GENERAL PRINCIPLES OF THE LOCAL FIELD THEORY AND THEIR EXPERIMENTAL CONSEQUENCES

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INTRODUCTION

The present report was written jointly with I.T. Todorov, Nguyen Van Hieu, and M. K. Polivanov, and constitutes a review of some works on the development of the axiomatic approach and studies on the relations between the scattering amplitudes at high energies on the basis of the general principles of quantum theory.

We will not consider all the works [1 – 21] presented at this section, since this would render the account excessively diversified and in the report will be published in the proceedings of the conference.

Below we shall review the results of the axiomatic approach.

I. NEW DEVELOPMENTS IN THE AXIOMATIC APPROACH

1. THE HAAG-RUELLE COLLISION THEORY

The axiomatic approach in quantum field theory underwent an intensive development approximately 10 years ago, when the difficulties of perturbation theory became clear. From the standpoint of mathematical rigor, the axiomatic approach was most consistently developed in the works of Wightman et al. [22 – 24]. The fundamental concept in this approach is the quantized Heisenberg field, which is defined as a generalized operator function. The field operators operate in a Hilbert space of state vectors H with a positive definite metric.

Further, the following basic requirements are imposed.

1. Invariance with respect to the inhomogenous Lorentz group.
2. The principle of a spectrum:
   a) there exists a complete system of physical states with positive energy ($p^2 \geqslant 0$, $p_0 \geqslant 0$);
   b) there exists a single invariant vacuum state;
   c) there exists a one-particle state with minimum mass larger than zero.
3. Field locality: the fields at spatially separated points are commutative or anticommutative.

This postulate represents the microcausality condition and imposes very stringent requirements on the theory.

The basic working apparatus in this approach are Wightman's functions – the vacuum averages of the products of the field operators. To formulate the scattering problem in the field approach it is necessary to introduce asymptotic free fields (in and out). For a long time it was not clear to what extent the existence of these fields is associated with the remaining postulates. An important contribution to this problem was made by Ruelle in 1962 [22], who developed Haag's idea [26].

This result can be simply explained as follows [27]. According to the Källén-Lehmann theorem, Wightman's two-point function of a scalar field $A(x)$ has the form

$$\langle 0|\, A(x)\, A(y)\,|0\rangle = \frac{1}{i} D_m^{(-)}(x-y) +$$

$$+\frac{1}{i}\int_{M^2}^{\infty} D_M^{(-)}(x-y)\, d\varrho(M^2); \quad M > m,$$

where

$$D_m^{(-)}(x) = \frac{i}{(2\pi)^3}\int e^{ikx}\theta(-k^0)\,\delta(k^2-m^2)\,dk.$$

We define an auxiliary (nonlocal) field $B(x)$ by the formula

$$\widetilde{B}(p) = h(p^2)\widetilde{A}(p),$$

where $\widetilde{A}(p)$ and $\widetilde{B}(p)$ are the Fourier transforms of the fields $A(x)$ and $B(x)$, respectively, and $h(p^2)$ is a real, infinitely smooth function, concentrated in a fairly small neighborhood of $p^2 = m^2$, where $h(m^2) = 1$. With this choice of h, the vacuum average of the product of two operators B will agree with the two-point function of the free field $D_m^{(-)}(x-y)$. Let f be a positive-frequency solution of the Klein-Gordon equation, differing from zero only in a finite region of the P-space. We introduce an operator (of the annihilation type) of the form

$$B(f, t) = i\int_{x_0=t}\left[f^*(x)\frac{\partial B(x)}{\partial x_0} - \frac{\partial f^*(x)}{\partial x_0}B(x)\right]d^3x.$$

We define the state vectors

$$\psi_t(f_1, \ldots, f_n) = B^*(f_1, t), \ldots, B^*(f_n, t)|0\rangle.$$

Then there exist strong bounds $\psi_{\text{in}}^{\text{out}}(f_1, \ldots, f_n)$, constituting n-particle asymptotic states

$$\lim_{t\to\pm\infty}\|\psi_t(f_1, \ldots, f_n) - \psi_{\substack{\text{out}\\\text{in}}}(f_1, \ldots, f_n)\| = 0.$$

This is Ruelle's basic theorem.

The elements of the scattering matrix are determined by the usual formula

$$S(f_1, \ldots, f_n; g_1, \ldots, g_k) =$$
$$= (\psi_{\text{out}}(f_1, \ldots, f_n), \psi_{\text{in}}(g_1, \ldots, g_k)).$$

Despite the great fundamental importance of the Haag-Ruelle results, it is currently felt that this method of determining the S-matrix is ineffective. A rigorously proved reduction formula of the Lehmann-Symanzik-Zimmermann type [28] is required, enabling us to express the elements of the scattering matrix in terms of the Wightman function. A useful step to this end was made by Hepp [3]. Here it was shown that the scattering amplitude has the property of decomposition into clusters; this has a simple physical meaning: the scattering amplitude of particles with nonzero mass is asymptotically independent of the presence of other separated particles.

With Ruelle's results (see above) and Dyson's representation [35] for the causal commutator, Hepp then found, using Wightman's approach, the retarded and advanced functions necessary for providing the dispersion relations.

We note that the basic problem of this approach – that of constructing an example which satisfies all postulates and leads to a nontrivial scattering matrix – still remains open. A preliminary study of this problem was carried out by Khristov [10].

