T-spectra & maintaining the properties of Boltzmann-Gibbs statistics in the limit of non-extensivity. Although the Tsallis approach remains very successful in explaining the low-p_T part of the spectra (except at very low values), it deviates from experimental data towards higher-p_T. This might be due to finite contributions from "soft excitation process" as well as "hard scattering process" to the spectrum of particles produced in heavy-ion collision [25–28]. In other work, considering the Tsallis distribution as a good approximation for p_T-spectra for soft processes [29–32], it has been derived that the corresponding particle multiplicity spectra will be of the form of a negative binomial distribution (NBD) [33]. However, it is observed that a single NBD doesn't explain particle produced in hard processes [29–32]. At the same time, QCD calculations suggest that the inverse power-law distribution successfully describes p_T-spectra for hard scattering processes. Hence, the Tsallis approach has not much to say about the hard-processes, where more generalized descriptions should be applied. Our proposal is to combine spectra from both processes and to explain them in a more generalized way, which could not be done in previous approaches.

As we discussed earlier, there does not exist a well established theory that can be used to explain the form of p_T-spectra for both soft & hard processes (usually hard process are explained using perturbative QCD). The perturbative QCD calculations suggest that transverse momentum spectra of particles produced in hard scattering processes will be of the form of an inverse power-law [34–38] given by the Eq. (7)

$$f(p_T) = \frac{1}{N}\frac{dN}{dp_T} = Ap_T\left(1+\frac{p_T}{p_0}\right)^{-n} \tag{7}$$

Here A is normalization constant and p_0, n are free parameters. The relationship in the above equation enforces a generalization in such a way that at low-p_T, we rely on statistical physics to explain the spectra, whereas at high-p_T we have a well established perturbative QCD theory that nicely explains transverse momentum spectra.

2. Pearson formalism

Our proposed method is a generalized approach which is in-line to the Tsallis statistics, which can explain the spectra at low-p_T well. The basic goal is to present a generalized approach to satisfy both "soft" and "hard" processes, keeping all thermodynamical properties preserved, and to develop a unified spectral distribution which is required for upcoming high-energy & high-luminosity experiments like High luminosity LHC, Future Circular Collider (FCC) etc., where a dominance of hard-processes is anticipated. Therefore, we introduce the "Pearson Distribution" as the master equation to obtain a generalized distribution of the subject under discussion. The *Pearson Distribution* was initially discussed by Karl Pearson in 1895 [39] and subsequently modified in 1901 and 1916. Pearson's proposal was to classify a distribution function based on the first four moments related to mean, standard deviation, skewness and kurtosis of the distribution. Having successful applications in many different fields like geophysics, statistics and financial marketing, the Pearson's distribution provides ample scope to explore generalizations of spectral analyses, in our case, particularly in momentum spectra. The Pearson distribution function is expressed in the form of a differential equation [40] as:

$$\frac{1}{p(x)}\frac{dp(x)}{dx} + \frac{a+x}{b_0+b_1x+b_2x^2} = 0 \tag{8}$$

where parameters a, b_0, b_1, b_2 can be related to first four moments as:

$$a = b_1 = \frac{m_3(m_4+3m_2^2)}{10m_2m_4 - 18m_2^3 - 12m_3^2} \tag{9}$$

$$b_0 = \frac{m_2(4m_2m_4 - 3m_3^2)}{10m_2m_4 - 18m_2^3 - 12m_3^2} \tag{10}$$

$$b_2 = \frac{2m_2m_4 - 6m_2^3 - 3m_3^2}{10m_2m_4 - 18m_2^3 - 12m_3^2} \tag{11}$$

here m_1, m_2, m_3 and m_4 are the first four central moments with $m_1 = 0$ by construction. Different conditions on parameters a, b_0, b_1, b_2 or more generally different types of roots of the quadratic equation in the denominator will give different distribution functions. There are 12 different distributions that comes under Pearson family of curves. The Pearson criteria which will decide the type of distribution is given as

$$k = \frac{b_1^2}{4b_0b_2} \tag{12}$$

This determines the type of roots of the quadratic equation in the denominator of Eq. 8. When the roots are real and have different signs (*i.e.* $k < 0$) we will get a Pearson distribution of type *I* also known as the β-distribution whereas real roots with identical signs (*i.e.* $k > 1$) correspond to type *VI*. Complex roots appear for $0 <$

