

Quantum entanglement and axion physics

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The axion particle is the outcome of the proposed Peccei–Quinn mechanism for solving the strong CP problem. Axion is also a popular dark matter candidate. Thus, there is an increased interest in establishing its existence. Axions couple to two photons and most experiments search for the transition of an axion into a photon, in the presence of a magnetic field. In our study, we examine the coupling of the axion into a pair of entangled photons. The presence of a magnetic field changes the polarization correlations of the entangled photons, thus offering an unambiguous signature for axion existence.

Keywords: Quantum entanglement; axion physics; axion two photon coupling.

The Standard Model of particle physics offers a successful description of the fundamental constituents of matter and the interactions among them. The discovery of the Higgs boson¹ confirmed our ideas about the mechanism of spontaneous symmetry breaking and the generation of masses of the weak gauge bosons. Yet there are unresolved issues and one of them is the strong CP problem. The strong CP problem arises from the non-Abelian nature of QCD. QCD vacuum allows the existence of a CP-violating term

$$L_\theta = \theta \frac{g_3}{32\pi^2} G_\alpha^{\mu\nu} \tilde{G}_{\alpha\mu\nu}, \quad (1)$$

where g_3 is the coupling constant of the strong force, $G_{\mu\nu}$ is the gluon field strength tensor, $\tilde{G}_{\mu\nu}$ is its dual and θ is the angle determining the strength of CP violation. From the absence of a measurable neutron electric dipole moment we infer that $\theta < 10^{-9}$. Here lies the strong CP problem: why this angle is exceedingly small?

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One may accept the small θ value by fiat, but Peccei and Quinn offered an elegant solution based on spontaneous symmetry breaking.² A global chiral symmetry, known as $U(1)_{\text{PQ}}$, is introduced. This symmetry is spontaneously broken when a scalar field develops a vacuum expectation value. The associated Goldstone boson couples to two gluons and at the minimum of the effective potential the total coefficient of $G\tilde{G}$ (shorthand of $G_{\alpha}^{\mu\nu}\tilde{G}_{\alpha\mu\nu}$) relaxes to zero. The physical particle, called the axion,^{3,4} replaces effectively θ and the Lagrangian no longer has a CP-violating term. Note that the axion is one of the leading candidates for dark matter.^{5,6} Furthermore, extensive studies are carried out on primordial axions and axions permeating the universe.^{7–12} It appears then that the search for axions is a highly important issue for fundamental physics.

Similar to the axion–two-gluon coupling there is an axion–two-photon coupling

$$L_{a\gamma\gamma} = -\frac{1}{4}gaF_{\mu\nu}\tilde{F}^{\mu\nu} = ga\mathbf{E} \cdot \mathbf{B}, \quad (2)$$

where a is the axion field, $F_{\mu\nu}$ ($\tilde{F}^{\mu\nu}$) the (dual) electromagnetic field strength tensor and g the photon–axion coupling constant. This implies that in the presence of an external magnetic field photons and axions may mix.^{13,14} Let us recapitulate the essentials of the photon–axion mixing. For a photon traveling in the z -direction, its polarization lies in the $x - y$ plane. The component of \mathbf{B} parallel to z -axis does not induce any mixing. Following Eq. (2), the transverse magnetic field \mathbf{B}_T couples to \mathbf{A}_{\parallel} , the photon polarization parallel to \mathbf{B}_T and decouples from \mathbf{A}_{\perp} , the photon polarization orthogonal to \mathbf{B}_T . The photon–axion mixing is governed by the following matrix:

$$M = \frac{(m_a^2 - m_{\gamma}^2)}{4E}N, \quad (3)$$

$$N = \begin{bmatrix} 1 & \frac{2gBE}{(m_a^2 - m_{\gamma}^2)} \\ \frac{2gBE}{(m_a^2 - m_{\gamma}^2)} & -1 \end{bmatrix} \quad (4)$$

with m_a (m_{γ}) the axion mass (effective photon mass). Defining

$$\tan 2\theta = \frac{2gBE}{(m_a^2 - m_{\gamma}^2)} \quad (5)$$

we find that the eigenvalues of N are

$$\lambda = \pm \frac{1}{\cos 2\theta} \quad (6)$$

and the corresponding eigenstates

$$V_1 = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \quad V_2 = \begin{pmatrix} \sin \theta \\ -\cos \theta \end{pmatrix}. \quad (7)$$