2. SCATTERING MATRIX. BOGOLYUBOV'S MICROCAUSALITY CONDITIONS AND THE ANALYTIC PROPERTIES OF THE SCATTERING AMPLITUDE

Since 1954 another axiomatic approach is being developed, in which the basic concept – Heisenberg's scattering matrix – is considered as a functional of the asymptotic out-fields

$$S = \sum_{n=0}^{\infty}\frac{1}{n!}\int dx_1, \ldots, dx_n\Phi^{(n)}\times$$
$$\times(x_1, \ldots, x_n) : \varphi(x_1), \ldots, \varphi(x_n)$$

$\phi(x)$, $\varphi(x)$, and is expanded beyond the mass surface (i.e., $(\Box - m^2)\,\varphi(x) =$ is not assumed).

Without such an expansion, it is impossible to construct nontrivial local operators and, in particular, to formulate the causality condition. The main investigation objects here are radiation operators, which are variational derivatives of the S-matrix of the form

$$S^n(x_1, \ldots, x_n) = \frac{\delta^n S}{\delta\varphi(x_1), \ldots, \delta\varphi(x_n)} S^+$$

and their matrix elements. In this approach a need of the Heisenberg field concept does not arise, since all the observable elements of the scattering matrix are expressed in terms of the vacuum averages of the radiation operators.

The basic postulates in the S-matrix approach are partly analogous to Wightman's postulates and, naturally, are divided into two groups: the group of general properties of relativistic invariance, spectrum unitarity of the S-matrix ($SS^+ = 1$), stability of the vacuum and one-particle states, and the group of special local properties: the requirement that the vacuum averages of the radiation operators with different arguments are generalized functions of moderate growth, and Bogolyubov's causality condition

$$\frac{\delta}{\delta\varphi(y)}\left(\frac{\delta S}{\delta\varphi(x)} S^+\right) = 0, \quad \text{for } y \lesssim x.$$

In the work of B.V. Medvedev and M. K. Polivanov [5], presented at the conference, it was shown that within the framework of these axioms the radiation operators S^n $(x_1, \ldots, x_n)$ are expressed by the so-called chronological representation in terms of a sequence of simpler current-like operators λ_n, which effectively depend only on a single point.

Let us mention a few words on what is known about the analytic properties of matrix elements.

As is known, from the causality condition it follows that the retarded function $F^{\text{ret}}(x)$ vanishes everywhere, except in the future light cone Γ^+, resulting in the analyticity of its Fourier transform $T^{\text{ret}}(k)$ in the tubular region

$$T^+ = \{k - p + iq;\ p \in R_4,\ q \in \Gamma^+\}.$$

Due to the condition of spectral character, different retarded and advanced functions agree with one another in some region of the real momenta, and Bogolyubov's theorem of "sharp wedges" [32 – 36] ensures the existence of a single analytic function which is holomorphic in some complex neighborhood of this region. The maximum analyticity region – the holomorphism domain – is found by means of Dyson's integral representation [35] only for the retarded and advanced functions of one vector argument.

For the two-particle Green's function the general problem reduces to considering 32 analytic functions of three vector variables which are holomorphic in different tubular regions and satisfy Steinmann's six identities [36].

Bros, Epstein, and Glaser [25] considered the whole set of functions which are analytic continuations of some single function, and found the part of the holomorphism domain corresponding to the sum of the tubular regions (the whole holomorphism domain in the case with nonzero masses has not yet been found even for the vertex part). Their results are at present the most conclusive. Along with the known analytic properties, the analyticity of the elastic scattering amplitude with respect to two variables s and t in some neighborhood of the whole physical region was demonstrated. The analyticity of the partial amplitudes $T_l(s)$ in the neighborhood of the physical points (for $s > (M + m)^2$ was also demonstrated.

3. INFINITE SYSTEM OF NONLINEAR EQUATIONS FOR THE GREEN'S FUNCTION

As is known, in a number of cases the investigation of the analytic properties of the matrix elements makes it possible to obtain different spectral representations (in particular, dispersion relations). If it would be possible to set up simple spectral representations for the Green's functions with an arbitrary number of cut-offs, these representations and the unitarity conditions could hopefully be sufficient for the complete determination of the theory.

However, concrete investigations of even the simplest cases show that the analytic nature of the matrix elements is much more complicated, and therefore the realizability of a program based on what is known as the principle of maximum analyticity becomes doubtful.

However, on the other hand, already in the works of Lehmann et al. [28] and N.N. Bogolyubov et al. [29 – 31] it was shown that a formal use of the basic properties of unitarity and causality (without a detailed investigation of the analyticity) for obtaining infinite systems of nonlinear integral equations is possible. For the first time such a system of equations was written and studied in the works of Lehmann, Symanzik, and Zimmermann.

Three works dealing with different approaches to this problem were presented at this conference. In the work of B.V. Medvedev and M.K. Polivanov [5] the possibility of rigorously deriving a system of equations for the matrix elements of the current on the mass surface was shown. We note, however, that this is still insufficient for determining the physically interesting elements of the scattering matrix. Within the framework of the perturbation theory it is shown that this system of equations has a unique (up to the usual number of constants) solution.

A study of this system makes it possible to draw other interesting conclusions as well. It was shown that in the absence of counterterms, the system has only a trivial solution ($S \equiv 1$). A tentative estimate of the possible rates of growth (upon uniform extension of the momenta) leads to results known from perturbation theory. In this estimate an assumption of the absence of compensation for the rates of growth in the sum over the intermediate states was made and was justified only by analogy with the perturbation theory.

