CDF/ANAL/BOTTOM/CDFR/7510
Release 2.0
May 16, 2005

Tagging power of the muon + two-track trigger sample

J. Beringer, A. Cerri, A. Deisher, H.-C. Fang, J. Mülmenstädt, M.D. Shapiro

Lawrence Berkeley National Laboratory

Abstract

Beginning in summer 2004 CDF has implemented a trigger requiring a pair of displaced tracks and a muon. This trigger, called the *muon + two-track trigger* ($\mu+$ TT), offers the possibility of collecting a data set enhanced in hadronic B meson decays while ensuring that nearly every event contains a flavor-tagging object. The advantage of the $\mu+\mathrm{TT}$ trigger over the two-track trigger is that it allows CDF to collect a data sample with an event rate several factors smaller than the two-track sample but of comparable statistical power for a mixing analysis. In this note we demonstrate that this hope is realized by showing that in the $B^{\pm}\to D(\to K\pi)\pi$ decay channel the Likelihood Soft Muon Tagger achieves a raw flavor-tagging $N\epsilon D^2$ of 17.7 ± 8.7 (and an ϵD^2 of $5.2\pm2.6\%$). We then compare the statistical power (*i.e.*, $N\epsilon D^2$) of the $\mu+\mathrm{TT}$ data sample with that of the TTT data sample acquired during the same run period and find that the samples provide comparable statistical power. Finally we recommend changes to the trigger that will allow it to remain unprescaled in the 2005 run.

Contents

1 Introduction **2**
- 1.1 Data set description . . . 3
- 1.2 Trigger description . . . 4

2 Statistical power of the $\mu+\mathrm{TT}$ sample **4**
- 2.1 Candidate reconstruction . . . 5
- 2.2 Offline confirmation . . . 6
- 2.3 Measurement of the tagging efficiency and tagging dilution . . . 7
- 2.4 Statistical power of the $\mu+\mathrm{TT}$ sample; overlap with TTT . . . 8

3 Improvements to the trigger **9**
- 3.1 Three-track requirement . . . 9
- 3.2 Muon cuts . . . 11
- 3.3 $\Delta\phi$ cut . . . 11
- 3.4 But which one is *right*? . . . 11

4 Conclusions and recommendations **12**

A Estimating ϵ, D, ϵD^2 and $N\epsilon D^2$ **13**
A.1 Defining ϵ, D, ϵD^2 and $N\epsilon D^2$ 13
A.2 Estimating ϵ, D, ϵD^2 and $N\epsilon D^2$ safely 15
A.3 Analytic approximations 16

List of Figures

1 L2 trigger cross sections of the $\mu+$TT triggers 5
2 Invariant mass distribution of reconstructed B candidates 6
3 Relative statistical power of $\mu+$TT and TTT 10
4 L2 rate reduction for various cuts 12

List of Tables

1 Fraction of offline objects in events that satisfy a $\mu+$TT trigger 7
2 Efficiency, dilution, ϵD^2 and $N\epsilon D^2$ by muon system 8
3 $N\epsilon$ for various cuts 11

1 Introduction

The B_s mixing reach of CDF depends on collecting a sample of events in which the lifetime and initial and final flavor of B_s mesons can be reconstructed. The CDF detector currently employs several trigger strategies for acquiring high-statistics samples enriched in B physics events. One of these strategies, the two-track trigger (TTT), is of particular importance to the mixing measurement. This trigger requires two tracks displaced from the beamline but does not require a lepton; these requirements provide a sample enriched in hadronic B decays, where the measurement of the B lifetime is more precise than in semileptonic decays.

Currently multiple instantiations of the TTT strategy exist, differing in the looseness of the trigger requirements. They are prescaled to cope with data acquisition bandwidth limits. Since the trigger requirements do not ensure that every event be taggable, the efficiency of flavor taggers on the TTT data sample is low. Thus the detector is randomly rejecting potentially useful events because the data acquisition system is filled with events of which a large fraction is not usable in B mixing analyses.

A proposed remedy to this situation is to require tagging objects at the trigger level. Since summer 2004 CDF has run with triggers requiring a pair of displaced tracks and a muon. This trigger, called the *muon + two-track trigger* ($\mu+$TT), offers the possibility of collecting a hadronic B data set on which the soft muon tagger efficiency is high. At the same time the muon requirement sufficiently reduces the event rate compared to the TTT event rate that this trigger (after minor improvements) can run unprescaled.

This note describes a study we have performed on the $\mu + \mathrm{TT}$ data set collected in 2004. We have measured the flavor-tagging efficiencies ϵ and dilutions D achieved with the Likelihood Soft Muon Tagger (LSMT) [1] in the decay channel $B^{\pm} \to D(\to K\pi)\pi$. We calculate the effective number of events $N(\mu + \mathrm{TT})\epsilon D^2$ in the $\mu + \mathrm{TT}$ sample and compare it to the $N(\mathrm{TTT})\epsilon D^2$ of the TTT sample. From this we conclude that the statistical power of the $\mu + \mathrm{TT}$ sample is comparable to the statistical power of the LSMT-taggable part of the TTT sample obtained in the summer 2004 data-taking period.

Because the average prescale factor on the TTT will increase in the higher-luminosity environment in 2005, an unprescaled $\mu + \mathrm{TT}$ will contribute to the mixing analysis in 2005 that would otherwise not be achievable. We describe two approaches to reducing the $\mu + \mathrm{TT}$ data rate sufficiently so that the trigger will remain unprescaled. One method, a distinctness requirement on the three trigger tracks, reduces the data rate by a factor of two while rejecting few taggable B decays. The second, a cut on the $\Delta\phi$ between the two-track pair and the muon, reduces the data rate further but also rejects taggable B decays.

