the kinematical cut in the q^2 plane. When you work in the W plane this is out in the open. It is true that you can find functions which are functions only of q^2 but we prefer to work with the whole amplitude because that is the one which has the simple unitarity properties. I do not contend that you can not work in the q^2 plane; I am sure you can, but I do not feel it will afford any advantages to us in our partial wave procedure.

OPPENHEIMER : One gains the impression that by this time next year some comparison between the various methods, all of which are trying to do the same thing and adopt the same philosophy, will be in order, but at the moment it is a little premature to do that.

WICK : Regarding Hamilton's method, is this the same thing that Oehme wrote about in the Physical Review Letters?

HAMILTON : The singularities inside the circle correspond to the singularities in Oehme's second sheet and in order to find the discontinuity across these singularities you have to use the second sheet for the purpose of crossing, so really effectively what I have done is equivalent to Oehme's dispersion relation.

SHIRKOV : I only want to stress that surely the choice of variable is not the principal point because you can always change the variable; but q^2 is, I believe, the most convenient one because you do not have such a complicated situation in the complex plane. The second point I want to stress is that the problem of the approximation in the unphysical region arises here also, and here I return to the point from which I started some hours ago; in order to get the well-behaved equation we did not integrate over all this region to get the partial wave equations, but we expanded the amplitude of the first process near the point C = -1.

THE $N\bar{N}$-$\pi\pi$ AMPLITUDE (*)

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In the same spirit as the dispersion approach to the pion-pion and the pion-nucleon problems, I shall now describe the determination of the $N\bar{N}$ to $\pi\pi$ amplitude in terms of known singularities of the function. I should mention that this work is done in collaboration with Ball of Berkeley. We are mainly concerned with the region where the energy of the nucleon-antinucleon system is not too far from twice the pion mass. This region is of some immediate interest since it is expected to give a substantial contribution to the absorptive parts of the nucleon form factor, the pion-nucleon amplitude and also the nucleon-nucleon amplitude. Let us denote the square of the nucleon-antinucleon center-of-mass energy by t. For any partial wave of a given angular momentum and spin, the singularities in the t-plane are : the branch cut due to the exchange of a single nucleon starting at $t = 4\mu^2(1-\mu^2/4m^2)$; the exchange

(*) This work was done under the auspices of the U.S. Atomic Energy Commission.

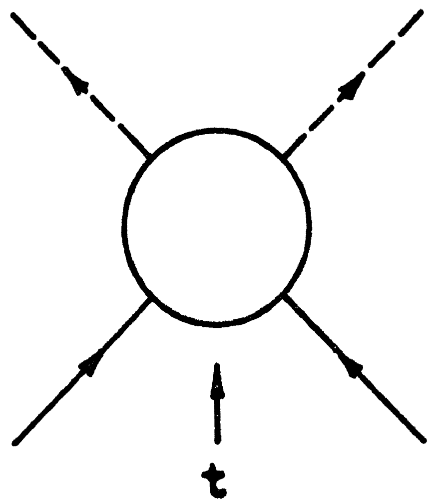

Fig. 1 Feynman diagram for $N\bar{N}-\pi\pi$.

of a nucleon plus a pion starting at $t=0$; etc. The threshold for the production of two pions is at $t=4\mu^2$. From this threshold to the production of more than two pions, the $\langle N\bar{N}|\pi\pi\rangle$ amplitude has the same phase as the $\langle\pi\pi|\pi\pi\rangle$ amplitude, since

$$\text{Im}\,\langle N\bar{N}|\pi\pi\rangle \simeq \langle N\bar{N}|\pi\pi\rangle^* \langle\pi\pi|\pi\pi\rangle$$

in this region.