In the work of V.Ya. Fainberg [4], along with the usual requirements of the local theory, the condition of minimum singularity of the commutator of the field and current operators for equal times is imposed. In the case of the theory of a scalar field $\varphi(t, x)$, this condition has the form

$$[\varphi(t, \mathbf{x})\, \varphi(t, \mathbf{x}')] = 0,$$

where

$$\varphi(x) = \varphi_{\text{in}}(x) = \int D^{\text{ret}}(x - x')\, j(x')\, dx'.$$

Apparently, the additional postulate of Fainberg reduces to a selection of the minimum rates of growth of the matrix elements, which provide a nontrivial solution. The passage beyond the mass surface is effected in one variable only – the square of the momentum of one of the particles. The system of equations considered is very close to that proposed in the work of B.V. Medvedev and M.K. Polivanov [5]. The main difference, aside from the method of derivation, is that Fainberg arrives at integral differential equations by using differentiation (in momentum space) in order to eliminate arbitrary subtractive terms. The number of arbitrary constants involved in the theory agrees with the number of matrix elements which do not vanish at infinity.

The problem of the existence of solutions to the field equations is dealt with in Taylor's work [6]. This is a very important problem. It is unfortunate that the presented text does not contain any formula, so that it is difficult to get an idea to what extent the relevant problem is solved.

II. ASYMPTOTIC RELATIONS BETWEEN THE AMPLITUDES OF ELASTIC AND INELASTIC PROCESSES

1. GENERAL REQUIREMENTS IMPOSED ON THE SCATTERING MATRIX, AND THE GROWTH OF THE SCATTERING AMPLITUDE IN MOMENTUM SPACE

Among the fundamental postulates of quantum field theory there is a condition of a mathematical character: it is required that the elements of the scattering matrix be generalized functions of moderate growth, which ensures polynomial boundedness of the Fourier transform of the retarded amplitude (see, for example, [32], theorem 1). This condition is naturally sufficient from the physical standpoint, since it reflects the equal justification of the momentum (p) and coordinate (x) representations and enables us to speak of local properties of the matrix elements in both representations. Without this assumption even the usual dispersion relations with a finite number of subtractions cannot be obtained.

A question arises – to what extent is this requirement independent of the remaining principles of the local theory and, in particular, of the microcausality principle? This problem is interesting from the theoretical standpoint, since the formal perturbation theory indicates that in the case of the nonrenormalizable theory, the amplitude in momentum space grows faster than any polynomial [37]. Simple one-dimensional examples with functions growing in the upper energy half-plane faster than some exponent show that only the analyticity of the Fourier transform of the retarded function is insufficient to make this function satisfy the microcausality principle [38].

In [19] arguments of a more general character are given in favor of the affirmation that for a noncontradictory formulation of the microcausality principle it must be assumed that the amplitude grows more slowly than any exponent as a function of the energy.

$$|T(\omega)| < A_\varepsilon e^{\varepsilon|\omega|}, \quad \frac{I_m(\omega)}{|\omega|} \geqslant \delta > 0, \qquad (2.1)$$

where ϵ is any positive number.

If a faster than exponential growth is allowed in momentum space, then the formulation of the problem of the local properties in x-space becomes meaningless.

If the differential cross sections for a pair of cross processes do not increase as a function energy (in the physical region) faster than some polynomial, then from the microcausality condition, including equation (2.1) it follows that the amplitudes of these processes are polynomially bounded in the whole complex plane of the energy. This follows from the analyticity of the amplitudes in the upper energy half-plane and the Phragmén-Lindelöf theorem in the theory of analytic functions.

The assumption of polynomial boundedness of the differential cross sections in the physical region (which is fairly well justified in experiment) can be obtained from the following, weaker assumption

$$|T(s, t)| < Ae^{as^n}, \quad s > 0, \qquad (2.2)$$

when the transfer of the momentum t runs through the Lehmann ellipse [39].

Then, using the analyticity of the amplitude in the Lehmann ellipse and the unitarity condition, it can be shown, following Greenberg and Low [40], that in the physical region ($t \leqslant 0$) the amplitude $T(s, t)$ does not increase faster than a polynomial of s, when $s \longrightarrow \infty$.

Thus we see that in the case of very weak and natural assumptions, the requirement of polynomial boundedness of the amplitude follows from the remaining physical principles of local field theory. This makes probable the assumption that in nonrenormalizable local theory (if such a theory exists at all) the amplitude is nonanalytic with respect to the coupling constant in the vicinity of zero. This is also indicated by the models reported at the conference by B.Z. Arbuzov et al. [8] based on the quasipotential approach to quantum field theory (see the work of Lee [41]. This is also confirmed by the results of a recent work of Schroer [42], in which the author, assuming nonrenormalizability of the theory and analyticity with respect to the coupling constant, concludes that the theory is nonlocal.

2. ASYMPTOTIC EQUATIONS OF THE DIFFERENTAL CROSS SECTIONS FOR CROSS SCATTERING AND PHOTOPRODUCTION PROCESSES

The combination of the properties of analyticity, cross symmetry, and bounded growth appears to make it possible to attain asymptotic relations between the amplitudes of crossed processes. The first relation of this kind – the equality of the total interaction cross sections of a particle and an antiparticle – was obtained by I.Ya. Pomeranchuk [43]. Subsequently, a number of works dealt with a more complete proof and a better formulation of the theorem of Pomeranchuk. We should mention here the work of Sugawara and Kanazawa [44], in which the Phragmén-Lindelöf theorem was actually "rediscovered" and newly proved. In the following work Meiman [45] drew attention to the fact that here we are dealing with a classical theorem from the theory of analytic functions. On the basis of this theorem Sugawara [44, 46] and Meiman [45] gave a simple and natural proof of Pomeranchuk's theorem under fairly general conditions. Subsequently, Van Hove and the Dubna theoreticians [15-17, 47-49] obtained asymptotic relations between the differential cross sections for cross scattering and photoproduction processes, as well as between polarization effects.