We have structured this note as follows. In Sec. 2 we present our measurement of the statistical power of the $\mu + \mathrm{TT}$ sample. In Sec. 3 we discuss the possible improvements to the trigger. We conclude and make recommendations in Sec. 4.

1.1 Data set description

The events used for this study come from the `B_HADRONIC` sample collected during summer 2004 (`xbhd0e`) with run numbers greater than 182629. The integrated luminosity of the data set is approximately 120 pb^{-1}. In our studies we use two samples:

- all events that arrived on a $\mu + \mathrm{TT}$ trigger path
- events with fully reconstructed B mesons in the $B^{\pm} \to D(\to K\pi)\pi$ channel that arrived on any of the TTT paths or a $\mu + \mathrm{TT}$ trigger path.

The $\mu + \mathrm{TT}$ trigger requirements are described in Sec. 1.2.

The data was processed and skimmed with the 5.3.1 release of the CDF offline software. The B reconstruction described in Sec. 2 was performed with the 5.3.4 release. Runs from the COT compromised period [179056, 182843] and COT recovery period [184062, 184208] have been excluded from this study. The SVX and SVT online status bits are also required to be good. The CMU and CMX status bits are required to be good for events in the respective flavors of $\mu + \mathrm{TT}$ trigger samples.

1.2 Trigger description

There are two trigger paths for events in the $\mu + \mathrm{TT}$ sample, one for CMU muons and one for CMX muons. Both require a L1 muon in addition to an unprescaled L1 TTT trigger bit. The CMU muon stub and its XFT match must have $p_T > 1.5$ GeV/c. The CMX muon stub must have $p_T > 1.5$ GeV/c, and its XFT match must have $p_T > 2.0$ GeV/c. The L1 trigger bits are currently combined with a logical and, so the trigger muon may be one of the XFT tracks. The L2 and L3 triggers use only the low-p_T TTT requirements. There are no cuts on the muon after L1.

The two trigger paths for events satisfying the $\mu + \mathrm{TT}$ are given below.

- `B_CHARM_LOWPT_L1_CMU1.5_PT1.5`
  - `L1_TWO_TRK2_DPHI90&L1_CMU1.5_PT1.5`
  - `L2_B_CHARM_LOWPT_L1_CMU1.5_PT1.5`
  - `L3_TWO_TRACK_B_CHARM_LOWPT`
- `B_CHARM_LOWPT_L1_CMX1.5_PT2`
  - `L1_TWO_TRK2_DPHI90&L1_CMX1.5_PT2_CSX`
  - `L2_B_CHARM_LOWPT_L1_CMX1.5_PT2`
  - `L3_TWO_TRACK_B_CHARM_LOWPT`

The current $\mu + \mathrm{TT}$ trigger and TTT have `fitType` set to `1` in the L3 instance of `L3-TwoTrackFilterModule`. This `fitType` setting requires the two L3 tracks to pass d_0 and d_0 significance cuts. If `fitType` is set to `2`, the two L3 tracks must also be matched to SVT tracks. The latter L3 cut is implemented in the B semileptonic triggers.

Fig. 1 shows the current L2 cross sections for the μ+TT triggers. The bandwidth required by the μ+TT triggers at high luminosity is not negligible. For example at $\mathcal{L} = 10^{32}$ cm^{-2}s^{-1} the $\mu + \mathrm{TT}$ trigger has a cross section of 500 nb, while the Scenario C (high p_T TTT with fixed prescale 2) cross section is 600 nb. However, the next section of this note demonstrates that the μ+TT trigger will be valuable for B_s mixing. We suggest a number of improvements that will lower the cross section and help the upgraded $\mu + \mathrm{TT}$ avoid future prescale.

2 Statistical power of the $\mu + \mathrm{TT}$ sample

We now turn to the measurement of the statistical power of the $\mu + \mathrm{TT}$ data sample. For this measurement we reconstruct the decay $B^+ \to \overline{D}^0(\to K^+\pi^-)\pi^+$ (and its conjugate) and tag the flavor of the B candidates using the LSMT. We compare the charge of the B candidate to the decision of the LSMT to determine the raw dilution (*i.e.*, with no correction

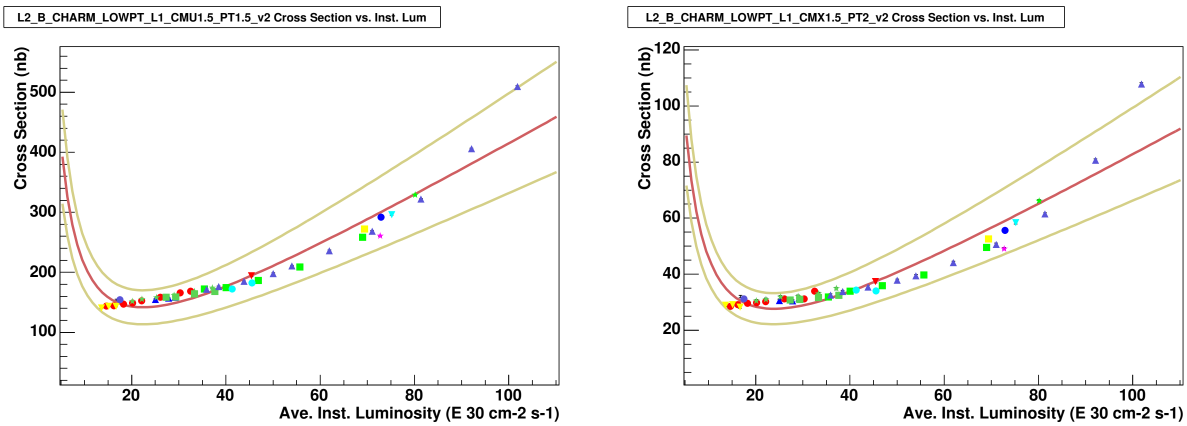


Figure 1: L2 trigger cross sections of the $\mu+\mathrm{TT}$ triggers. The $\mu+\mathrm{TT}$ flavor requiring CMU muons is shown on the left, the flavor requiring CMX muons is shown on the right. The cross section fits are the official fits [2].

for mixing on the opposite side) of the tagger on the $\mu + \mathrm{TT}$ sample; we use the number of cases in which the tagger returns no decision to determine the efficiency. The details of this calculation, and the large ϵD^2 that we measure, are described in Sec. 2.3.