Frazer and Fulco [1] were the first to write down partial wave dispersion relations for the $N\bar{N}$ to $\pi\pi$ amplitude. They considered the two p-wave amplitudes in some detail. For convenience, they took the combination

$$\Gamma_1=\alpha_1\langle+|+\rangle+\beta_1\langle+|-\rangle$$

$$\Gamma_2=\alpha_2\langle+|+\rangle+\beta_2\langle+|-\rangle$$

where the symbol $\langle\pm|\pm\rangle$ denotes the nucleon-anti-nucleon annihilation amplitude in a given helicity

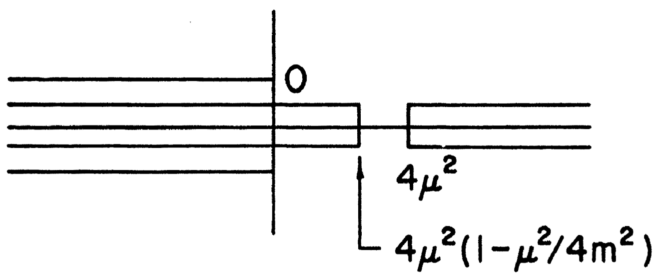

Fig. 2 Singularities in t-plane.

state and α, β are kinematic factors which are known functions of t. They have shown that Γ_1 contributes to the nucleon charge form factor and Γ_2 contributes to the magnetic form factor. Frazer and Fulco took a " one-pole " approximation for the $\pi\pi$ p-wave and determined the phase of the $\langle N\bar{N}|\pi\pi\rangle$ amplitude on such basis. They considered the nucleon pole term exactly and estimated the " rescattering " cut (the exchange of a nucleon plus a pion) by a δ-function approximation for the pion-nucleon (3, 3) amplitude. They found it necessary to introduce a cut-off on the left-hand cut in order to obtain convergent integrals. It turned out that the magnetic amplitude (Γ_2) is insensitive to the cut-off. They then chose the p-wave $\pi\pi$ parameters to fit the magnetic form factor. However, the charge amplitude was quite dependent on the cut-off. What I would like to report first today are two modifications of the Frazer-Fulco solution. (1) A " two-pole " approximation is taken for the $\pi\pi$ p-wave with a repulsive outer region plus an attractive inner region as discussed by Chew. (2) The values of Γ_1 and Γ_2 in the neighborhood of $t=0$ are determined by the pion-nucleon fixed-momentum transfer dispersion relation in the neighborhood of forward scattering. The left-hand cut of Γ is calculated in the same manner as Frazer and Fulco except that we terminate the rescattering cut at $t=-26\mu^2$ (where the πN partial wave expansion diverges) and replace all remaining cuts by a pole which is adjusted to give the correct value of Γ in the neighborhood of $t=0$. Of course, this phenomenological pole can also compensate for part of the inaccuracy in the rescattering cut.

I will now give the value and derivative of Γ_1 and Γ_2 at $t=0$ as calculated from a one-subtraction pion-nucleon dispersion relation. The subtraction constants are related to pion-nucleon scattering lengths and the dispersion integrals involve pion-nucleon partial cross sections which are expressed in terms of combinations of total cross sections and the (3, 3) amplitude in such a way that the $J=3/2$ states ($p_{3/2}$ and $d_{3/2}$) are taken into account exactly. We believe that such a combination is more accurate than the (3, 3) amplitude alone since the second resonance is probably in the $J=3/2$ d-state. The following table is a summary of the value and derivative of the Γ's and the contributions from various terms :

	Born Term	Scattering Lengths			Dispersion Integral	Total
		s	p	d		
$\Gamma_1(0)$	−0.1055	−0.0339	—	—	−0.0012	−0.141
$\Gamma_2(0)$	0.0238	−0.0002	−0.0319	—	0.0041	−0.0042
$\Gamma_1'(0)$	0.00535	0.00135	0.03820	—	−0.00186	0.0430
$\Gamma_2'(0)$	−0.00562	−0.00010	−0.00258	?	−0.00036	?

The scattering lengths are taken from the analysis of Barnes *et al.*[2)] and Hamilton and Woolcock[3)]. It is clear that $\Gamma_1(0)$ is most accurately determined. The uncertainty in $\Gamma_2(0)$ and $\Gamma_1'(0)$ mainly comes from the inaccuracy of the small p-wave scattering lengths. If the small p-waves are ignored, we find $\Gamma_2(0) = 0.0033$ and $\Gamma_1'(0) = 0.0348$. The uncertainty from the dispersion integral is considerably smaller. We have checked this point by a comparison with the corresponding integrals where only the (3, 3) p-wave is kept. Since the d-wave scattering lengths are yet unknown, we can only estimate the order of magnitude of $\Gamma_2'(0)$ from the no-subtraction formula which gives $\Gamma_2'(0) \simeq -0.005$. Fortunately, it turns out that the phenomenological pole in the Γ_2 amplitude is very weak and the amplitude on the right is quite insensitive to the position of this pole. Hence we can adjust the position and residue of the pole in Γ_1 to give the normalized value and derivative at $t = 0$, and adjust only the residue of the Γ_2 pole to fit the normalized value leaving the position arbitrary as long as it is beyond ~ -15.