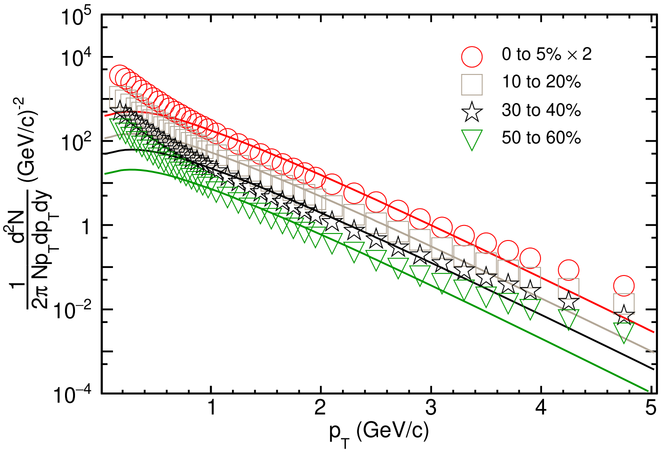

Fig. 1. Boltzmann fit of charged particle p_T-spectra produced in $PbPb$ collision for four different centrality bins at 2.76 TeV measured in ALICE at LHC.

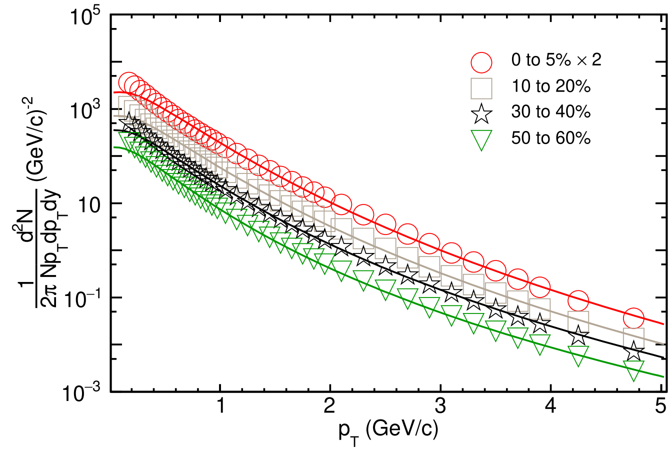

Fig. 2. Tsallis fit of charged particle p_T-spectra produced in $PbPb$ collision for four different centrality bins at 2.76 TeV measured in ALICE at LHC.

$k < 1$ and the corresponding distribution is classified as type *IV*. A detailed discussion of Pearson criteria can be found in Ref. [41,42]. Upon solving Eq. (8), we get:

$$p(x) = C(e + x)^f (g + x)^h \tag{13}$$

$$p(x) = B\left(1 + \frac{x}{e}\right)^f \left(1 + \frac{x}{g}\right)^h \tag{14}$$

upto some normalization constant $B = Ce^f g^h$. If we replace $g = \frac{T}{q-1}$, $h = -\frac{q}{q-1}$, $f = -n$ and $e = p_0$ we will get:

$$\frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy} = B\left(1 + \frac{p_T}{p_0}\right)^{-n}\left(1 + (q-1)\frac{p_T}{T}\right)^{-\frac{q}{q-1}} \tag{15}$$

where

$$B = C\frac{1}{(p_0)^n}\left(\frac{T}{q-1}\right)^{-\frac{q}{q-1}} \tag{16}$$

Interestingly, the backward compatibility can be proved by reducing the Pearson distribution to Tsallis distribution within the limit $n = -1$ and $p_0 = 0$, which indeed preserves all thermodynamic properties of the Tsallis distribution. It is therefore, a generalized version of the Tsallis distribution which has the extra advantage of explaining the spectra at higher-$p_T$which infers hard scattering processes. A detailed thermodynamical consistency check is beyond the scope of this article and will appear in follow-up work. However, we discuss outcomes of this approach by analyzing heavy-ion collision data.