We shall illustrate how these relations are obtained in the example of scalar particles. Consider the pair of processes

$$a_1 + b_1 \to a_2 + b_2; \tag{I}$$

$$\bar{a}_2 + b_1 \to \bar{a}_1 + b_2, \tag{II}$$

whose amplitudes $T^{\mathrm{I}}(s,\ t)$ and $T^{\mathrm{II}}(s, t)$ are boundary values of the same analytic function $T(s,\ t)$ on different cut boundaries. These amplitudes are interconnected by the relation of cross symmetry (or Low's substitution rule)

$$\begin{aligned} T^{\mathrm{I}}(u,\ t) &= T^{\mathrm{II}}(s,\ t)^*; \\ s+t+u &= \sum_i m_i^2. \end{aligned} \tag{2.3}$$

Let us assume that the particle masses are such that from the principles of the local theory follows that analyticity of the amplitude $T(s,\ t)$ for fixed t in the complex plane of s with cuts along the real axis. In order to give the class of amplitudes a fairly general asymptotic behavior, we introduce an auxiliary concept. The function $\varphi(s,\ t)$ is called permissible if $1/\varphi\,(s,\ t)$ is analytic, does not exceed any exponent (2.1) of $|s|$ in the upper half-plane, is continuous on the real axis, and if

$$\lim_{s\to\infty} \frac{\varphi\,(s,\ t)}{\varphi\,(-s,\ t)} = e^{-i\pi\alpha(t)}, \tag{2.4}$$

where $\alpha(t)$ is an arbitrary real function of t An example of a permissible function may be one of the form

$$\begin{aligned} \varphi\,(s,\ t) = (s+i)^{\alpha(t)}\,[\ln(s+i)]^{\beta(t)} \times \\ \times [\ln\ln(s+i)]^{\gamma(t)} \end{aligned} \tag{2.5}$$

The basic result is contained in the following theorem.

Theorem. Suppose for some permissible function finite bounds exist

$$V^{\mathrm{I}}(t) = \lim_{s\to\infty} \frac{T^{\mathrm{I}}(s,\ t)}{\varphi\,(s,\ t)}\,, \quad V^{\mathrm{II}}(t) = \lim_{s\to\infty} \frac{T^{\mathrm{II}}(s,\ t)^*}{\varphi\,(-s,\ t)}\,.$$

Then in the local theory these bounds are equal to one another

$$V^{\mathrm{I}}(t)=V^{\mathrm{II}}(t),$$

whence follows the asymptotic equality of the differential cross sections of the processes

$$\lim_{s\to\infty}\frac{\dfrac{d\sigma^{\mathrm{I}}(s,\,t)}{dt}}{\dfrac{d\sigma^{\mathrm{II}}(s,\,t)}{dt}}=1. \qquad (2.6)$$

An elementary proof of this theorem reduces to applying the Phragmén-Lindelöf theorem to the function

$$V(s,\,t)=\frac{T(s,\,t)}{\varphi(s,\,t)}\,.$$

The assumption of the existence of the bounds (2.5) of this function for $s\longrightarrow\pm\infty$ roughly speaking means that the amplitude $T(s,\,t)$ has a definite growth, and does not oscillate.

3. PHRAGMEN-LINDELOF THEOREM

Let us consider a particular case of the theorem which is of relevance for our purposes.

Let $f(t)$ be an analytic function of $z = re^{i\theta}$, which is regular in the upper half-plane of t and tends to finite bounds a and b along the real axis for $z\longrightarrow\pm\infty$. Then, if $a\neq b$, a sequence of points with moduli $r_n\longrightarrow\infty$ is found, such that

$$\max_{\substack{|z|=r_n\\ 0<\theta<\pi}}|f(z)|\geqslant e^{\nu r_n},\quad \nu>0.$$

In our case the function $V(s,\,t)$ is analytic in the upper half-plane of s (for fixed t) and grows slower than any exponent. Its bounds for $s\longrightarrow\pm\infty$ therefore should agree.

Let us consider elastic scattering, assuming that

$$\alpha(0)=1. \qquad (2.7)$$

In this case the amplitudes of elastic forward scattering of truly neutral particles are purely imaginary and as a consequence of the optical theorem, we have

$$\left.\frac{d\sigma}{dt}\right|_{t=0}\sim\frac{1}{16\pi}\,[\sigma_{\mathrm{tot}}]^2. \qquad (2.8)$$

We note that in the case of elastic scattering one can get rid of the assumption of the absence of amplitude oscillation by assuming that the imaginary parts of the amplitudes in the asymptotic behavior are non-negative (this is automatically satisfied for all energies for $t = 0$). Then to obtain (2.6) it is difficult to assume the absence of oscillations only in the differential cross sections.

