

# Dirac dark matter in $U(1)_{B-L}$ with the Stueckelberg mechanism

Chengcheng Han<sup>a,b</sup>, M.L. López-Ibañez<sup>c,d</sup>, Bo Peng<sup>c,d,\*</sup>,  
Jin Min Yang<sup>c,d,e</sup>

<sup>a</sup> School of Physics, Sun Yat-Sen University, Guangzhou 510275, China

<sup>b</sup> School of Physics, KIAS, 85 Hoegiro, Seoul 02455, Republic of Korea

<sup>c</sup> CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

<sup>d</sup> School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

<sup>e</sup> Department of Physics, Tohoku University, Sendai 980-8578, Japan

Received 18 May 2020; received in revised form 28 July 2020; accepted 20 August 2020

Available online 26 August 2020

Editor: Hong-Jian He

---

## Abstract

We investigate a  $U(1)_{B-L}$  gauge extension of the Standard Model (SM) where the gauge boson mass is generated by the Stueckelberg mechanism. Three right-handed neutrinos are added to cancel the gauge anomaly and hence the neutrino masses can be explained. A new Dirac fermion could be a WIMP dark matter whose interaction with the SM sector is mediated by the new gauge boson. Assuming the perturbativity of the gauge coupling up to the Planck scale, we find that only the resonance region is feasible for the dark matter abundance. After applying the  $\Delta N_{eff}$  constraints from the current Planck experiment, the collider search constraints as well as the dark matter direct detection limits, we observe that the  $B-L$  charge of dark matter satisfies  $|Q_\chi| > 0.11$ . Such a scenario might be probed conclusively by the projected CMB-S4 experiment, assuming the right-handed neutrinos are thermalized with the SM sector in the early universe. © 2020 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP<sup>3</sup>.

---

---

\* Corresponding author.

E-mail address: [pengbo@mail.itp.ac.cn](mailto:pengbo@mail.itp.ac.cn) (B. Peng).

## 1. Introduction

The discovery of neutrino oscillations [1,2] indicates that neutrinos should have tiny but non-zero masses, which can not be explained in the framework of the Standard Model (SM). One of the compelling solutions to neutrino mass problem is to introduce three right-handed neutrinos which directly couple to the SM sector through Yukawa interactions. At the same time, the gauge sector can be extended with an additional anomaly free  $U(1)_{B-L}$ . Another fact demanding the presence of new physics is the existence of dark matter (DM) which constitutes about 27% of the global energy budget in the universe. Therefore, it is intriguing to explain these phenomena in a same framework.<sup>1</sup>

In this article, we investigate a Dirac fermionic dark matter in a  $B-L$  gauge extension of the SM where the new gauge boson  $Z'$  obtains mass via the Stueckelberg mechanism. Such a model is first proposed by [3] and here we will give a more comprehensive study. In this model the neutrinos are Dirac fermions and a vector-like Dirac particle  $\chi$  charged under  $U(1)_{B-L}$  is assumed to be a WIMP dark matter candidate. In the early universe, DM is in thermal equilibrium with the SM plasma by exchanging the  $Z'$  boson and then freezes out when the expansion rate of the universe exceeds its annihilation rate. Finally, the current DM relic abundance needs to be consistent with the Planck data [20].

On the other hand, since the right-handed neutrinos interact with the new gauge boson, they are also in the thermal equilibrium with the SM sector in the early universe. When the temperature goes much below the gauge boson mass, they decouple and become the hot relic. Similar to the neutrinos and photons, they contribute to the radiation energy density which is usually described as the effective number of neutrino species  $N_{eff}$ , which is predicted to be 3.043 in the SM [21–25]. The radiation density can be probed by the observation of the anisotropies in the cosmic microwave background (CMB), which was proposed long time ago [26]. The recent result from the Planck satellite shows  $N_{eff} = 2.99^{+0.34}_{-0.33}$ , providing a strong constraint on the extensions of SM where the massless or light particles are present. We will show that this value already gives a very strong limit on our scenario. Particularly, it is pointed out in a recent article [27] that the projected CMB-S4 experiment will provide serious constraints for almost all Dirac-neutrino models, especially those addressing the origin of small neutrino masses.