From the measured efficiency and dilution we determine the effective number of B candidates (*i.e.* $N\epsilon D^2$) in the $\mu + \mathrm{TT}$ sample. We then determine $N\epsilon D^2$ in the TTT sample collected during the same period. We divide our results further into three categories: the effective number of candidates present only in the TTT sample; the effective number of candidates present in both the TTT and the $\mu + \mathrm{TT}$ sample; and the effective number of candidates present only in the $\mu+\mathrm{TT}$ sample. Our result indicates that from the standpoint of muon tagging the $\mu + \mathrm{TT}$ trigger provides statistical power comparable to the TTT even though the raw number of candidates in the $\mu+\mathrm{TT}$ sample is a small fraction ($\sim 10\%$) of the number of candidates in the TTT sample and although the $\mu + \mathrm{TT}$ trigger rate is a several times smaller than the TTT rate. Our result further indicates that, due to the prescales applied to the TTT, there are a significant number of effective candidates in the $\mu + \mathrm{TT}$ sample that are not present in the TTT sample. If the $\mu + \mathrm{TT}$ trigger continues to run unprescaled, the relative size of the subsample that can only be collected with the $\mu + \mathrm{TT}$ trigger will increase as the average TTT prescale factor increases during higher-luminosity running.

2.1 Candidate reconstruction

The B candidates are reconstructed using primary vertexing and mass constraints. The procedure is described in detail in CDF note 7512[3]. We apply the following background-

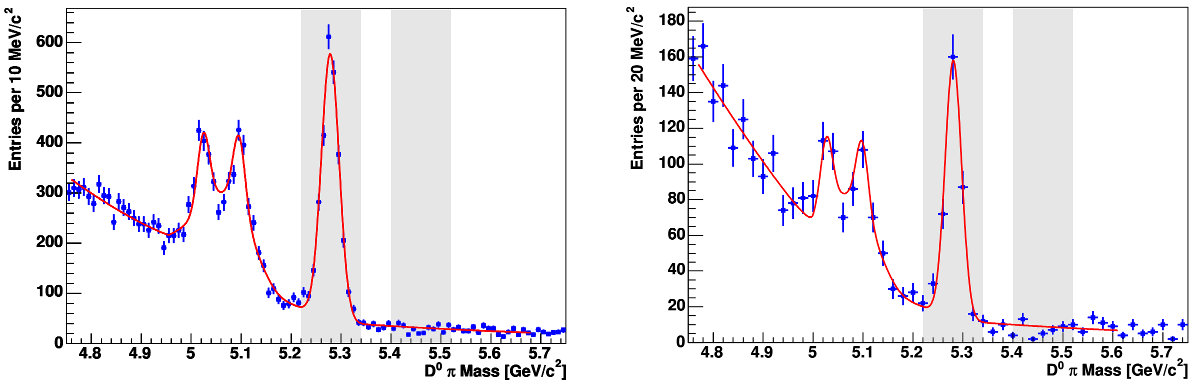


Figure 2: Invariant mass distribution of the reconstructed B candidates. The left plot shows all candidates in the data sample; the right plot shows only those from events that fired the $\mu +$ TT trigger. The peak (5.22 GeV $< m <$ 5.34 GeV) and sideband (5.40 GeV $< m <$ 5.52 GeV) regions are indicated.

reducing cuts on the candidates: $L_{xy} > 300$ µm, isolation cone size $\Delta R > 0.5$, and B vertex χ^2 probability per degree of freedom > 0.10.[1] A plot of the invariant mass distribution of the B candidates reconstructed in the summer 2004 B hadronic data is shown in the left plot of Fig. 2. The right plot includes candidates only from the events that satisfied at least one of the $\mu +$ TT triggers.

2.2 Offline confirmation

In order to study the dilution of the sample, we need to examine the offline counterparts of the muons and SVT pairs that triggered the events. The muons considered in this study are `CdfMuon` objects. The "SVT pairs" to which we will refer consist of two SVT tracks in the event record that satisfy the TTT requirements. We require that each leg of the SVT pair must be matched to a `CdfTrack` with SVT-matching $\chi^2 < 25$ in curvature and ϕ . The "triplets" to which we will refer consist of a SVT pair and a muon that is not matched to either leg of the pair. The distinctness requirement is made by comparing the `CdfTrack` IDs. The fraction of $\mu +$ TT events with zero, one, and greater than one muon, SVT pair, and triplet are given in Tbl. 1.

[1] χ^2 is rescaled according to [3].

SVT pairs	0	9.9%	Offline	0	7.4%	μ + TT	0	57.6%
	1	78.3%	muons	1	75.5%	triplets	1	31.5%
	> 1	11.8%		> 1	17.1%		> 1	10.9%

Table 1: Fraction of offline objects in events that satisfy a μ + TT trigger. There were 9,704,751 μ + TT events in the `xbhd0e` dataset.

2.3 Measurement of the tagging efficiency and tagging dilution

For each reconstructed B candidate in an event that fired the μ + TT trigger we apply the 5.3.4-release LSMT algorithm. If the tagger returns no decision we classify the candidate as untagged. Otherwise we compare the decision of the LSMT to the charge of the reconstructed B to classify the candidate as correctly or incorrectly tagged.