Other conditions under which the differential cross sections are equal are discussed in [18]. When the amplitude $T(s,t)$ has a finite number of zeros in the plane s, the result of this work can be formulated as follows. Let there exist a bound

$$\lim_{s\to\infty}\left|\frac{T^{\mathrm{I}}(s,\,t)}{T^{\mathrm{II}}(s,\,t)}\right|=\gamma.$$

Then $\gamma = 1$, if the argument (phase) of the ratio $T^{\mathrm{I}}(s,\,t)\,/\,T^{\mathrm{II}}(s,\,t)$ increases (decreases) more slowly than $\ln s$ ($-\ln s$) for $s\longrightarrow\infty$; γ has a finite positive value, different from unity, if the phase of this ratio increases (decreases) as $\ln s$ ($-\ln s$); γ equals 0 or ∞, if the phase increases (decreases) faster than $\ln s$ ($-\ln s$).

In [15 – 17] the asymptotic relations between the scattering amplitudes for particles with spin were established: meson-baryon scattering, baryon-baryon scattering and photoproduction. Different relations are obtained between the differential cross sections, total cross sections, polarization effects, etc. In particular, there is asymptotic equality of the differential cross sections of the following processes:

processes of the type $0 + 1/2\longrightarrow 0 + 1/2$:

$$\pi^+ + p\longrightarrow\pi^+ + p\quad\text{and}\quad\pi^- + p\longrightarrow\pi^- + p,$$
$$K^+ + p\to K^+ + p\quad\text{and}\quad K^- + p\longrightarrow K^- + p,$$
$$\pi^+ + p\longrightarrow K^+ + \Sigma^+\quad\text{and}\quad K^- + p\longrightarrow\pi^- + \Sigma^+,$$

$$\pi^- + p \to K^0 + \Lambda \quad \text{and} \quad \bar{K}^0 + p \to \pi^+ + \Lambda,$$

$$\Sigma^+ + H_e \to p + H_{e\Lambda} \quad \text{and} \quad \bar{p} + H_e \to \bar{\Sigma}^+ + H_{e\Lambda};$$

processes of the type $1/2 + 1/2 \to 1/2 + 1/2$:

$$p + p \to p + p \quad \text{and} \quad \bar{p} + p \to \bar{p} + p,$$

$$\Sigma^+ + p \to \Sigma^+ + p \quad \text{and} \quad \bar{\Sigma}^+ + p \to \bar{\Sigma}^+ + p,$$

$$\Sigma^+ + p \to p + \Sigma^+ \quad \text{and} \quad \bar{p} + p \to \bar{\Sigma}^+ + \Sigma^+;$$

processes of the type $\gamma + 1/2 \to 0 + 1/2$:

$$\gamma + p \to \pi^+ + n \quad \text{and} \quad \gamma + n \to \pi^- + p.$$

As in the case of scalar particles, if (2.7) holds, the amplitude of elastic forward scattering of the K_2^0 meson on a proton is purely imaginary and satisfies the approximate equality

$$\left.\frac{d\sigma\,(K_2^0 p \to K_2^0 p)}{dt}\right|_{t=0} \sim \frac{1}{16\pi}\,[\sigma_{\text{tot}}\,(K_2^0 p)]^2. \quad (2.9)$$

If we allow for isotopic invariance, then for pion-nucleon scattering we obtain

$$\left[\frac{d\sigma\,(\pi^{\pm} p \to \pi^{\pm} p)}{dt} - \frac{1}{2}\,\frac{d\sigma\,(\pi p \to \pi^0 n)}{dt}\right]_{t=0} \sim$$

$$\sim \frac{1}{16\pi}\,[\sigma_{\text{tot}}\,(\pi^{\pm} p)]^2. \quad (2.10)$$

Different relations between polarization effects in the considered processes can also be established. In particular, the polarization of recoil protons in the scattering of π^+ and π^- mesons (see also [50]) or a proton and an antiproton on a proton are asymptotically equal in magnitude and opposite in sign, and the polarization of the recoil proton in the scattering of a K_2^0 meson vanishes.

In models with external symmetries (unitary symmetry, symmetry of the group G_2) relations exist between the amplitudes of the different processes. These relations, however, are fairly complicated and it is not always possible to obtain a sufficient amount of information from them. With the aid of the asymptotic relations between the amplitudes of cross processes it is possible to simplify the relations following from the symmetry properties and obtain additional equalities between the cross sections. We write these relations in the form of a table. For comparison we also include in the table some relations associated only with higher symmetries. For the designation of the models we use the following notation: I – isotopic invariance; T – Sakata's triplet model; O – octet model with R-invariance; G_2 – G_2 model; $F-L$ – Phragmén-Lindelöf theorem.

3. ASYMPTOTIC RELATIONS FOR THE CROSS SECTIONS OF PROCESSES WITH FORMATION OF PARTICLES

The asymptotic relations between the amplitudes of the considered binary processes can be obtained even in the case when the usual dispersion relations are not valid for these amplitudes. For this, in accordance with Meiman [19], it is sufficient to assume that the Fourier transform of the retarded amplitude, for example, of the scattering of a meson on a nucleon

$$T^{\text{ret}}(\omega, \mathbf{p}) = \int e^{i(\omega x^0 - \sqrt{\omega^2 - \mu^2 - \mathbf{p}^2}\,\mathbf{ex})} \times$$

$$\times F^{\text{ret}}(x)\,d^4x, \quad (2.11)$$

where ω is the meson energy and $\mathbf{p}$ is the 3-momentum of the nucleon in Breit's system, can be written for the asymptotic condition $\omega^2 \gg \mathbf{p}^2 + \mu^2$ in the form

$$T_\infty^{\text{ret}}(\omega, \mathbf{p}) = \int e^{i\omega(x^0 - \mathbf{ex})}\,F^{\text{ret}}(x)\,d^4x. \quad (2.12)$$