The photon and the axion states are expressed in terms of the eigenstates. After a travel of distance z within the magnet the photon amplitude is modified to

$$A_{\parallel}(z) = [\cos^2 \theta e^{-i\omega_+ z} + \sin^2 \theta e^{-i\omega_- z}] A_{\parallel}(0) \quad (8)$$

with

$$\phi = \omega_+ - \omega_- = \Delta, \quad (9)$$

$$\Delta = \frac{1}{2E} [(m_a^2 - m_\gamma^2)^2 + 4g^2 B^2 E^2]^{1/2}. \quad (10)$$

Most experiments are looking for sources of axions (solar axions, halo axions). In that case, helioscopes or haloscopes are trying to check the amplitude

$$A(a \rightarrow A_{\parallel}) = \cos \theta \sin \theta (e^{-i\omega_+ z} - e^{-i\omega_- z}). \quad (11)$$

In our proposal, we study the coupling of the axion to a pair of entangled photons. Quantum entanglement is a distinct feature of Quantum Mechanics, supporting its nonlocal character.^{15–18} The wave function describing the entangled photons, named photon 1 and photon 2, is

$$|\Psi\rangle = \frac{1}{\sqrt{2}} [|x_1, x_2\rangle + |y_1, y_2\rangle]. \quad (12)$$

Clearly, the wave function is not factorized into a product of the individual particles. For a measurement of photon's polarization along a specific axis, the result is $+1$ (-1) if the polarization is found parallel to the axis (or perpendicular to the axis). We decide to measure the polarization of photon 1 along the x_1 -axis and the polarization of photon 2 along an axis in the $x_2 - y_2$ plane forming an angle β with the x_2 -axis. For a photon with polarization along the β -axis the wave function is

$$|\mathcal{E}_\beta\rangle_+ = \cos \beta |x_2\rangle + \sin \beta |y_2\rangle. \quad (13)$$

For a photon with polarization perpendicular to the β -axis the wave function is

$$|\mathcal{E}_\beta\rangle_- = -\sin \beta |x_2\rangle + \cos \beta |y_2\rangle. \quad (14)$$

Let us consider the result for the probability of joint polarizations P_{++} , that is, photon 1 with polarization parallel to the x_1 -axis and photon 2 with polarization parallel to the β -axis. The answer is obtained from the overlap of the initial wave function (the first term in Eq. (12)) with the wave functions of the individual photons. The overlap amplitude is $\frac{1}{\sqrt{2}} \cos \beta$ and therefore the probability is

$$P_{++} = \frac{1}{2} \cos^2 \beta. \quad (15a)$$

For the probability of the joint polarizations P_{+-} , we have again to consider the overlap of the initial wave function (the first term) with the wave functions of

A. Nicolaidis

the individual photons. The overlap between the initial state and the final states is $-\frac{1}{\sqrt{2}} \sin \beta$ and the probability is

$$P_{+-} = \frac{1}{2} \sin^2 \beta. \quad (15b)$$

The probability P_{-+} is obtained similarly. This time we have to consider the overlap of the second term in Eq. (12) with the wave functions describing the individual photons. The amplitude is $\frac{1}{\sqrt{2}} \sin \beta$ and for the probability we obtain

$$P_{-+} = \frac{1}{2} \sin^2 \beta. \quad (15c)$$

Finally, for the P_{--} we take into account the overlap of the initial state, the second term in Eq. (12), with the final photon states. The amplitude is $\frac{1}{\sqrt{2}} \cos \beta$ and the probability is

$$P_{--} = \frac{1}{2} \cos^2 \beta. \quad (15d)$$

Next, we introduce the magnetic field B in the x_1 -direction. The interaction equation (2) will change the photon amplitude according to Eq. (8). The $|x_1\rangle$ state is replaced by

$$|x_1\rangle_a = [\cos^2 \theta e^{-i\omega_+ z} + \sin^2 \theta e^{-i\omega_- z}] |x_1\rangle. \quad (16)$$