3. Result and discussion

A detailed analysis was performed on the invariant yield of charged hadrons produced in a $PbPb$ collision at 2.76 TeV at LHC measured by ALICE experiment [43]. This dataset is chosen because of its better resolution and availability of the larger p_T range for all centrality bins which enable a better qualitative analysis and a better comparison between various models. We performed a numerical fitting by sliding the ranges in such a way that the models could predict their best fits over the entire range of spectra. The Boltzmann, Tsallis and Pearson fits to the transverse momentum spectra are shown in Figs. 1, 2 and 3 respectively. From the fitting analysis it is evident the Bolzmann statistics shows a poor agreement in explaining the spectra, Tsallis statistics fits only at the intermediate p_T range whereas Pearson gives a best fit results by expressing entire p_T range. Goodness of fit in terms of

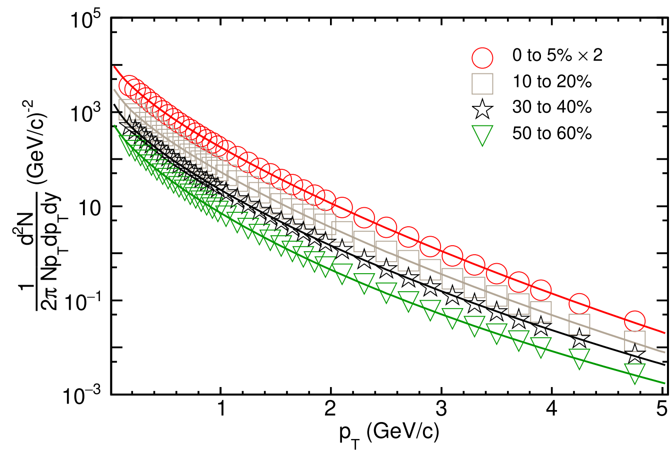

Fig. 3. Pearson fit of charged particle p_T-spectra produced in $PbPb$ collision for four different centrality bins at 2.76 TeV measured in ALICE at LHC.

Table 1
Best fit value of χ^2/NDF for different centrality bins.

Centrality	Boltzmann	Tsallis	Pearson
0 to 5%	25.3451	1.99445	0.10100
5 to 10%	25.5971	1.86747	0.08545
10 to 20%	26.5224	1.75271	0.08609
20 to 30%	27.6911	1.57784	0.08423
30 to 40%	28.3606	1.34457	0.06994
40 to 50%	29.8191	1.1226	0.05170
50 to 60%	29.4844	0.88907	0.03901
60 to 70%	27.9139	0.65552	0.02568

χ^2/NDF has been provided in Table 1 for different centrality bins and we observe that the values are minimum for Pearson distribution which support our claim of best fit using Pearson distribution. We have already discussed that the q parameter is a measure of deviation from thermal equilibrium, so the fitted values of q suggest a departure from equilibrium with an increase in centrality, where central collisions are more toward the equilibrium as compared to peripheral collisions. We observed a decreasing trend in q-parameter with centrality, which matches well with the observations in Tsallis Blast Wave (TBW) fits [44] and q-Weibull fits [45].