Denchev established general conditions for which T^{ret} and T_∞^{ret} asymptotically agree, i.e.,

$$\lim_{\omega\to\infty} \frac{T^{\text{ret}}(\omega, \mathbf{p})}{T_\infty^{\text{ret}}(\omega, p)}\,.$$

This equality is automatically satisfied if the amplitude does not oscillate too rapidly and if for any $\epsilon > 0$ and fairly large ω it is bounded by the inequality

$$B_\varepsilon e^{-\varepsilon|\omega|} \leqslant |T^{\text{ret}}(\omega, \mathbf{p})| \leqslant A_\varepsilon e^{\varepsilon|\omega|}.$$

Thus, to establish the asymptotic relations, analyticity of the asymptotic amplitude alone, which is a direct consequence of the causality principle, is sufficient. Applying the Phragmén-Lindelöf theorem to this asymptotic amplitude, one can obtain all the above-enumerated asymptotic relations between the scattering amplitudes of a nucleon on a nucleon (where the dispersion relations are established only from the perturbation theory) and of a hyperon on a nucleon (where the usual dispersion relations are not valid even for the simplest Feynman diagrams).

In [20] a wide class of inelastic processes with formation of particles is considered. The amplitudes of these processes have complex singularities. Also in this case, however, it can be shown that at high energies the scattering amplitude can be replaced by an asymptotic amplitude, which is analytic in the upper half-

Relations	Model and method	Reference
$\sigma(\pi^\pm p \to \pi^\pm p) - \frac{1}{2}\sigma(\pi^- p \to \pi^0 n) \geqslant 0$	I, F–L	[17]
$\sigma(\pi^\pm p \to \pi^\pm p) - \frac{1}{2}\sigma(\pi^- p \to \pi^0 n) \sim \frac{1}{16\pi}[\sigma_{\text{tot}}(\pi^\pm p)]^2,\ t=0$	I, F–L	[15]
$\sigma(K^- p \to K^0\Xi^0) - \frac{1}{4}\sigma(K^- p \to K^+\Xi^-) \geqslant 0$	I, F–L	[17]
$\sigma(K^- p \to K^0 n) = \sigma(K^- p \to \pi^-\Sigma^+)$	O, G_2	[51–54]
$\sigma(\bar{K}^0 p \to K^+\Xi^0) = \sigma(\pi^- p \to K^+\Sigma^-)$	O, G_2	[51–54]
$\sigma(\pi^\pm p \to \pi^\pm p) = \sigma(K^\pm n \to K^\pm n)$	O, G_2	[51–54]
$\sigma_{\text{tot}}(\pi^\pm p) = \sigma_{\text{tot}}(K^\pm n)$	O, G_2	[51–54]
$\sigma(K_2^0 p \to K_1^0 p) = \frac{1}{2}\sigma(\pi^- p \to \pi^0 n)$	O, G_2, F–L	[17]
$\sigma(\pi^\pm p \to \pi^\pm p) = \sigma(K_2^0 p \to K_2^0 p) + \sigma(K_2^0 p \to K_1^0 p)$	O, G_2, F–L	[17]
$\sigma(\pi^- p \to K^0\Sigma^0) - \frac{1}{4}\sigma(\pi^- p \to \pi^0 n) \geqslant 0$	O, F–L	[17]
$\sigma(\pi^- p \to K^0\Lambda) - \frac{3}{4}\sigma(\pi^- p \to \pi^0 n) \geqslant 0$	O, F–L	[17]
$\sigma(\pi^\pm p \to \pi^\pm p) = \sigma(K^\pm p \to K^\pm p)$	T	[51–54]
$\sigma_{\text{tot}}(\pi^\pm p) = \sigma_{\text{tot}}(K^\pm p)$	T	[51–54]
$\sigma(K^- p \to \bar{K}^0 n) = \sigma(\pi^- p \to K^0\Lambda)$	T	[51–54]
$\sigma(K^+ n \to K^+ n) = \sigma(K^- n \to K^- n)$	T	[51–54]
$\sigma_{\text{tot}}(K^+ n) = \sigma_{\text{tot}}(K^- n)$	T	[51–54]
$\sigma(K_2^0 p \to K_2^0 p) = \sigma(K^+ n \to K^+ n)$	T	[17]
$\sigma_{\text{tot}}(K_2^0 p) = \sigma_{\text{tot}}(K^+ n)$	T	[17]
$\sigma(K_2^0 p \to K_1^0 p) \geqslant 0$	T	[17]

plane. On this basis asymptotic equalities of the differential cross sections for inelastic processes can also be obtained. As an example we consider π-meson formation in π-meson-nucleon collision.

$$\pi + N \longrightarrow \pi' + \pi'' + N'; \qquad \text{(I)}$$
$$\bar{\pi} + N' \longrightarrow \bar{\pi}' + \bar{\pi}'' + N. \qquad \text{(II)}$$

Let us denote the 4-momenta of the nucleons in the initial and final states by p and p', respectively, and the 4-momenta of the π mesons by q, q', and q''. We chose as independent variables the following invariants: $t=(p-p')^2$, $t''=(q-q'')^2$, $W^2=(q'+q'')^2$;

$$e^{2\xi}=\frac{q'(p+p')}{q''(p+p')}, \quad \omega=\frac{q(p+p')}{4\,\mathrm{ch}\,\xi\sqrt{M^2-\frac{t}{4}}}.$$