This paper is organized as follows. In Section 2, we introduce the  $U(1)_{B-L}$  Stueckelberg extension of the SM. Next, in Section 3, the calculation of the shift in the effective number of neutrino species,  $\Delta N_{eff}$ , and the current and future experimental bounds are discussed. In Section 4, we examine various constraints on the parameter space of the model. At last, we draw our conclusions in Section 5.

## 2. The model

The gauge group of the model is

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}. \quad (1)$$

The Stueckelberg mechanism as an alternative to the Higgs mechanism can give mass to abelian vector bosons without breaking gauge invariance [28–37]. The Lagrangian related to the Stueckelberg mechanism is given by

<sup>1</sup> In Ref. [3–19], a connection between the DM candidate and the origin of neutrino masses has been explored in detail.

$$\mathcal{L}_{St} = -\frac{1}{4}Z'^{\mu\nu}Z'_{\mu\nu} - \frac{1}{2}(M_{Z'}Z'_\mu + \partial_\mu\sigma)^2 \quad (2)$$

which is invariant under the following transformation

$$\delta Z'_\mu = \partial_\mu\epsilon(x), \quad \delta\sigma = -M_{Z'}\epsilon(x). \quad (3)$$

In the quantum theory, a gauge fixing term

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}(\partial_\mu Z'^\mu + \xi M_{Z'}\sigma)^2 \quad (4)$$

should be added to the total Lagrangian so that the new gauge boson becomes massive while the field  $\sigma$  decouples. Note that the scalar field  $\sigma$  can have Stueckelberg couplings to all abelian gauge bosons, including the hypercharge vector boson  $B$  in the SM [38–42]. However, in this work, we only focus on the pure Stueckelberg sector in the absence of the mass mixing of the gauge boson  $B$  with the  $U(1)_{B-L}$  gauge boson  $Z'$  for simplicity. Then the  $B-L$  vector current  $J'$  coupling to the gauge boson  $Z'$  is given as

$$\mathcal{L}_{St}^{int} = -g'Z'_\mu J'^\mu \quad (5)$$

where  $g'$  is the  $B-L$  gauge coupling and  $J'$  comes from quarks, leptons and DM.

In the Stueckelberg scenario, neutrinos are Dirac-type and their masses can be generated by Yukawa interactions via the Higgs mechanism,

$$\mathcal{L}_\nu = -Y_\nu \bar{l}_L i\sigma_2 H^* \nu_R + \text{h.c.} \quad (6)$$

For a sub-eV neutrino mass, the coupling  $Y_\nu$  should be generally smaller than  $10^{-12}$ . We can add a Dirac fermion  $\chi$  which only takes the  $B-L$  charge and can be a dark matter candidate (its stability can be guaranteed if its  $B-L$  charge  $Q_\chi$  is not equal to  $\pm 1$ , otherwise it will mix with the right-handed neutrino and decay). The relevant Lagrangian for the DM is then written as

$$\mathcal{L}_{DM} = i\bar{\chi}\gamma^\mu D_\mu\chi - M_\chi\bar{\chi}\chi. \quad (7)$$

The total Lagrangian in the model can be summarized as

$$\mathcal{L}_{tot} = \mathcal{L}_{SM} + \mathcal{L}_{St} + \mathcal{L}_{gf} + \mathcal{L}_{St}^{int} + \mathcal{L}_\nu + \mathcal{L}_{DM}. \quad (8)$$

Based on the above Lagrangian, for  $M_\chi \gg m_f$ , DM can annihilate into  $e_i^+e_i^-$ ,  $\bar{\nu}_i\nu_i$ ,  $\bar{u}_i u_i$ ,  $\bar{d}_i d_i$  ( $i$  denotes three families of quarks and leptons) via the gauge boson  $Z'$ . The non-relativistic form for these annihilation cross sections is

$$\begin{aligned} \sigma v(\chi\bar{\chi} \rightarrow f\bar{f}) &= \frac{N_f^C Q_\chi^2 Q_f^2 g'^4}{2\pi} \sqrt{1 - \frac{m_f^2}{M_\chi^2} \frac{2M_\chi^2 + m_f^2}{(4M_\chi^2 - M_{Z'}^2)^2 + M_{Z'}^2 \Gamma_{Z'}^2}} \\ &\approx \frac{N_f^C Q_\chi^2 Q_f^2 g'^4 M_\chi^2}{\pi[(4M_\chi^2 - M_{Z'}^2)^2 + M_{Z'}^2 \Gamma_{Z'}^2]} \end{aligned} \quad (9)$$