One possible reason for obtaining the best fit using the Pearson distribution is the presence of higher order moments as parameters whereas other distribution functions use only the mean and standard deviation as parameters. It was discussed earlier that the

Tsallis distribution is not fitting higher p_T values which form the tail part of the distribution. The tail of a distribution is sensitive to higher order moments, so a distribution function which depends on higher moments can fit the tail part in a more precise way as in our case. It can be clearly seen that the Pearson fitting is compatible with both low-p_T and high-p_T.

Upon a careful observation of the fitting results, it can be seen that while the numerical value of n is of the order 10^{-2}, the q produces a value close to unity. This shows a rapid decay of the Tsallis component of Eq. (15) and a slow decay of hard scattering processes. This also supports our argument that the first part of the Eq. (15) will be dominant in higher p_T region of the spectra.

The Pearson distribution is more physically realistic over many two component models that explain data as usually two component models fit the data separately in low-p_T and high-p_T missing a unified explanation. The Pearson distribution is a single function which is thermodynamically consistent and preserves both the Tsallis and Boltzmann-Gibbs distributions in different limiting cases as well as explain both low and high-p_T regions. Although the Pearson distribution nicely describes the momentum spectra, we have kept the parameters as free parameters and an exploration of those parameters will be the subject of a separate work to suffice our examinations.

With the successful reproduction of the momentum spectra of high energy collisions at the LHC, we examined other data interpretations, for example, a scaling of flow parameters in heavy-ion collisions. The flow parameters basically explain azimuthal anisotropies in particle production and are directly related to the fluctuation in the initial fireball produced in the collision, which plays a key role in understanding the dynamics of QGP evolution which in turn contains information of QCD phase structure [46]. The flow parameters appear in a Fourier series expansion of the azimuthal distribution of particles. Particle yield in terms of Fourier expansion is given as [47,48]

$$E\frac{d^3N}{dp^3} = \frac{1}{p_T}\frac{d^2N}{dp_T dy}\frac{N}{2\pi}[1 + 2\sum_n v_n cos\{n(\phi - \psi)\}] \quad (17)$$

Here, E is the energy of the particle, p_T is transverse momentum, v_n represents the nth order flow coefficient, y is rapidity, ψ represent the reaction plane angle and ϕ is the azimuthal angle. Flow coefficients are expressed as

$$v_n = \langle cos[n(\phi - \psi)]\rangle \quad (18)$$

where the angle brackets represent an average over all of the particles in all events. Since the differential flow depends directly on p_T & y, it is natural to estimate the flow by integrating the flow coefficient over p_T and y. We further explore the correlation of flow coefficients to different Pearson parameters. This is due to the fact that the flow-coefficient depends on the azimuthal anisotropy of momentum distribution and thus an imprint of flow must be present in the p_T-spectra of particles produced in the heavy-ion collision. Since the flow-coefficients directly depend on the initial geometry of the collision, there must be a centrality dependence of Pearson parameter which are connected to these flow-coefficient.

In the spirit of successful spectral analysis, we examined the correlation of various parameters with the flow coefficient. To verify the potential correlation, we investigated the charged hadron elliptic flow coefficient ($v_2\{2\}$ extracted using two particle cumulant method) integrated over p_T from 0.2 to 5 GeV/c with a rapidity interval of $|\eta| < 0.8$ for a $PbPb$ collision at 2.76 TeV for different centrality bins [49]. Further, we analyzed the p_T-spectra of charged hadrons in a similar range of p_T, η & centralities and obtained the values of parameters. Fig. 4 shows the Pearson parameter f versus centrality and observed that the behavior of