We distinguish between B candidates whose invariant mass lies in the peak region (5.22 GeV $< m <$ 5.34 GeV) and candidates whose invariant mass lies in the sideband region (5.40 GeV $< m <$ 5.52 GeV). PR, PW and PU denote the number of correctly tagged ("right-sign"), incorrectly tagged ("wrong-sign") and untagged B candidates in the peak region, and SR, SW and SU denote the number of correctly tagged, incorrectly tagged and untagged B candidates in the sideband region.

We further classify candidates according to the muon system in which the corresponding tag muon was found. The categories we consider are CMU muons, CMUP muons and CMX muons; CMUP muons are not counted in the CMU category. (For the purposes of calculating the efficiency of CMX muons, a candidate tagged with a non-CMX muon counts as untagged, and equivalently for the other muon types.) Further classification in the spirit of [1] is not possible because of the smallness of our data set.

From the values of PR, PW, PU, SR, SW and SU measured on the data sample we obtain an unbiased estimate of the efficiency

$$\epsilon = \left\langle \frac{R+W}{R+W+U} \right\rangle \tag{2·1}$$

and dilution

$$D = \left\langle \frac{R-W}{R+W} \right\rangle \tag{2·2}$$

by the method described in Sec. A.2. R, W and U are the sideband-subtracted quantities

$$\begin{aligned} R &= PR - SR \\ W &= PW - SW \\ U &= PU - SU \end{aligned}$$

and the expectation value is taken over an ensemble of pseudo-experiments with equal tagging probabilities and expected number of B candidates. The result for the various muon types

Muon system	$N\epsilon$	$\epsilon(\%)$	$D(\%)$	$\epsilon D^2(\%)$	$N\epsilon D^2$
$\mu + $TT trigger sample					(337 B candidates)
CMU	57	16.9 ± 2.4	33.6 ± 15.4	2.3 ± 1.8	7.6 ± 6.0
CMUP	23	6.8 ± 1.4	47.5 ± 19.1	1.8 ± 1.3	6.1 ± 4.3
CMX	26	7.7 ± 1.6	45.5 ± 18.9	1.9 ± 1.4	6.4 ± 4.6
CMU + CMUP	80	23.7 ± 2.6	37.6 ± 12.2	3.7 ± 2.2	12.4 ± 7.4
CMU + CMUP + CMX	106	31.5 ± 2.8	39.6 ± 10.3	5.3 ± 2.6	17.7 ± 8.7
TTT sample					(2540 B candidates)
CMU	38	1.5 ± 0.3	15.4 ± 21.3	0.1 ± 0.1	2.6 ± 3.3
CMUP	19	0.7 ± 0.2	36.2 ± 22.7	0.1 ± 0.1	3.5 ± 3.2
CMX	45	1.8 ± 0.3	24.1 ± 15.6	0.1 ± 0.1	3.7 ± 3.6
CMU + CMUP	57	2.2 ± 0.4	22.7 ± 15.8	0.2 ± 0.2	4.3 ± 4.4
CMU + CMUP + CMX	102	4.0 ± 0.4	23.4 ± 11.1	0.3 ± 0.2	6.9 ± 5.5

Table 2: Efficiency, dilution, ϵD^2 and $N\epsilon D^2$ by muon system measured in the $\mu + $TT trigger sample (top); and the TTT sample acquired over the same run period (bottom). Sec. A.2 describes the method used to estimate ϵ, D, ϵD^2 and $N\epsilon D^2$. The number of candidates indicated in parentheses is sideband-subtracted. See Sec. 2.3 and Sec. 2.4 for detailed descriptions of each sample.

is listed in the top sub-table of Tbl. 2. Even if no weighting of muons by system is done, the combined LSMT ϵD^2 of the $\mu + $TT sample exceeds the LSMT ϵD^2 of the TTT sample by an order of magnitude.

2.4 Statistical power of the $\mu + $TT sample; overlap with TTT

The statistical significance of a mixing amplitude measured in a sample of tagged B candidates scales as $\sqrt{N\epsilon D^2}$. Therefore it is useful to speak of $N\epsilon D^2$ as the *effective number of candidates* in the sample. We now discuss the $N\epsilon D^2$ we have measured in the several samples. The first sample is the $\mu + $TT trigger sample; the second is the TTT sample from the same run period; the third consists of events that fired both the $\mu + $TT trigger and the TTT. The fourth sample consist of $\mu + $TT events that did not fire the TTT; the fifth sample consists of events TTT that did not fire the $\mu + $TT trigger. The Venn diagram in Fig. 3 shows the five samples. More detailed results on the statistical power of the TTT sample can be found in the central sub-table of Tbl. 2.

From the number of effective candidates in each sample we can draw the following conclusions about the 2004 data.

1. Most of the events of importance to the LSMT fired both the $\mu + $TT trigger and the TTT, even though the TTT sample contains an order of magnitude more B candidates

than the $\mu + \mathrm{TT}$ sample.

2. The events that did not fire the $\mu + \mathrm{TT}$ trigger are of little value to the LSMT.

3. A non-negligible number of useful events would have been discarded by the prescaled TTT but were rescued by the unprescaled $\mu + \mathrm{TT}$.

As the average prescale applied to the TTT increases in the upcoming higher-luminosity running period, events will move out of the overlap region and only appear in the $\mu + \mathrm{TT}$ data set. Therefore it is essential that the $\mu + \mathrm{TT}$ triggers remain unprescaled.