where  $v$  is the relative velocity of the annihilating DM pair,  $N_f^C$  is the number of colors of the final state SM fermions,  $Q_\chi$  and  $Q_f$  represent the  $B-L$  charges of DM and SM fermions, and  $\Gamma_{Z'}$  is the decay width of  $Z'$  boson given by

$$\Gamma_{Z'} = \sum_f \theta(M_{Z'} - 2m_f) \frac{N_f^C Q_f^2 g'^2 M_{Z'}}{12\pi} \sqrt{1 - \frac{4m_f^2}{M_{Z'}^2}} \left(1 + \frac{2m_f^2}{M_{Z'}^2}\right) + \theta(M_{Z'} - 2m_\chi) \frac{Q_\chi^2 g'^2 M_{Z'}}{12\pi} \sqrt{1 - \frac{4m_\chi^2}{M_{Z'}^2}} \left(1 + \frac{2m_\chi^2}{M_{Z'}^2}\right). \quad (10)$$

From Eq. (9) it can be seen that the ratio of the contribution of a quark to the total DM annihilation cross section and the contribution of a lepton is about 1 : 3. Besides, DM is also able to annihilate into two  $Z'$  bosons when DM is heavier than  $Z'$ . The annihilation cross section of this channel is

$$\sigma v(\chi\bar{\chi} \rightarrow Z'Z') = \frac{Q_\chi^4 g'^4}{16\pi M_\chi^2} \left(1 - \frac{M_{Z'}^2}{M_\chi^2}\right)^{3/2} \left(1 - \frac{M_{Z'}^2}{2M_\chi^2}\right)^{-2}. \quad (11)$$

### 3. The effective number of neutrino species in cosmology

Three additional right-handed neutrinos can be in thermal equilibrium with the SM plasma via the exchange of  $Z'$  boson in the early universe so that they can contribute to the expansion rate of the universe. However, due to their weak interactions,<sup>2</sup> such particles decouple earlier from the plasma than the left-handed neutrinos and therefore their contribution to the energy density of the universe is suppressed compared to that of the left-handed neutrinos. The extra radiation energy density is usually expressed in terms of an effective number of neutrinos,  $N_{eff} = (8/7)(11/4)^{4/3} \rho_\nu / \rho_\gamma$ . The SM prediction of this value is 3.043.

In this scenario, the extra contribution of the right-handed neutrinos to the effective number of neutrino species is given as

$$\Delta N_{eff} = N_{\nu_R} \left(\frac{T_{\nu_R}}{T_{\nu_L}}\right)^4 = N_{\nu_R} \left(\frac{g_*(T_{\nu_R}^{dec})}{g_*(T_{\nu_L}^{dec})}\right)^{-\frac{4}{3}} \quad (12)$$

where  $N_{\nu_R}$  represents the number of relativistic right-handed neutrinos,  $g_*(T) = g_B(T) + \frac{7}{8}g_F(T)$  with  $g_{B,F}(T)$  being the number of bosonic and fermionic relativistic degrees of freedom in equilibrium at the temperature  $T$ . The second equality is obtained from taking into account of the isentropic heating of the rest of the plasma between  $T_{\nu_R}^{dec}$  and  $T_{\nu_L}^{dec}$  decoupling temperatures. Taking three active neutrinos,  $e^\pm$  and photon into account, we have  $g_*(T_{\nu_L}^{dec}) = 43/4$  at  $T_{\nu_L}^{dec} \approx 2.3$  MeV [43].

The effective number of neutrino species has a strong relation with the temperature at which the right-handed neutrinos decouple from the SM plasma, which can be decided by

$$H(T_{\nu_R}^{dec}) \simeq \Gamma(T_{\nu_R}^{dec}). \quad (13)$$

Here  $H(T)$  is the Hubble expansion parameter which is estimated by  $1.66 g_*^{1/2}(T) \frac{T^2}{M_{Pl}}$  where  $g_*(T)$  is the effective degree of freedom [44,45] including the contribution of right-handed neutrinos.  $\Gamma_{\nu_R}(T)$  is the right-handed neutrino interaction rate which can be calculated by  $\sum_f n_{\nu_R}(T) \langle \sigma(\bar{\nu}_R \nu_R \rightarrow \bar{f} f) v \rangle$ .