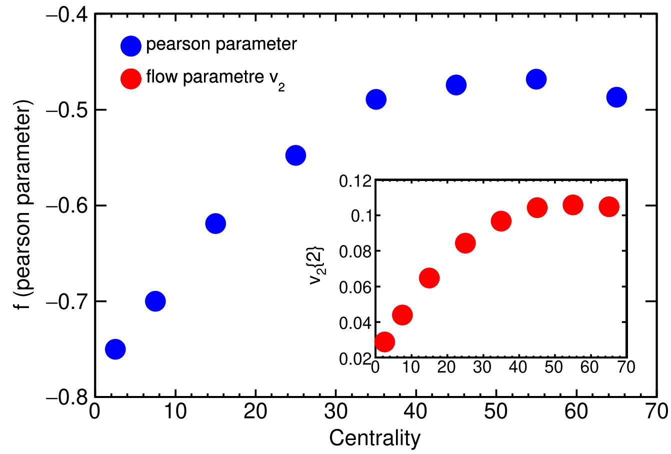

Fig. 4. Pearson parameter f versus centrality (%) for charged hadrons at 2.76 TeV and inlay shows the v2 at same energy.

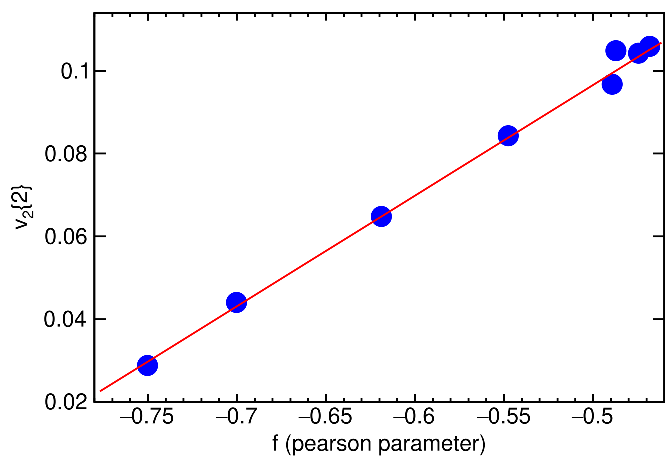

Fig. 5. Elliptic flow coefficient versus Pearson parameter f at 2.76 TeV $PbPb$ collision and the curve is fitted with a linear equation.

curves for integrated elliptic flow and Pearson f are of similar nature. More interestingly, both of them peak at the $50 - 60\%$ centrality bin. We also observed that there is a linear relation between f and $v_2\{2\}$ as in Fig. 5 fitted by $v_2 = 0.267426f + 0.230294$ and the Pearson correlation coefficient is 0.9976 which proves the linear relation between these two variables.

These exciting results envisage that the v_2 can be obtained using the Pearson's f parameter which requires more data for estimating the energy dependent correlation. The above results establish a direct correlation of anisotropic flow to the parameters of the Pearson distribution as the flow coefficients are directly obtained from transverse momentum spectra instead of a conventional flow analysis.

4. Conclusion

To conclude, our study presents a complete solution for the explanation of transverse momentum spectra of particles produced in high energy collisions. We examine a single particle Pearson distribution function, which in turn introduces a generalized definition of entropy. In this work, we have demonstrated that the study of the Pearson distribution and its application has the unique potential to resolve particle production through hard-processes in high-energy collisions. The particular strength of our findings is that they apply to particles which are produced throughout spectrum from low to high-p_T. To receive the ultimate benefit of the proposed method requires sufficient and wide testing of different observables, which requires more events samples. The Pearson formalism provides a strong motivation for the studies of high-p_T particle production in heavy-ion collisions and this work should be expanded in the future to include larger statistics and data from a similar range of results for v_2 and p_T.

This approach opens up a new avenue to test our formalism, for example, there would be substantial prospects to study chiral symmetry restoration within the framework of Pearson distribution. This is due to the fact that the effective potential depends on the momentum distribution and in turn, the symmetry restoration also depends on momentum distribution. Thus, the generalized power like distribution may change the value related to chiral symmetry. We hope that this work may be helpful for the community to study the thermal system of QGP within the framework of the Pearson generalized statistics. The Pearson distribution can also be tested in many fields of physics such as Bose-Einstein condensation, black-body radiation, neutron star, early universe, superconductivity etc.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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