3 Improvements to the trigger

Because the tagging power of sample collected by the $\mu + \mathrm{TT}$ trigger hinges on warding off a prescale, we now study improvements to the trigger. We consider the effect of three proposed improvements: requiring that the trigger muon be distinct from the TTT track pair (Sec. 3.1); cutting on muon quality after L1 (Sec. 3.2); and imposing an angular cut between the trigger muon and the TTT track pair (Sec. 3.3).

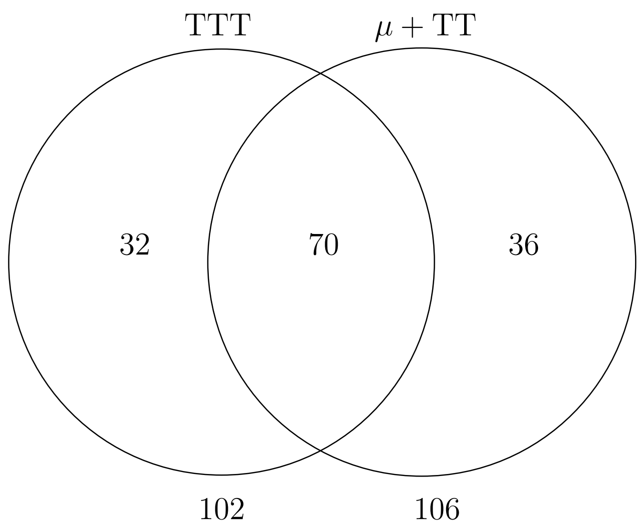
We estimate the L2 rates of modified $\mu + \mathrm{TT}$ triggers by using `svtfilter` to cut on L2 quantities. The events we use for this study are L3 auto-accepted, since we cannot assume that the efficiency of the L3 cuts will be the same for the current and modified, heavy-flavor-enhanced $\mu + \mathrm{TT}$ triggers. In addition to satisfying the L1 and L2 $\mu + \mathrm{TT}$ trigger requirements, the events must satisfy a prescaled, L3 auto-accepted TTT in the `B_BACKUP` dataset. The details of that trigger path are listed below:

- `B_HAD_L3TAG_L2_TWO_TRK2_D100`
  - `L1_TWO_TRK2_PS200`
  - `L2_TWO_TRK2_D100_PS20`
  - `L3_NULL`

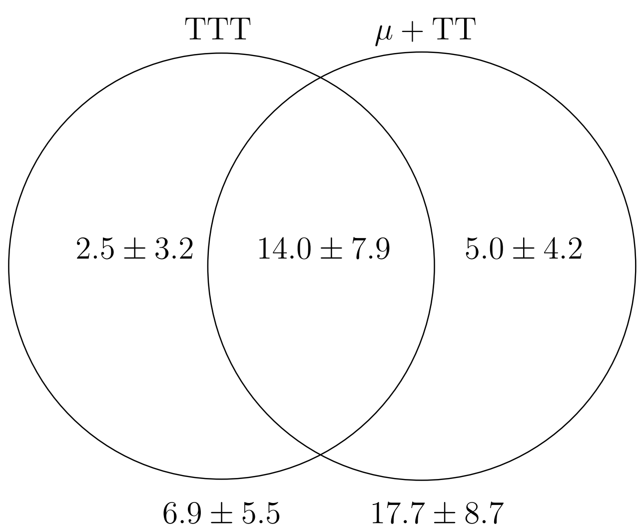
To estimate $N\epsilon$ in the samples collected with the modified $\mu + \mathrm{TT}$, we fit the invariant mass plot of LSMT-tagged B candidates using a gaussian on top of an exponential background. (This procedure differs from the background-subtraction procedure in Sec. 2.) Because of small statistics we do not attempt to fit the B candidates from the CMX+TT sample.

3.1 Three-track requirement

Since we would like to use this trigger to find hadronic B decays taggable by the LSMT, we wish to trigger on events with muons that are not part of the TTT pair. The current $\mu + \mathrm{TT}$ triggers do not require the muon to be distinct from the pair that satisfies the TTT. If we



(a)



(b)

Figure 3: Comparison of statistical power in the $\mu + \mathrm{TT}$ and TTT samples. (a) shows the number of tagged candidates ($N\epsilon$) in the $\mu + \mathrm{TT}$ sample; in the TTT sample; in the overlap between $\mu + \mathrm{TT}$ and TTT; in the part of the $\mu + \mathrm{TT}$ sample that did not fire a TTT; and in the part of the TTT sample that did not fire a $\mu + \mathrm{TT}$ trigger. (b) shows the $N\epsilon D^2$ measured in each category.

CMU + TT	2004	distinctness	$\Delta\phi$	muon	$\Delta\phi$ + muon
$\mu +$ TT only	28±14	26±13	20±12	15±11	12±9
$\mu +$ TT/TTT overlap	62±10	58±9	37±8	35±8	23±6

Table 3: $N\epsilon$ for various cuts.

require the muon to be an XFT track distinct from the legs of the TTT pair, we find the L2 rate decreases by approximately 40%. See the upper left plot in Fig. 4. The decrease in $N\epsilon$ is small.

3.2 Muon cuts

The summer 2004 μ + TT triggers did not include a cut on the muon quality after L1. Implementation of a three-track requirement would ensure the presence of a L2 muon. We have also studied the effect of requiring the l2 muon to have a matching SVT track. The L2 rate reduction of 60% is show in the upper right plot of Fig. 4; $N\epsilon$ decreases by 40%.

3.3 $\Delta\phi$ cut

To prepare for the higher luminosity environment of summer 2005, an angle cut has already been implemented. In its current form it requires a L2 instance of `MuonSVT` with the muon matched to an SVT track and the angle between the muon and a 2 GeV SVT track satisfying $\pi/2 < \Delta\phi < \pi$. The estimated rate reduction is shown in the lower right plot of Fig. 4 and is a combination of the triviality, muon, and angle cuts. The trigger rate is reduced to 20%, which agrees with the XMon plots of current trigger performance. The rate reduction of a trigger with the angle and triviality cuts alone is shown in the lower left plot of Fig. 4.