<sup>2</sup> For a dark matter mass around TeV, the dark matter relic abundance generally requires the gauge boson mass not much beyond that.

In our paper we examine the bounds on the parameters of the model by using present and prospective experimental data. The current Planck CMB measurement gives the result  $N_{eff} = 2.99^{+0.34}_{-0.33}$  including baryon acoustic oscillation (BAO) data [20]. Combining with the data given above,  $N_{eff}^{SM} = 3.043$ , we adopt a conservative limit  $\Delta N_{eff} < 0.283$  and then the bound with respect to the  $B - L$  gauge coupling  $g'$  and  $Z'$  boson mass  $M_{Z'}$  is given as  $M_{Z'}/g' > 10.4$  TeV. It gives a very strong limit on the parameter space, as we will show later.

Besides, there are several experiments with better sensitivities which are underway or projected. The South Pole Telescope (SPT-3G), which is a ground-based telescope in operation at present, will have a sensitivity of  $\sigma(\Delta N_{eff}) = 0.058$  [46]. The CMB Simons Observatory (SO), which will see first light in 2021 and start a five-year survey in 2022, is expected to reach a similar sensitivity in the range of  $\sigma(\Delta N_{eff}) = 0.05 - 0.07$  [47]. The CMB Stage IV (CMB-S4) experiment will have the potential to constrain  $\Delta N_{eff} = 0.06$  at 95% C.L. as a single parameter extension to  $\Lambda$ CDM [48]. Importantly, according to Eq. (12), the minimal shift in the effective number of neutrino species in our scenario can be evaluated to acquire  $\Delta N_{eff} \geq 0.141$  when  $T_{\nu R}^{dec}$  is high enough. Hence, the future CMB-S4 experiment will be able to probe this scenario for arbitrary decoupling temperatures conclusively as long as the right-handed neutrinos have a thermalization with the SM plasma in the early universe.

#### 4. Numerical results and discussions

According to the Lagrangian in Eq. (8), in the dark matter sector there are only four relevant parameters: the  $U(1)_{B-L}$  gauge boson mass  $M_{Z'}$ , the DM mass  $M_\chi$ , the  $B - L$  charge of the DM  $Q_\chi$  and the gauge coupling  $g'$ . By definition the  $B - L$  charges of quarks and leptons in SM are  $+1/3$  and  $-1$  respectively and DM could have arbitrary charge except for  $\pm 1$ . The gauge coupling should satisfy  $g' < 2\sqrt{\pi}$  to ensure the perturbativity of the theory.

In this work, we consider the constraints from the DM relic density  $\Omega_\chi h^2$ , the shift in the effective number of neutrino species  $\Delta N_{eff}$ , the dark matter direct detection limits as well as the collider search limits for the  $Z'$ . We also require the gauge coupling to keep perturbativity up to the Planck scale  $M_{pl}$ .

The one-loop  $\beta$  function of the  $U(1)_{B-L}$  gauge coupling is given by

$$\beta(g') = \frac{g'^3}{16\pi^2} \sum_i Q_i^2 = \frac{(60 + 9Q_\chi^2)g'^3}{144\pi^2} = \beta_0 g'^3 \quad (14)$$

where  $i$  sums over all particles that carry  $B - L$  charge and  $\beta_0$  is a function of  $Q_\chi$ . Assuming that the Landau pole does not occur below the Planck scale, then we get

$$g'(\mu) \lesssim \left( 2\beta_0 \ln \frac{M_{pl}}{\mu} \right)^{-\frac{1}{2}} \quad (15)$$

with the renormalization scale  $\mu = M_{Z'}$ .