The same quantum mechanical rules offers now the modified probabilities

$$P_{++}^a = \frac{1}{2} \cos^2 \beta \left[1 - \left(\sin 2\theta \sin \frac{\phi z}{2} \right)^2 \right], \quad (17a)$$

$$P_{+-}^a = \frac{1}{2} \sin^2 \beta \left[1 - \left(\sin 2\theta \sin \frac{\phi z}{2} \right)^2 \right], \quad (17b)$$

$$P_{-+}^a = \frac{1}{2} \sin^2 \beta, \quad (17c)$$

$$P_{--}^a = \frac{1}{2} \cos^2 \beta. \quad (17d)$$

Note that the probabilities P_{++}^a and P_{--}^a remain unchanged since they involve a photon with polarization vertical to the magnetic field.

The ratios

$$\frac{P_{++}^a}{P_{--}^a} = \frac{P_{+-}^a}{P_{-+}^a} = \left[1 - \left(\sin 2\theta \sin \frac{\phi z}{2} \right)^2 \right] \quad (18)$$

are sensitive only to the axion parameters and the experimental setup. Thus, the correlations among the polarizations of entangled photons may reveal the existence of axions. Note that the second term within the bracket represents the transition

probability for a photon to become an axion and therefore Eq. (18) is a manifestation of the probability conservation

$$P(\gamma \rightarrow \gamma) + P(\gamma \rightarrow a) = 1.0. \quad (19)$$

The probability $P(\gamma \rightarrow a)$ is simplified when $\phi z \ll 1$

$$P(\gamma \rightarrow a) \simeq \frac{g^2}{4}(Bz)^2. \quad (20)$$

There are other proposals searching for signatures of axions. Astrophysical objects with strong magnetic fields, like neutron stars, white dwarfs, supernovas, may display modified photon spectra because of the photon–axion coupling.^{19–21} Helioscopes are trying to check the existence of solar axions, while haloscopes are looking for halo axions.⁶ In these experiments, the final outcome is dependent on models of solar dynamics or hypotheses on axion dark matter distribution. Our proposal has the distinctive feature that does not depend upon an incoming axion flux and expresses solely the photon–axion interaction, Eq. (2). There is also the “light shining through a wall” experiments based on the fundamental axion–two-photon interaction. However, one has to pay the price for a tiny conversion, the probability $P(\gamma \rightarrow a)P(a \rightarrow \gamma)$.²² From another direction the axion–two-photon interaction provides magnetic birefringence, a difference in the index of refraction Δn and a difference in the imaginary part of the index of refraction $\Delta \kappa$, leading to dichroism.^{13,23}

$$\Delta n \simeq \frac{g^2}{3}(Bz)^2 \left(\frac{m_a}{4E} \right)^2, \quad (21)$$

$$\Delta \kappa \simeq \frac{g^2}{8}(Bz)^2 \frac{1}{Ez}. \quad (22)$$

For a magnet of length $z \simeq 10$ m, an energy of the laser beam $E \simeq 2\text{--}10$ eV and an axion mass m_a within the accepted limits, both terms, Eqs. (21) and (22), are orders of magnitude smaller compared to the term in Eq. (20). We should notice also that the advances in laser light technology paves the way for an increased sensitivity of the proposed experiment. The experiment with the entangled photons may take place within a lab. We may consider also the propagation of satellite-based entangled photons to ground stations.²⁴

Quantum entanglement lies at the heart of quantum information, quantum communication and quantum computation.²⁵ In another direction, quantum entanglement on a cosmological scale creates a geometry, which explains the important aspects of our universe.²⁶ In this study, we suggest to use quantum entanglement in order to reveal features of particle and astroparticle physics.

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