The modified μ+TT triggers (called `B_CHARM_LOWPT_CMU_v-1` and `B_CHARM_LOWPT_CMX_v-1`) were first tested in the table `PHYSICS_TEST_3_01` for run 194541 and has replaced the summer 2004 version in trigger tables after March 30, 2005 (it first appeared in `PHYSICS_3_01_v2`).

3.4 But which one is *right*?

Which combination of cuts is appropriate depends on the required rate reduction and hence depends on luminosity. The 2005 run provides a natural point for switching between cuts: the transition between the high-luminosity and low-luminosity trigger tables that occurs around $\mathcal{L} = 90 \cdot 10^{30}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$. For the low-luminosity table, where no rate reduction is required, we recommend that the 2004 μ + TT be reinstated. At high luminosity, the convenient choices are `B_CHARM_LOWPT_CMU_v-1` and `B_CHARM_LOWPT_CMX_v-1`, which have already been written and tested. The expense of having to maintain two separate μ + TT trigger paths will buy an increased muon-taggable B sample.

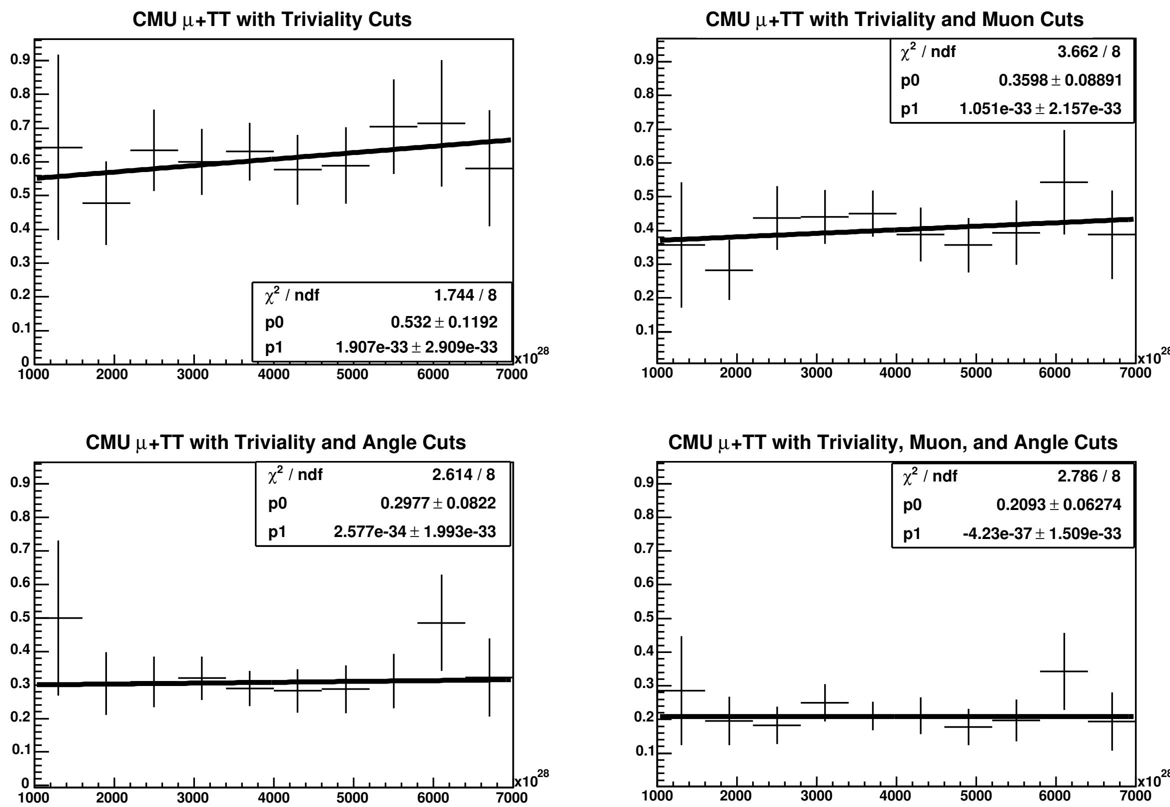


Figure 4: Fraction of events passing L2 cuts that may be implemented in future μ + TT triggers.

4 Conclusions and recommendations

Our study of the statistical power of the μ + TT data sample has led to the following conclusions.

Data rate The summer combined L2 μ + TT trigger rate is roughly one quarter of the dynamically prescaled Scenario A (`B_CHARM_L1_DPS`) rate.

ϵ, D, ϵD^2 The tagging efficiency of the LSMT applied to the $\mu + $TT data set is an order of magnitude higher than applied to the TTT data set, and the dilution is slightly higher. The ϵD^2 achieved in the $\mu + $TT is $5.2 \pm 2.6\%$.

LSMT Statistical power Despite the smaller size of the μ + TT sample, its statistical power is comparable to the TTT sample. Most of the statistical power of the TTT sample is found in the overlap between μ+TT and TTT. A non-negligible contribution to the statistical power of the μ + TT sample comes from events that were discarded by the prescaled TTT but rescued by the unprescaled μ + TT trigger.

Expected statistical power in the 2005 run In the higher-luminosity environment expected for 2005 the fraction of taggable events that are discarded by the TTT but salvaged by the μ + TT triggers will increase.

From these considerations we realize the importance of the unprescaled μ + TT. We recommend the following to maximize the size of the μ + TT sample:

Low-luminosity trigger table The 2004 μ+TT trigger should be kept in the low-luminosity table.