The first three variables have a straightforward physical meaning, the fifth variable, ω, is proportional to the square of the total reaction energy s and the fourth asymptotically for $s \longrightarrow \infty$ characterizes the ratio of the energy of the nucleon and one of the π mesons in the final state to the total energy

$$e^{2\xi} \longrightarrow \frac{s'}{s-s'}, \quad s'=(p'+q')^2. \qquad (2.14)$$

This choice of variables is convenient in that by fixing t, t'', W^2 and ξ, we can make ω tend to infinity, thus remaining in the physical region. The cross symmetry relation for the total amplitude has the form

$$T^{\mathrm{I}}(p', -q; p', q', q'')= = P_{ss'}T^{\mathrm{II}}(p', -q; p, -q', -q'')^*. \qquad (2.15)$$

From this relation and with the aid of the Phragmén-Lindelöf theorem it is possible to verify the asymptotic equality of the differential cross sections of processes (I) and (II)

$$\lim_{s\to\infty}\frac{\dfrac{d^4\sigma^{\mathrm{I}}}{dt\,dt''\,dW^2\,d\xi}}{\dfrac{d^4\sigma^{\mathrm{II}}}{dt\,dt''\,dW^2\,d\xi}}=1.$$

We give some of these pair processes below:

$$\pi^+ + p \longrightarrow \pi^+ + \pi^0 + p, \quad \pi^- + p \longrightarrow \pi^- + \pi^0 + p;$$
$$K^+ + p \longrightarrow K^+ + \pi^0 + p, \quad K^- + p \longrightarrow K^- + \pi^0 + p;$$
$$K^+ + p \longrightarrow K^0 + \pi^+ + p, \quad K^- + p \longrightarrow \bar{K}^0 + \pi^- + p;$$
$$\pi^- + p \longrightarrow K^+ + K^- + n, \quad \pi^+ + n \longrightarrow K^- + K^+ + p;$$
$$\gamma + p \longrightarrow \pi^+ + \pi^0 + n, \quad \gamma + n \longrightarrow \pi^- + \pi^0 + p;$$
$$p + p \longrightarrow n + \pi^+ + p, \quad \bar{n} + p \longrightarrow \bar{p} + \pi^+ + p.$$

When two of the three final particles are formed in a resonant state, we have equality of the cross sections for the following processes:

$$\pi^+ + p \longrightarrow \varrho^+ + p, \quad \pi^- + p \longrightarrow \varrho^- + p;$$
$$K^+ + p \longrightarrow K^{*+} + p, \quad K^- + p \longrightarrow K^{*-} + p;$$
$$\pi^- + p \longrightarrow \varphi + n, \quad \pi^+ + n \longrightarrow \varphi + p;$$
$$\gamma + p \longrightarrow \varrho^+ + n, \quad \gamma + n \longrightarrow \varrho^- + p;$$
$$p + p \longrightarrow n + \Delta^{++}, \quad \bar{n} + p \longrightarrow \bar{p} + \Delta^{++}, \text{ etc.}$$

In the work of L.D. Solov'ev [55] it was shown that allowance for electromagnetic interactions does not alter the result that the differential cross sections of particles and antiparticles are equal at high energies (if they are measured by equipment with the same, fairly good, energy resolution).

It is natural to expect that asymptotic behavior starts at energies exceeding the mass of the particles and the resonances. If in this energy range equality of the cross sections for particles and antiparticles does not occur, this will constitute a firm basis for considering the violation of the microcausality principle at small distances.

We therefore regard as very important the experimental verification of the asymptotic relations with sufficiently high accuracy.

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DISCUSSION

N. N. Meiman

The following three properties of the scattering amplitude $A(s, u, t)$ are sufficient for deriving asymptotic relations for the cross sections of a particle and an antiparticle: analyticity of $A(s, u, t)$ with respect to the variable s for complex s of sufficiently large modulus; a slower increase of $|A(s, u, t)|$ for $s \longrightarrow \infty$ in the complex domain than any linear exponent; cross symmetry. In particular, the existence of an arbitrary number of complex singularities in the amplitude does not affect the asymptotic relations.

Pomeranchuk's theorem and its various generalizations remain in force also when the first and second conditions are violated, if two auxiliary functions $A_{\infty}^{\mathrm{adv}}(s,t)$ and $A_{\infty}^{\mathrm{ret}}(s,t)$

with the following properties exist:

1. $A^{\text{ret}}_{\infty}(s, t)$ asymptotically agrees with the amplitude $\overset{\infty}{A}(s, u, t)$ in the physical range of the reaction, and $A^{\text{adv}}_{\infty}(s,t)$ asymptotically agrees with the amplitude in the physical range of the cross reaction.

2. $A^{\text{ret}}_{\infty}(s,t)$ and $A^{\text{adv}}_{\infty}(s, t)$ are analytic respectively in the upper and lower half-planes and increase at infinity slower than any linear exponent.

3. The cross-symmetry $A^{\text{ret}}_{\infty}(s, t) = A^{\text{adv}}_{\infty}(s^*, t)^*$, $t \leqslant 0$ is satisfied.*

On the basis of the microcausality in Bogolyubov's form the structure of the permissible generalized function is obtained. These functions are not generalized functions of moderate growth and may contain derivatives of the δ-function of all orders.