Now that we have the formalism set up, we can compute the Γ's and the two-pion contribution to the vector part of the nucleon form factors for any given set of $\pi\pi$ parameters in much the same way as the Frazer-Fulco calculation. A typical set that gives the observed magnetic moment form factor is : $\nu_1 = 60$, $\nu_2 = 4$, $A_1 = 0.3$, $A_2 = 0.31$ (in pion units) where we have taken the p-wave $\pi\pi$ amplitude to be

$$\sqrt{\frac{\nu+\mu^2}{\nu^3}}e^{i\delta}\sin\delta = N/D$$

with

$$\nu = \frac{t}{4}-\mu^2 ,$$

$$N = \frac{A_1\nu_1}{\nu+\nu_1}-\frac{A_2\nu_2}{\nu+\nu_2} ,$$

$$D = 1-\nu[A_1\nu_1 K(\nu_1, -\nu)-A_2\nu_2 K(\nu_2, -\nu)]$$

and $K(a, b)$ is the kernel defined by Chew and Mandelstam. This set of parameters gives a resonance at $t{\sim}14$. The two pion contribution to the charge turns out to be $\sim 20\%$ of the total charge. The smallness of this charge is due to the cancellation of the phenomenological pole and all other terms in the normalized Γ_1 amplitude. The pole is found to be situated at a very high energy region but gives far greater (negative) contribution to the electric charge than the contribution (positive) from the rescattering cut. This leads to our belief that although the Γ_1 amplitude is still quite sensitive to the uncertainty in the normalization, we have at least obtained a $\Gamma_1(t)$ qualitatively more reliable than the function given by Frazer and Fulco.

I shall now turn to the question of the compatibility of our $\pi\pi$ parameters with the dispersion theory of Chew and Mandelstam[4)]. For any given set of p-wave parameters, we can determine one or more sets of s-wave parameters, by using the so-called " almost exact " crossing conditions in the neighborhood of the $\pi\pi$ symmetry point.

Our present solution with $\nu_1 = 60$ and $\nu_2 = 4$ gives a negative p-wave amplitude at the symmetry point and is inconsistent with the Chew-Mandelstam theory. However, this situation may be improved by moving the attractive pole farther to the left.

Should we be able to obtain an s-wave $\pi\pi$ solution in this way or in any other way, we can immediately construct the s-wave $\langle N\bar{N}|\pi\pi\rangle$ amplitude in much the same manner as the p-wave problem. The normalization at $t = 0$ plays an even more important role in the s-wave problem. In fact it serves to suppress the nucleon pole term which is known to give a superfluously large fourth order potential in the nucleon-nucleon scattering problem. In closing, I should mention that it is quite probable that a p-wave $\pi\pi$ resonance will give a substantial contribution to the " medium range " attractive force between two nucleons but it is unlikely that the resonance will ever produce a repulsive core. However, there is still a possibility that a one-subtraction formula including the one- and two-pion exchange terms in the nucleon-nucleon problem may simulate the effect of a hard core in the physical region.

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DISCUSSION

CHEW : Wong passed over rather quickly a fairly serious embarrassment for the calculation, and that is the fact that the dynamical solutions of the p-wave problem which Mandelstam and I have obtained have never given a sharp resonance of as small a width as implied by the calculations of Ball and Wong. The point is, as Frazer and Fulco emphasized in the original calculation, that the contribution to the magnetic moment goes inversely with the width of the resonance and you need quite a narrow resonance if you want to get the full anomalous moment from the two-pion state. This fact is reflected in Wong's result that λ_1 wants to be negative; that means that the pion p-wave amplitude is negative a little bit to the left of the origin and increasing rapidly, which means a very sharp resonance. We cannot possibly get that kind of behavior out of the dynamical solution. We have to be content with a width that gives only about half of the magnetic moment, I would say, and this is a serious difficulty with the present scheme.

OPPENHEIMER : This leaves room for the four-pion state.