High-luminosity trigger table We endorse the trigger strategy with distinctness and angle cuts for the high-luminosity table. At the same time we believe that additional statistics can be gained by exploring the viability of less restricted strategies. As the high-luminosity data is processed, it should be possible to make accurate predictions for the trigger rates of the various strategies at high luminosity.

A Estimating ϵ, D, ϵD^2 and $N\epsilon D^2$

The determination of ϵ, D, ϵD^2 and $N\epsilon D^2$ and their errors by counting correctly and incorrectly tagged B candidates is surprisingly difficult. This appendix is our attempt at clarifying matters. We begin by defining what is meant by ϵ, D, ϵD^2 and $N\epsilon D^2$ and their errors. Then we point out the pitfalls in making a measurement and how to avoid them. Finally we give analytic approximations for estimating the error on ϵ, D, ϵD^2 and $N\epsilon D^2$. These formulæ are intended as a correction of the formulæ appearing in CDF note 3065 [4].

Throughout this appendix R, W and U are the number of correctly tagged ("right-sign"), incorrectly tagged ("wrong-sign") and untagged B candidates in a sample of $N = R+W+U$ candidates.

A.1 Defining ϵ, D, ϵD^2 and $N\epsilon D^2$

ϵ, D, ϵD^2 and $N\epsilon D^2$ are quantities we use to characterize a tagger. They depend on a number of parameters α. In the case of soft-muon tagging those parameters are the p_{T} of the muon relative to the B jet and the muon system that detected the muon, among others. We measure them on one sample with the intention of predicting the tagger performance on a different sample with the same set of parameters. ϵ, D, ϵD^2 and $N\epsilon D^2$ are estimators of the unknown "true" values ϵ_0, D_0, $(\epsilon D^2)_0$ and $(N\epsilon D^2)_0$.

If we had a large number of data sets with the same parameters α, the following expressions would give us unbiased estimates of ϵ_0, D_0, $(\epsilon D^2)_0$ and $(N\epsilon D^2)_0$:

$$\epsilon = \left\langle \frac{R+W}{R+W+U} \right\rangle, \tag{A·1}$$

$$D = \left\langle \frac{R-W}{R+W} \right\rangle, \tag{A·2}$$

$$\epsilon D^2 = \left\langle \frac{(R-W)^2}{(R+W+U)(R+W)} \right\rangle \tag{A·3}$$

and

$$N\epsilon D^2 = \left\langle \frac{(R-W)^2}{R+W} \right\rangle. \tag{A·4}$$

R, W and U are the number of right-tagged, wrong-tagged and untagged candidates in each particular experiment; the expectation value is taken over the ensemble of data sets with parameters α. Accordingly we let (A·1)–(A·4) define ϵ, D, ϵD^2 and $N\epsilon D^2$.

In the same vein we can define the variance of our estimate of ϵ, D, ϵD^2 and $N\epsilon D^2$:

$$\sigma_\epsilon^2 = \left\langle \left(\frac{R+W}{R+W+U} \right)^2 \right\rangle - \epsilon^2, \tag{A·5}$$

$$\sigma_D^2 = \left\langle \left(\frac{R-W}{R+W} \right)^2 \right\rangle - D^2, \tag{A·6}$$

$$\sigma_{\epsilon D^2}^2 = \left\langle \left(\frac{(R-W)^2}{(R+W+U)(R+W)} \right)^2 \right\rangle - \left(\epsilon D^2\right)^2 \tag{A·7}$$

and

$$\sigma_{N\epsilon D^2}^2 = \left\langle \left(\frac{(R-W)^2}{R+W} \right)^2 \right\rangle - \left(N\epsilon D^2\right)^2. \tag{A·8}$$

The question that remains to be addressed is what p.d.f.s to use for R, W and U. Tagging is a multinomial process: for every candidate one of three outcomes must occur, and the probability of each outcome is fixed. The appropriate p.d.f. for the number of candidates N in the case of a tagging study performed on a data set corresponding to a fixed integrated luminosity is a Poisson distribution. It can be shown that multinomial-distributed R, W and U with Poisson-distributed sum is equivalent to uncorrelated Poisson-distributed R, W and U (whose sum is then also Poisson-distributed).

A.2 Estimating ϵ, D, ϵD^2 and $N\epsilon D^2$ safely

The trivial statement that for a general function f of R, W and U that is non-linear in R, W and U the inequality

$$\langle f(R,W,U)\rangle \neq f\left(\langle R\rangle , \langle W\rangle , \langle U\rangle\right) \tag{A·9}$$

holds, advises us to use caution when estimating ϵD^2. Since

$$\epsilon D^2 = \left\langle \frac{(R-W)^2}{(R+W+U)(R+W)} \right\rangle \neq \frac{(\langle R\rangle - \langle W\rangle)^2}{(\langle R\rangle + \langle W\rangle + \langle U\rangle)(\langle R\rangle + \langle W\rangle)} \tag{A·10}$$

we cannot estimate ϵD^2 by simply inserting the values of R, W and U obtained in our single physical experiment into the expression $(R-W)^2/\left[(R+W+U)(R+W)\right]$. Instead we have to treat R, W and U as estimators of parameters of their respective Poisson p.d.f.s; we can then either derive the p.d.f. of ϵD^2 analytically (which is cumbersome) or determine its mean and variance numerically by sampling the p.d.f.s for R, W and U. The same argument applies for D and $N\epsilon D^2$.

The procedure we advocate (and have used in this note) is the following.

1. Generate Poisson-distributed R, W and U. Call the values generated in the i-th pseudo-experiment R_i, W_i and U_i.

2. Insert R_i, W_i and U_i into the expressions whose expectation values are taken in (A·1)–(A·4); call the values thus calculated ϵ_i, D_i, $(\epsilon D^2)_i$, $(N\epsilon D^2)_i$.