In our numerical calculation, we use LanHEP 3.2.0 [49] to generate the Feynman rules of the model and apply MicrOMEGAs 5.0.9 [50] to compute the DM relic abundance and DM-nucleon scattering cross-section. In Fig. 1, we show all the relevant constraints on the plane of  $M_{Z'}$  versus  $M_\chi$  for  $Q_\chi = 1/3$  (left) and  $4/3$  (right). In the gray region in the upper left corner the dark matter is overabundant because the dark matter annihilation cross section is too small. It also shows that the direct detections and collider experiments can exclude some regions where the gauge boson mass or dark matter mass is relatively light. However,  $\Delta N_{eff}$  gives a stronger constraint than

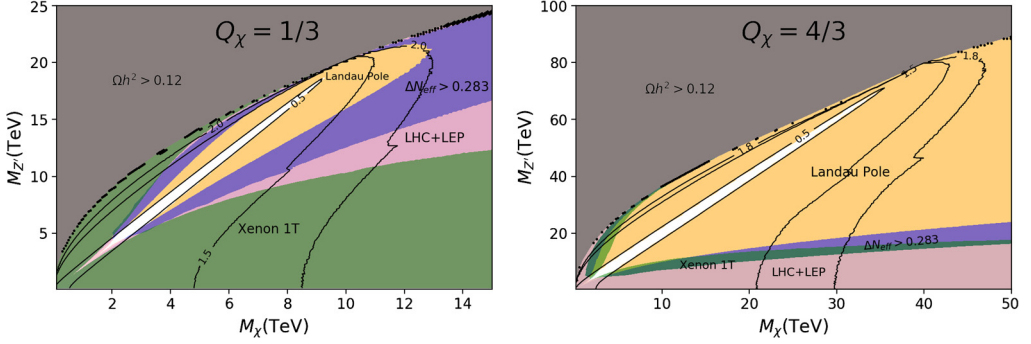


Fig. 1. Various constraints on the parameter space of  $M_\chi$  versus  $M_{Z'}$  for  $Q_\chi = 1/3$  (left) and  $4/3$  (right). All the regions, except for the gray region which overproduces DM, satisfy the DM relic density,  $\Omega_\chi h^2 = 0.12$ . The solid black curve represents different  $g'$ s that are tuned to get the correct DM relic density. The green, pink, blue and orange regions are excluded by the Xenon 1T [51], LHC+LEP [52,53],  $\Delta N_{eff}$  [20] and Landau pole, respectively. Black points satisfy the DM relic abundance but their gauge couplings are larger than  $2\sqrt{\pi}$ . The remained blank region survives all these constraints. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

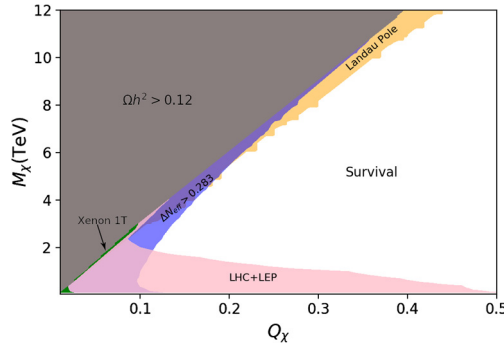


Fig. 2. Same as Fig. 1, but showing  $M_\chi$  versus  $Q_\chi$  in the resonant region  $M_{Z'} = 2M_\chi$ .

LHC+LEP and Xenon 1T. For  $Q_\chi = 1/3$ , the cosmological constraint can give an upper bound on  $Z'$  boson mass and DM mass:  $M_{Z'} \leq 22$  TeV and  $M_\chi \leq 13$  TeV, which is consistent with the result in Ref. [3]. For a larger  $Q_\chi$ , this constraint can be weakened because a smaller  $g'$  can still achieve the correct DM relic abundance. Assuming the gauge coupling perturbativity up to the Planck mass scale which generally requires  $g' \lesssim 0.5$  for  $Q_\chi = 1/3$  or  $4/3$ , the survival region is restricted to the blank area where the DM annihilation cross-section is enlarged significantly via the resonance  $M_{Z'} \simeq 2M_\chi$ . From this figure, we conclude that the gauge boson mass is in general of tens of TeV after we impose the Landau pole constraint and hence the gauge boson might be possibly accessible at the future 100 TeV hadron collider. Besides, the small bulge which can be observed at about  $M_\chi \approx 11.5(39)$  TeV in the left (right) panel originates from the contribution of the  $\chi\bar{\chi} \rightarrow Z'Z'$  channel to the relic density.