We gave the necessary and sufficient conditions, under which the ratio of the differential cross sections of a particle and an antiparticle tends for $s \to \infty$ to unity, to a finite limit $\gamma^2 \neq 1$, and to infinity or zero.**

If $A(s, u, t)$ has only a finite number of zeros in the plane s (this number may grow together with t), then the first case occurs when the phase of the ratio $A(s, u, t)/A(u, s, t)$ increases or decreases more slowly than $\ln s$; the second case occurs when the phase of this ration increases or decreases with $\ln s$, and the third case when the phase increases or decreases faster than $\ln s$.

The most difficult and interesting problems seem to be those associated with the oscillation, especially when the amplitude $A(s, t)$ has an infinite number of zeros.**† The formulations in this case become somewhat more complicated.

Yu. M. Lomsadze

In connection with the problems discussed in the report of Prof. Logunov, as well as in another connection, it is of definite interest to consider the behavior of the amplitude at infinity in the s-plane in the case of potential scattering. Some hints in this respect for the case of nonrelativistic scattering were already made at the Rochester conference.†† At present this problem can be regarded as solved rigorously for all types of potential scattering – nonrelativistic and relativistic, quantum mechanical and Bethe-Salpeter.‡

* JETP 46, 4 (1964); Preprint IEF, No 252, 1964

** JETP, 46, 3 (1964).

† JETP, 46, 4 (1964); Preprint IEF, No 252, 1964.

†† Proc. 1962 Internat. Conf. on High-Energy Physics, CERN, Geneva (1962): Discussion after Regge's report.

‡ Lomsadze, Yu. M. Proceedings of the University of Uzhgorod, p. 60, May, 1964.

We will apply the term "nonphysical" to a potential $V(r)$, where r is the three-dimensional (in the case of the nonrelativistic or relativistic Schrödinger equation) or the four-dimensional (in the case of the Bethe-Salpeter equation) distance between the interacting particles, if for $r \to 0$ the potential tends to $+\infty$ more strongly than r^{-n} where n is the maximum order of the derivative in the corresponding equation. A potential with a smaller singularity at zero will be called physical. Then in general form, omitting details, it can be affirmed that the partial amplitude $f(l, s)$ of scattering on an arbitrary physical potential cannot have an essential singularity at infinity in the s-plane, whereas in the case of an arbitrary nonphysical potential, it inevitably will have an essential singularity there, rendering impossible either Mandelstam's representation, or even one-dimensional dispersion relations, at least in the generally accepted form. A rigorous proof of this affirmation, for example, in the nonrelativistic case, assumes that the nonphysical potential satisfies the condition $V(r) \geqslant c \ln^4 r/f^2, r \to \infty$. From the general principles of the local theory and the assumption of polynomial behavior of the amplitude when s increases along the real axis, it thus follows that the real interaction of elementary particles can be modeled only by physical potentials.

The following example illustrates which results can be extracted from the above-mentioned fact. It is well known that the field theoretical interaction Hamiltonians of class II and higher lead to such Bethe-Salpeter potentials, for which the coefficients of power expansion (in the square of the coupling constant g of the fields), at least starting from some number, possess a nonphysical behavior for $r \to 0$. On the basis of the most general considerations it therefore can be concluded that this is only a consequence of the abnormal behavior of the given power of expansion for $r \to 0$. In other words, the radius of convergence of this expansion, if it is not equal to zero for all r, should inevitably tend to zero for $r \to 0$.*

Since the amplitude $f(l, s)$ of scattering on an arbitrary physical potential $gV(r)$ is free from a stationary singularity in the g-plane at the point $g = 0$, and in the case of a nonphysical potential $gV(r)$ it inevitably has at this point a stationary (independent of s and l) singularity (of the square root and only of the square root type), a mutually single-valued correlation appears between: 1) the physical character or the nonphysical character of the exact Bethe-Salpeter potential, 2) the absence or presence in the amplitude $f_g(l, s)$ of an essential singularity in the s-plane at infinity, and 3) the absence of presence in $f_g(l, s)$ of a stationary singularity in the g-plane at the point $g = 0$. There are also grounds for con-

* Lomsadze, Yu. M. et al., Preprint of Uzhgorod University.

sidering this correlation to be closely related to the renormalizability or nonrenormalizability of the corresponding variant of the theory.

Nguyen Van Hieu

My remark refers to the asymptotic behavior of the cross sections for inelastic processes with formation of resonances. Among the inelastic processes there exist several quasi-elastic ones, whose cross sections have the same asymptotic behavior as the cross sections for elastic scattering, for example:

$$\pi^+ + p \to \pi^+ + N^{*+},\ \pi^- + p \to \pi^- + N^{*+},$$
$$p + p \to p + N^{*+},\ \bar{p} + p \to \bar{p} + N^{*+},$$

where N^* is a nucleonic isobar with asymptotic spin $I = 1/2$. For these processes in the t-channel exist states with vacuum quantum numbers similar to those for elastic processes. If for elastic processes some singularity of a partial amplitude of a state with vacuum quantum numbers (in the t-channel) determines the asymptotic behavior of the cross section in the s-channel, then for the considered quasi-elastic processes the asymptotic behavior of the cross section is apparently also determined by a similar singularity of a partial amplitude with vacuum quantum numbers (Regge vacuum pole, for example). This means that the cross sections of the quasi-elastic processes under consideration have the same behavior. This conclusion also holds for processes:

$$\gamma + p \to \varrho^0 + p,$$
$$\gamma + p \to \omega + p.$$