3. Keep track of ϵ_i, D_i, $(\epsilon D^2)_i$, $(N\epsilon D^2)_i$ by histogramming them or keeping running sums.

4. Repeat 1.–3. for a sufficiently large number n of pseudo-experiments, keeping in mind that the errors on x and σ_x ($x \in \{\epsilon, D, \epsilon D^2, N\epsilon D^2\}$) are $\mathcal{O}(\sigma_x/\sqrt{n})$.

The mean and variance of the distributions formed in 3. are unbiased estimators of the mean and variance of ϵ, D, ϵD^2 and $N\epsilon D^2$.

This method is easily extended to cases where R, W and U are not Poisson variables. In the analysis presented in this note R, W and U are background-subtracted quantities; the number of events in each tagging category in the peak region (PR, PW and PU) and in the sideband region (SR, SW and SU) is Poisson-distributed, but

$$\begin{aligned} R &= PR - SR \\ W &= PW - SW \\ U &= PU - SU \end{aligned} \tag{A·11}$$

are not. Instead of generating Poisson-distributed R, W and U in step 1. above we generate Poisson-distributed PR, PW, PU, SR, SW and SU, calculate R, W and U from (A·11) and proceed as usual.

A.3 Analytic approximations

In some cases it may be desirable to perform quick estimates of the statistical power of a tagger without employing the machinery of the previous section. The formulæ in this section are useful for order-of-magnitude calculations. As before R, W and U are the number of correctly tagged, incorrectly tagged and untagged B candidates in a sample of $N = R+W+U$ candidates. Additionally we assume the R, W and U are Poisson variables, are not correlated with each other, and are disjoint subsets of N.[2]

A.3.1 Error on ϵ

The efficiency ϵ is estimated as

$$\epsilon = \frac{R+W}{N} = \frac{R+W}{R+W+U}. \tag{A·12}$$

Its variance is

$$\begin{aligned} \sigma_\epsilon^2 &= \left(\frac{\partial\epsilon}{\partial R}\right)^2 \sigma_R^2 + \left(\frac{\partial\epsilon}{\partial W}\right)^2 \sigma_W^2 + \left(\frac{\partial\epsilon}{\partial U}\right)^2 \sigma_U^2 \\ &= \left(\frac{1-\epsilon}{N}\right)^2 \left(\sigma_R^2 + \sigma_W^2\right) + \left(\frac{\epsilon}{N}\right)^2 \sigma_U^2, \end{aligned} \tag{A·13}$$

which, when Poisson errors are inserted for σ_R^2, σ_W^2 and σ_U^2, simplifies to

$$\sigma_\epsilon = \sqrt{\frac{\epsilon(1-\epsilon)}{N}}. \tag{A·14}$$

This result agrees with the expectation for a Bernoulli process. Note that (A·12) and (A·14) are the only unbiased results obtained in this section (Sec. A.3).

A.3.2 Error on D

A biased estimator for the dilution D is

$$D = \frac{R-W}{R+W}. \tag{A·15}$$

An estimator for its variance is

$$\begin{aligned} \sigma_D^2 &= \left(\frac{\partial D}{\partial R}\right)^2 \sigma_R^2 + \left(\frac{\partial D}{\partial W}\right)^2 \sigma_W^2 \\ &= \frac{4}{N^4} \left(W^2\sigma_R^2 + R^2\sigma_W^2\right). \end{aligned} \tag{A·16}$$

[2]This means that the intermediate formulæ (A·13), (A·16) and (A·20) are not applicable if R, W and U are not a Poisson variables or if R, W and U have non-zero correlations.

This reduces to

$$\sigma_D = \sqrt{\frac{4RW}{(R+W)^3}}. \tag{A·17}$$

A.3.3 Error on ϵD^2

A biased estimator for ϵD^2 is:

$$\epsilon D^2 = \frac{(R-W)^2}{(R+W+U)(R+W)}. \tag{A·18}$$

An estimator for its variance is

$$\sigma^2_{\epsilon D^2} = \left(\frac{\partial \epsilon D^2}{\partial R}\right)^2 \sigma_R^2 + \left(\frac{\partial \epsilon D^2}{\partial W}\right)^2 \sigma_W^2 + \left(\frac{\partial \epsilon D^2}{\partial U}\right)^2 \sigma_U^2 \tag{A·19}$$

or

$$\begin{aligned} \frac{\sigma^2_{\epsilon D^2}}{(\epsilon D^2)^2} &= \left(\frac{1}{R+W} + \frac{1}{N} - \frac{2}{R-W}\right)^2 \sigma_R^2 \\ &+ \left(\frac{1}{R+W} + \frac{1}{N} + \frac{2}{R-W}\right)^2 \sigma_W^2 + \frac{1}{N^2}\sigma_U^2. \end{aligned} \tag{A·20}$$

Amid cries of agony enough terms eliminate each other to yield

$$\sigma^2_{\epsilon D^2} = \left(\epsilon D^2\right)^2 \left(4\frac{R+W}{(R-W)^2} - \frac{3}{R+W} - \frac{1}{N}\right). \tag{A·21}$$

References

[1] G. Giurgiu, V. Tiwari, M. Paulini, and J. Russ, Muon B flavor tagging — a likelihood approach, CDF note 7043.

[2] TDWG, `http://www-cdfonline.fnal.gov/~cdf_val/CrossSectionFits/main.html`.

[3] M. D. Shapiro *et al.*, Performance of the event-by-event primary vertex finder on$B$ physics data at cdf, CDF note 7512.

[4] M. D. Peters, M. Paulini, M. D. Shapiro, and H. Wenzel, A Study of B-Flavor Tagging Using Soft Electrons, CDF note 3065.