In Fig. 2 we show the parameter space of  $M_\chi$  versus  $Q_\chi$  by setting  $M_{Z'} = 2M_\chi$  and changing  $g'$  to meet the condition  $\Omega_\chi h^2 = 0.12$  (note that we only scanned the positive  $Q_\chi$ ). It shows that the Xenon-1T experiment has a weaker constraint than others and collider experiments mainly restrict the lower bound on DM mass while  $\Delta N_{eff}$  and the Landau pole primarily restrict the upper bound. As  $Q_\chi$  increases, the restrictions of collider experiments and  $\Delta N_{eff}$  get weakened

due to the decrease of  $g'$ . From this plot, we can see that the  $B - L$  charge of DM must be larger than 0.11 to evade all the constraints. For  $Q_\chi = 1/3$ ,  $M_\chi$  should be heavier than 1 TeV. For the region  $Q_\chi > 0.5$ , the current experimental data does not give limits on the parameter space due to a small  $g'$ . Nevertheless, the future CMB-S4 experiment could be able to cover almost all the parameter space of this scenario even in the resonant region.

At last, it needs to be stressed that there are several differences between our model and the traditional  $B - L$  gauge symmetry model. Firstly, in the traditional  $B - L$  gauge symmetry model, an additional  $\Phi$  with a  $B - L$  charge 2 is introduced to break the  $B - L$  symmetry, while the right-handed neutrino can get a sizable Majorana mass through interacting with  $\Phi$ . In this model, however, the light neutrinos are Dirac fermions, therefore the three additional right-handed neutrinos can contribute to the radiation energy density of the early universe and furtherly affect the observation of the anisotropies in the CMB. In addition, if  $\Phi$  is present in the model, depending on the charge of the DM, there might be extra DM annihilation channels such as  $\bar{\chi}\chi \rightarrow \Phi\Phi^*$ , or  $\bar{\chi}\chi \rightarrow Z'\Phi$ , the parameter space of the model would be larger depending on the spectrum. On the other hand, this kind of model could also be hard to test if all the masses are much beyond TeV.

We also note that for a usual Dirac neutrino model, the  $\Delta N_{eff}$  limit can be relaxed by assuming the  $Z'$  mass much heavier than the reheating temperature to avoid the thermalization of right-handed neutrinos with SM particles. However, to realize a WIMP dark matter, such thermalization can not be avoided. Thus the constraint from the effective number of neutrino species in cosmology is a unique signature of this model.

## 5. Conclusion

In this work, we studied dark matter in a  $U(1)_{B-L}$  gauge extension of the Standard Model where the  $B - L$  gauge boson gains mass via the Stueckelberg mechanism. Three right-handed neutrinos are added to cancel the gauge anomaly and the neutrino masses can be thus explained. A new Dirac fermion plays the role of WIMP dark matter while its interaction with Standard Model sector is mediated by the new gauge boson. Assuming the perturbativity of the gauge coupling up to the Planck scale, we found that only the resonance region is available for the dark matter abundance. After applying the  $\Delta N_{eff}$  constraints from the current Planck experiment, the collider search constraints as well as the dark matter direct detection limits, we observed that the  $B - L$  charge of dark matter satisfies  $|Q_\chi| > 0.11$ . The projected CMB-S4 experiment might be able to probe this scenario conclusively, assuming the right-handed neutrinos are thermalized with the Standard Model sector in the early universe.

## CRedit authorship contribution statement

**Chengcheng Han:** Conceptualization, Methodology, Writing - review & editing. **M.L. López-Ibáñez:** Data curation, Formal analysis, Resources. **Bo Peng:** Conceptualization, Methodology, Software, Visualization, Writing - original draft. **Jin Min Yang:** Funding acquisition, Supervision, Writing - review & editing.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Acknowledgement

This work was supported by the National Natural Science Foundation of China (NNSFC) under grant Nos. 11675242, 11821505, and 11851303, by Peng-Huan-Wu Theoretical Physics Innovation Center (11947302), by the CAS Center for Excellence in Particle Physics (CCEPP), by the CAS Key Research Program of Frontier Sciences and by a Key R&D Program of Ministry of Science and Technology under number 2017YFA0402204. CH acknowledges support from the Sun Yat-Sen University Science Foundation.

## References

- [1] Y. Fukuda, et al., Super-Kamiokande Collaboration, *Phys. Rev. Lett.* 81 (1998) 1562, arXiv:hep-ex/9807003.
- [2] Q.R. Ahmad, et al., SNO Collaboration, *Phys. Rev. Lett.* 89 (2002) 011301, arXiv:nucl-ex/0204008.
- [3] P. Fileviez Perez, C. Murgui, A.D. Plascencia, *Phys. Rev. D* 100 (3) (2019) 035041, <https://doi.org/10.1103/PhysRevD.100.035041>, arXiv:1905.06344 [hep-ph].
- [4] L.M. Krauss, S. Nasri, M. Trodden, *Phys. Rev. D* 67 (2003) 085002, arXiv:hep-ph/0210389.
- [5] T. Asaka, S. Blanchet, M. Shaposhnikov, *Phys. Lett. B* 631 (2005) 151–156.
- [6] E. Ma, *Phys. Rev. D* 73 (2006) 077301, arXiv:hep-ph/0601225.
- [7] C. Boehm, Y. Farzan, T. Hambye, S. Palomares-Ruiz, S. Pascoli, *Phys. Rev. D* 77 (2008) 043516, arXiv:hep-ph/0612228.
- [8] J. Kubo, E. Ma, D. Suematsu, *Phys. Lett. B* 642 (2006) 18, arXiv:hep-ph/0604114.
- [9] M. Aoki, S. Kanemura, O. Seto, *Phys. Rev. Lett.* 102 (2009) 051805, arXiv:0807.0361 [hep-ph].
- [10] Y. Farzan, S. Pascoli, M.A. Schmidt, *J. High Energy Phys.* 1010 (2010) 111, arXiv:1005.5323 [hep-ph].
- [11] L. Canetti, M. Drewes, T. Frossard, M. Shaposhnikov, *Phys. Rev. D* 87 (2013) 093006, arXiv:1208.4607 [hep-ph].
- [12] C. Bonilla, E. Ma, E. Peinado, J.W.F. Valle, *Phys. Lett. B* 762 (2016) 214, arXiv:1607.03931 [hep-ph].
- [13] K. Kaneta, Z. Kang, H.S. Lee, *J. High Energy Phys.* 1702 (2017) 031, arXiv:1606.09317 [hep-ph].
- [14] B. Batell, T. Han, B. Shams Es Haghi, *Phys. Rev. D* 97 (9) (2018) 095020, arXiv:1704.08708 [hep-ph].
- [15] P. Cox, C. Han, T.T. Yanagida, *J. Cosmol. Astropart. Phys.* 1801 (2018) 029, arXiv:1710.01585 [hep-ph].
- [16] S. Baumholzer, V. Brdar, P. Schwaller, *J. High Energy Phys.* 1808 (2018) 067, arXiv:1806.06864 [hep-ph].
- [17] M. Blennow, E. Fernandez-Martinez, A. Olivares-Del Campo, S. Pascoli, S. Rosaura-Alcaraz, A.V. Titov, *Eur. Phys. J. C* 79 (7) (2019) 555, arXiv:1903.00006 [hep-ph].
- [18] C. Bonilla, E. Peinado, R. Srivastava, *Lett. High Energy Phys.* 124 (2019) 1, <https://doi.org/10.31526/lhep.1.2019>, arXiv:1903.01477 [hep-ph].
- [19] D. Nanda, D. Borah, arXiv:1911.04703 [hep-ph].
- [20] N. Aghanim, et al., Planck Collaboration, arXiv:1807.06209 [astro-ph.CO].
- [21] G. Mangano, G. Miele, S. Pastor, T. Pinto, O. Pisanti, P.D. Serpico, *Nucl. Phys. B* 729 (2005) 221, arXiv:hep-ph/0506164.
- [22] E. Grohs, G.M. Fuller, C.T. Kishimoto, M.W. Paris, A. Vlasenko, *Phys. Rev. D* 93 (8) (2016) 083522, arXiv:1512.02205 [astro-ph.CO].
- [23] P.F. de Salas, S. Pastor, *J. Cosmol. Astropart. Phys.* 1607 (2016) 051, arXiv:1606.06986 [hep-ph].
- [24] S. Gariazzo, P.F. de Salas, S. Pastor, *J. Cosmol. Astropart. Phys.* 1907 (2019) 014, arXiv:1905.11290 [astro-ph.CO].
- [25] J.J. Bennett, G. Buldgen, M. Drewes, Y.Y. Wong, arXiv:1911.04504 [hep-ph].
- [26] G. Jungman, M. Kamionkowski, A. Kosowsky, D.N. Spergel, *Phys. Rev. D* 54 (1996) 1332, arXiv:astro-ph/9512139.
- [27] K.N. Abazajian, J. Heeck, arXiv:1908.03286 [hep-ph].
- [28] E.C.G. Stueckelberg, *Helv. Phys. Acta* 11 (1938) 225.
- [29] M. Kalb, P. Ramond, *Phys. Rev. D* 9 (1974) 2273.
- [30] T.J. Allen, M.J. Bowick, A. Lahiri, *Mod. Phys. Lett. A* 6 (1991) 559.
- [31] B. Kors, P. Nath, *Phys. Lett. B* 586 (2004) 366, arXiv:hep-ph/0402047.
- [32] B. Kors, P. Nath, *J. High Energy Phys.* 0412 (2004) 005, arXiv:hep-ph/0406167.
- [33] B. Kors, P. Nath, *J. High Energy Phys.* 0507 (2005) 069, arXiv:hep-ph/0503208.
- [34] H. Ruegg, M. Ruiz-Altaba, *Int. J. Mod. Phys. A* 19 (2004) 3265, arXiv:hep-th/0304245.
- [35] D. Feldman, P. Fileviez Perez, P. Nath, *J. High Energy Phys.* 1201 (2012) 038, arXiv:1109.2901 [hep-ph].
- [36] D. Feldman, Z. Liu, P. Nath, *J. High Energy Phys.* 0611 (2006) 007, arXiv:hep-ph/0606294.
- [37] M. Duerr, P. Fileviez Perez, J. Smirnov, *Phys. Rev. D* 92 (8) (2015) 083521, <https://doi.org/10.1103/PhysRevD.92.083521>, arXiv:1506.05107 [hep-ph].



- [38] D. Feldman, B. Kors, P. Nath, Phys. Rev. D 75 (2007) 023503, arXiv:hep-ph/0610133.
- [39] D. Feldman, Z. Liu, P. Nath, Phys. Rev. D 75 (2007) 115001, arXiv:hep-ph/0702123 [HEP-PH].
- [40] S.A. Abel, M.D. Goodsell, J. Jaeckel, V.V. Khoze, A. Ringwald, J. High Energy Phys. 0807 (2008) 124, arXiv:0803.1449 [hep-ph].
- [41] Y. Zhang, Chin. Phys. Lett. 26 (2009) 081102.
- [42] C.P. Burgess, J.P. Conlon, L.Y. Hung, C.H. Kom, A. Maharana, F. Quevedo, J. High Energy Phys. 0807 (2008) 073, arXiv:0805.4037 [hep-th].
- [43] K. Enqvist, K. Kainulainen, V. Semikoz, Nucl. Phys. B 374 (1992) 392.
- [44] A. Bazavov, et al., Phys. Rev. D 85 (2012) 054503, arXiv:1111.1710 [hep-lat].
- [45] S. Borsanyi, et al., Nature 539 (7627) (2016) 69, arXiv:1606.07494 [hep-lat].
- [46] B.A. Benson, et al., SPT-3G Collaboration, Proc. SPIE Int. Soc. Opt. Eng. 9153 (2014) 91531P, arXiv:1407.2973 [astro-ph.IM].
- [47] M.H. Abitbol, et al., Simons Observatory Collaboration, arXiv:1907.08284 [astro-ph.IM].
- [48] K. Abazajian, et al., arXiv:1907.04473 [astro-ph.IM].
- [49] A. Semenov, Comput. Phys. Commun. 201 (2016) 167, arXiv:1412.5016 [physics.comp-ph].
- [50] G. Belanger, F. Boudjema, A. Goudelis, A. Pukhov, B. Zaldivar, Comput. Phys. Commun. 231 (2018) 173, <https://doi.org/10.1016/j.cpc.2018.04.027>, arXiv:1801.03509 [hep-ph].
- [51] E. Aprile, et al., XENON Collaboration, Phys. Rev. Lett. 119 (18) (2017) 181301, arXiv:1705.06655 [astro-ph.CO].
- [52] M. Aaboud, et al., ATLAS Collaboration, J. High Energy Phys. 1710 (2017) 182, arXiv:1707.02424 [hep-ex].
- [53] S. Alioli, M. Farina, D. Pappadopulo, J.T. Ruderman, Phys. Rev. Lett. 120 (10) (2018) 101801, arXiv:1712.02347 [hep-ph].